

Mathematica 11.3 Integration Test Results

Test results for the 1378 problems in "7.3.6 Exponentials of inverse hyperbolic tangent functions.m"

Problem 15: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 12 leaves, 3 steps):

$$\operatorname{Log}[x] - 2 \operatorname{Log}[1 - a x]$$

Result (type 3, 25 leaves):

$$\operatorname{Log}\left[1 - e^{2 \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 + e^{2 \operatorname{ArcTanh}[a x]}\right]$$

Problem 47: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2 \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 11 leaves, 3 steps):

$$\operatorname{Log}[x] - 2 \operatorname{Log}[1 + a x]$$

Result (type 3, 25 leaves):

$$\operatorname{Log}\left[1 - e^{-2 \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcTanh}[a x]}\right]$$

Problem 60: Unable to integrate problem.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{1}{4}, -\frac{1}{4}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Problem 61: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \text{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 282 leaves, 15 steps):

$$\begin{aligned} & -\frac{3(1-ax)^{3/4}(1+ax)^{1/4}}{8a^3} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{12a^3} \\ & - \frac{x(1-ax)^{3/4}(1+ax)^{5/4}}{3a^2} + \frac{3 \text{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^3} - \frac{3 \text{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^3} \\ & - \frac{3 \text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{16\sqrt{2}a^3} + \frac{3 \text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{16\sqrt{2}a^3} \end{aligned}$$

Result (type 7, 93 leaves):

$$\begin{aligned} & \frac{1}{96a^3} \left(-\frac{8e^{\frac{1}{2} \text{ArcTanh}[a x]} (9 + 6e^{2 \text{ArcTanh}[a x]} + 29e^{4 \text{ArcTanh}[a x]})}{(1 + e^{2 \text{ArcTanh}[a x]})^3} - \right. \\ & \left. 9 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right) \end{aligned}$$

Problem 62: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \text{ArcTanh}[a x]} x dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$\begin{aligned} & -\frac{(1-ax)^{3/4}(1+ax)^{1/4}}{4a^2} - \frac{(1-ax)^{3/4}(1+ax)^{5/4}}{2a^2} + \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} \\ & - \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} - \frac{\text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2} + \frac{\text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2} \end{aligned}$$

Result (type 7, 83 leaves):

$$\begin{aligned} & \frac{1}{16a^2} \left(-\frac{8e^{\frac{1}{2} \text{ArcTanh}[a x]} (1 + 5e^{2 \text{ArcTanh}[a x]})}{(1 + e^{2 \text{ArcTanh}[a x]})^2} + \right. \\ & \left. \text{RootSum}\left[1 + \#1^4 \&, \frac{-\text{ArcTanh}[a x] + 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right) \end{aligned}$$

Problem 63: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} dx$$

Optimal (type 3, 222 leaves, 13 steps):

$$-\frac{(1-ax)^{3/4} (1+ax)^{1/4}}{a} + \frac{\operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2} a} - \frac{\operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2} a} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2} a} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2} a}$$

Result (type 7, 71 leaves):

$$\frac{-\frac{8 e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}}{1+e^{2 \operatorname{ArcTanh}[a x]}} + \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]}{4 a}$$

Problem 64: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 227 leaves, 17 steps):

$$-2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - 2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 87 leaves):

$$-2 \operatorname{ArcTan}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] - \operatorname{Log}\left[1 + e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{-\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right]$$

Problem 70: Unable to integrate problem.

$$\int e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{3}{4}, -\frac{3}{4}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{3}{2} \text{ArcTanh}[a x]} x^m dx$$

Problem 71: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} \text{ArcTanh}[a x]} x^3 dx$$

Optimal (type 3, 290 leaves, 15 steps):

$$\begin{aligned} & - \frac{41 (1 - a x)^{1/4} (1 + a x)^{3/4}}{64 a^4} - \frac{x^2 (1 - a x)^{1/4} (1 + a x)^{7/4}}{4 a^2} - \\ & \frac{(1 - a x)^{1/4} (1 + a x)^{7/4} (11 + 4 a x)}{32 a^4} + \frac{123 \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} - \frac{123 \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \\ & \frac{123 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4} - \frac{123 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4} \end{aligned}$$

Result (type 7, 103 leaves):

$$\begin{aligned} & \frac{1}{256 a^4} \left(- \left(\left(8 e^{\frac{3}{2} \text{ArcTanh}[a x]} (41 + 183 e^{2 \text{ArcTanh}[a x]} + 147 e^{4 \text{ArcTanh}[a x]} + 133 e^{6 \text{ArcTanh}[a x]}) \right) / \right. \right. \\ & \left. \left. (1 + e^{2 \text{ArcTanh}[a x]})^4 - 123 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1} \&\right] \right) \right) \end{aligned}$$

Problem 72: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} \text{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 282 leaves, 15 steps):

$$\begin{aligned} & - \frac{17 (1 - a x)^{1/4} (1 + a x)^{3/4}}{24 a^3} - \frac{(1 - a x)^{1/4} (1 + a x)^{7/4}}{4 a^3} - \\ & \frac{x (1 - a x)^{1/4} (1 + a x)^{7/4}}{3 a^2} + \frac{17 \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \frac{17 \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \\ & \frac{17 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3} - \frac{17 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3} \end{aligned}$$

Result (type 7, 93 leaves):

$$\frac{1}{96 a^3} \left(-\frac{8 e^{\frac{3}{2} \text{ArcTanh}[a x]} (17 + 30 e^{2 \text{ArcTanh}[a x]} + 45 e^{4 \text{ArcTanh}[a x]})}{(1 + e^{2 \text{ArcTanh}[a x]})^3} - \right. \\ \left. 51 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \&\right)$$

Problem 73: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} \text{ArcTanh}[a x]} x \, dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$-\frac{3 (1 - a x)^{1/4} (1 + a x)^{3/4}}{4 a^2} - \frac{(1 - a x)^{1/4} (1 + a x)^{7/4}}{2 a^2} + \frac{9 \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} - \\ \frac{9 \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} + \frac{9 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2} - \frac{9 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2}$$

Result (type 7, 84 leaves):

$$\frac{1}{a^2} \left(-\frac{e^{\frac{3}{2} \text{ArcTanh}[a x]} (3 + 7 e^{2 \text{ArcTanh}[a x]})}{2 (1 + e^{2 \text{ArcTanh}[a x]})^2} - \right. \\ \left. \frac{9}{16} \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \&\right)$$

Problem 74: Result is not expressed in closed-form.

$$\int e^{\frac{3}{2} \text{ArcTanh}[a x]} \, dx$$

Optimal (type 3, 223 leaves, 13 steps):

$$-\frac{(1 - a x)^{1/4} (1 + a x)^{3/4}}{a} + \frac{3 \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{3 \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} + \\ \frac{3 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{2 \sqrt{2} a} - \frac{3 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{2 \sqrt{2} a}$$

Result (type 7, 72 leaves):

$$-\frac{2 e^{\frac{3}{2} \text{ArcTanh}[a x]}}{a (1 + e^{2 \text{ArcTanh}[a x]})} - \frac{3 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \&}{4 a}$$

Problem 75: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{3}{2} \text{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 227 leaves, 17 steps):

$$2 \text{ArcTan} \left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}} \right] + \sqrt{2} \text{ArcTan} \left[1 - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}} \right] - \sqrt{2} \text{ArcTan} \left[1 + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}} \right] -$$

$$2 \text{ArcTanh} \left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}} \right] + \frac{\text{Log} \left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}} \right]}{\sqrt{2}} - \frac{\text{Log} \left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2} (1-ax)^{1/4}}{(1+ax)^{1/4}} \right]}{\sqrt{2}}$$

Result (type 7, 87 leaves):

$$2 \text{ArcTan} \left[e^{\frac{1}{2} \text{ArcTanh}[a x]} \right] + \text{Log} \left[1 - e^{\frac{1}{2} \text{ArcTanh}[a x]} \right] - \text{Log} \left[1 + e^{\frac{1}{2} \text{ArcTanh}[a x]} \right] +$$

$$\frac{1}{2} \text{RootSum} \left[1 + \#1^4 \&, \frac{-\text{ArcTanh}[a x] + 2 \text{Log} \left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1 \right]}{\#1} \& \right]$$

Problem 80: Unable to integrate problem.

$$\int e^{\frac{5}{2} \text{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1} \left[1+m, \frac{5}{4}, -\frac{5}{4}, 2+m, ax, -ax \right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{5}{2} \text{ArcTanh}[a x]} x^m dx$$

Problem 81: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} \text{ArcTanh}[a x]} x^3 dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\begin{aligned}
 & \frac{475 (1 - a x)^{3/4} (1 + a x)^{1/4}}{64 a^4} + \frac{4 x^3 (1 + a x)^{5/4}}{a (1 - a x)^{1/4}} + \frac{17 x^2 (1 - a x)^{3/4} (1 + a x)^{5/4}}{4 a^2} + \\
 & \frac{(1 - a x)^{3/4} (1 + a x)^{5/4} (521 + 452 a x)}{96 a^4} - \frac{475 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \\
 & \frac{475 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{475 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4} - \frac{475 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4}
 \end{aligned}$$

Result (type 7, 114 leaves):

$$\begin{aligned}
 & \frac{1}{a^4} \\
 & \left(\left(e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} \left(1425 + 5415 e^{2 \operatorname{ArcTanh}[a x]} + 7483 e^{4 \operatorname{ArcTanh}[a x]} + 4645 e^{6 \operatorname{ArcTanh}[a x]} + 768 e^{8 \operatorname{ArcTanh}[a x]} \right) \right) / \right. \\
 & \left. \left(96 \left(1 + e^{2 \operatorname{ArcTanh}[a x]} \right)^4 \right) + \frac{475}{256} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right)
 \end{aligned}$$

Problem 82: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} \operatorname{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 305 leaves, 16 steps):

$$\begin{aligned}
 & \frac{55 (1 - a x)^{3/4} (1 + a x)^{1/4}}{8 a^3} + \frac{11 (1 - a x)^{3/4} (1 + a x)^{5/4}}{4 a^3} + \frac{2 (1 + a x)^{9/4}}{a^3 (1 - a x)^{1/4}} + \\
 & \frac{(1 - a x)^{3/4} (1 + a x)^{9/4}}{3 a^3} - \frac{55 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \frac{55 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \\
 & \frac{55 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3} - \frac{55 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3}
 \end{aligned}$$

Result (type 7, 104 leaves):

$$\begin{aligned}
 & \frac{1}{a^3} \left(\left(e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} \left(165 + 462 e^{2 \operatorname{ArcTanh}[a x]} + 425 e^{4 \operatorname{ArcTanh}[a x]} + 96 e^{6 \operatorname{ArcTanh}[a x]} \right) \right) / \right. \\
 & \left. \left(12 \left(1 + e^{2 \operatorname{ArcTanh}[a x]} \right)^3 \right) + \frac{55}{32} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right)
 \end{aligned}$$

Problem 83: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} \text{ArcTanh}[a x]} x \, dx$$

Optimal (type 3, 279 leaves, 15 steps):

$$\frac{25 (1 - a x)^{3/4} (1 + a x)^{1/4}}{4 a^2} + \frac{5 (1 - a x)^{3/4} (1 + a x)^{5/4}}{2 a^2} + \frac{2 (1 + a x)^{9/4}}{a^2 (1 - a x)^{1/4}} - \frac{25 \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} +$$

$$\frac{25 \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} + \frac{25 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2} - \frac{25 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2}$$

Result (type 7, 94 leaves):

$$\frac{1}{a^2} \left(\frac{e^{\frac{1}{2} \text{ArcTanh}[a x]} (25 + 45 e^{2 \text{ArcTanh}[a x]} + 16 e^{4 \text{ArcTanh}[a x]})}{2 (1 + e^{2 \text{ArcTanh}[a x]})^2} + \right.$$

$$\left. \frac{25}{16} \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 84: Result is not expressed in closed-form.

$$\int e^{\frac{5}{2} \text{ArcTanh}[a x]} \, dx$$

Optimal (type 3, 247 leaves, 14 steps):

$$\frac{5 (1 - a x)^{3/4} (1 + a x)^{1/4}}{a} + \frac{4 (1 + a x)^{5/4}}{a (1 - a x)^{1/4}} - \frac{5 \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} +$$

$$\frac{5 \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} + \frac{5 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{2 \sqrt{2} a} - \frac{5 \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{2 \sqrt{2} a}$$

Result (type 7, 83 leaves):

$$\frac{1}{4 a} \left(\frac{8 e^{\frac{1}{2} \text{ArcTanh}[a x]} (5 + 4 e^{2 \text{ArcTanh}[a x]})}{1 + e^{2 \text{ArcTanh}[a x]}} + \right.$$

$$\left. 5 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 85: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{5}{2} \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 248 leaves, 19 steps):

$$\begin{aligned} & \frac{8 (1 + a x)^{1/4}}{(1 - a x)^{1/4}} - 2 \operatorname{ArcTan}\left[\frac{(1 + a x)^{1/4}}{(1 - a x)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right] + \\ & \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right] - 2 \operatorname{ArcTanh}\left[\frac{(1 + a x)^{1/4}}{(1 - a x)^{1/4}}\right] + \\ & \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2}} \end{aligned}$$

Result (type 7, 97 leaves):

$$\begin{aligned} & 8 e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - 2 \operatorname{ArcTan}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] - \\ & \operatorname{Log}\left[1 + e^{\frac{1}{2} \operatorname{ArcTanh}[a x]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \end{aligned}$$

Problem 90: Unable to integrate problem.

$$\int e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1 + m, -\frac{1}{4}, \frac{1}{4}, 2 + m, a x, -a x\right]}{1 + m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} x^m dx$$

Problem 91: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} x^3 dx$$

Optimal (type 3, 290 leaves, 15 steps):

$$\begin{aligned}
 & - \frac{11 (1 - a x)^{1/4} (1 + a x)^{3/4}}{64 a^4} - \frac{x^2 (1 - a x)^{5/4} (1 + a x)^{3/4}}{4 a^2} - \\
 & \frac{(25 - 4 a x) (1 - a x)^{5/4} (1 + a x)^{3/4}}{96 a^4} - \frac{11 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{11 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a^4} - \\
 & \frac{11 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4} + \frac{11 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a^4}
 \end{aligned}$$

Result (type 7, 103 leaves):

$$\frac{1}{768 a^4} \left(- \left(\left(8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} \left(245 + 107 e^{2 \operatorname{ArcTanh}[a x]} + 279 e^{4 \operatorname{ArcTanh}[a x]} + 33 e^{6 \operatorname{ArcTanh}[a x]} \right) \right) / \right. \right. \\
 \left. \left. \left(1 + e^{2 \operatorname{ArcTanh}[a x]} \right)^4 \right) + 33 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 92: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 282 leaves, 15 steps):

$$\begin{aligned}
 & \frac{3 (1 - a x)^{1/4} (1 + a x)^{3/4}}{8 a^3} + \frac{(1 - a x)^{5/4} (1 + a x)^{3/4}}{12 a^3} - \\
 & \frac{x (1 - a x)^{5/4} (1 + a x)^{3/4}}{3 a^2} + \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \\
 & \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3} - \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3}
 \end{aligned}$$

Result (type 7, 93 leaves):

$$\frac{1}{96 a^3} \left(\frac{8 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} \left(29 + 6 e^{2 \operatorname{ArcTanh}[a x]} + 9 e^{4 \operatorname{ArcTanh}[a x]} \right)}{\left(1 + e^{2 \operatorname{ArcTanh}[a x]} \right)^3} - \right. \\
 \left. 9 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 93: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} x dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$\begin{aligned}
 & -\frac{(1-ax)^{1/4}(1+ax)^{3/4}}{4a^2} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}}{2a^2} - \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} + \\
 & \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a^2} - \frac{\text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2} + \frac{\text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a^2}
 \end{aligned}$$

Result (type 7, 79 leaves):

$$\frac{1}{16a^2} \left(-\frac{8e^{\frac{3}{2}\text{ArcTanh}[ax]}(5 + e^{2\text{ArcTanh}[ax]})}{(1 + e^{2\text{ArcTanh}[ax]})^2} + \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[ax] + 2\text{Log}\left[e^{-\frac{1}{2}\text{ArcTanh}[ax]} - \#1\right]}{\#1^3}\right] \& \right)$$

Problem 94: Result is not expressed in closed-form.

$$\int e^{-\frac{1}{2}\text{ArcTanh}[ax]} dx$$

Optimal (type 3, 221 leaves, 13 steps):

$$\begin{aligned}
 & \frac{(1-ax)^{1/4}(1+ax)^{3/4}}{a} + \frac{\text{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} - \frac{\text{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} + \\
 & \frac{\text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a} - \frac{\text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a}
 \end{aligned}$$

Result (type 7, 69 leaves):

$$-\frac{1}{4a} \left(-\frac{8e^{\frac{3}{2}\text{ArcTanh}[ax]}}{1 + e^{2\text{ArcTanh}[ax]}} + \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[ax] + 2\text{Log}\left[e^{-\frac{1}{2}\text{ArcTanh}[ax]} - \#1\right]}{\#1^3}\right] \& \right)$$

Problem 95: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{1}{2}\text{ArcTanh}[ax]}}{x} dx$$

Optimal (type 3, 227 leaves, 17 steps):

$$\begin{aligned}
 & 2\text{ArcTan}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] - \sqrt{2}\text{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] + \sqrt{2}\text{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] - \\
 & 2\text{ArcTanh}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] - \frac{\text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} + \frac{\text{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}}
 \end{aligned}$$

Result (type 7, 85 leaves):

$$-2 \operatorname{ArcTan}\left[e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]}\right] + \operatorname{Log}\left[1 - e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]}\right] - \operatorname{Log}\left[1 + e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]}\right] + \frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] + 2 \operatorname{Log}\left[e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right]$$

Problem 100: Unable to integrate problem.

$$\int e^{-\frac{3}{2}\operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, -\frac{3}{4}, \frac{3}{4}, 2+m, ax, -ax\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{3}{2}\operatorname{ArcTanh}[ax]} x^m dx$$

Problem 101: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2}\operatorname{ArcTanh}[ax]} x^3 dx$$

Optimal (type 3, 290 leaves, 15 steps):

$$\begin{aligned} & -\frac{41(1-ax)^{3/4}(1+ax)^{1/4}}{64a^4} - \frac{x^2(1-ax)^{7/4}(1+ax)^{1/4}}{4a^2} - \\ & \frac{(11-4ax)(1-ax)^{7/4}(1+ax)^{1/4}}{32a^4} - \frac{123 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{64\sqrt{2}a^4} + \frac{123 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{64\sqrt{2}a^4} + \\ & \frac{123 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{128\sqrt{2}a^4} - \frac{123 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{128\sqrt{2}a^4} \end{aligned}$$

Result (type 7, 103 leaves):

$$\frac{1}{256a^4} \left(- \left(\left(8 e^{\frac{1}{2}\operatorname{ArcTanh}[ax]} \left(133 + 147 e^{2\operatorname{ArcTanh}[ax]} + 183 e^{4\operatorname{ArcTanh}[ax]} + 41 e^{6\operatorname{ArcTanh}[ax]} \right) \right) / \left(1 + e^{2\operatorname{ArcTanh}[ax]} \right)^4 + 123 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] + 2 \operatorname{Log}\left[e^{-\frac{1}{2}\operatorname{ArcTanh}[ax]} - \#1\right]}{\#1} \&\right] \right)$$

Problem 102: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2}\operatorname{ArcTanh}[ax]} x^2 dx$$

Optimal (type 3, 282 leaves, 15 steps):

$$\frac{17 (1 - a x)^{3/4} (1 + a x)^{1/4}}{24 a^3} + \frac{(1 - a x)^{7/4} (1 + a x)^{1/4}}{4 a^3} -$$

$$\frac{x (1 - a x)^{7/4} (1 + a x)^{1/4}}{3 a^2} + \frac{17 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \frac{17 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^3} -$$

$$\frac{17 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3} + \frac{17 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{16 \sqrt{2} a^3}$$

Result (type 7, 93 leaves):

$$\frac{1}{96 a^3} \left(\frac{8 e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (45 + 30 e^{2 \operatorname{ArcTanh}[a x]} + 17 e^{4 \operatorname{ArcTanh}[a x]})}{(1 + e^{2 \operatorname{ArcTanh}[a x]})^3} - \right.$$

$$\left. 51 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right] - \#1}{\#1}\right] \& \right)$$

Problem 103: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]} x \, dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$-\frac{3 (1 - a x)^{3/4} (1 + a x)^{1/4}}{4 a^2} - \frac{(1 - a x)^{7/4} (1 + a x)^{1/4}}{2 a^2} - \frac{9 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} +$$

$$\frac{9 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a^2} + \frac{9 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2} - \frac{9 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a^2}$$

Result (type 7, 84 leaves):

$$\frac{1}{a^2} \left(-\frac{e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (7 + 3 e^{2 \operatorname{ArcTanh}[a x]})}{2 (1 + e^{2 \operatorname{ArcTanh}[a x]})^2} + \right.$$

$$\left. \frac{9}{16} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right] - \#1}{\#1}\right] \& \right)$$

Problem 104: Result is not expressed in closed-form.

$$\int e^{-\frac{3}{2} \operatorname{ArcTanh}[a x]} \, dx$$

Optimal (type 3, 222 leaves, 13 steps):

$$\frac{(1-ax)^{3/4}(1+ax)^{1/4}}{a} + \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} - \frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a} -$$

$$\frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a} + \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{2\sqrt{2}a}$$

Result (type 7, 72 leaves):

$$\frac{2 e^{-\frac{3}{2} \operatorname{ArcTanh}[ax]}}{a (1 + e^{-2 \operatorname{ArcTanh}[ax]})} - \frac{3 \operatorname{RootSum}\left[1 + \#1^4, \frac{\operatorname{ArcTanh}[ax] + 2 \operatorname{Log}\left[e^{-\frac{3}{2} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1}\right] \&}{4a}$$

Problem 105: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{3}{2} \operatorname{ArcTanh}[ax]}}{x} dx$$

Optimal (type 3, 227 leaves, 17 steps):

$$-2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] - \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right] -$$

$$2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right] + \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} - \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{1-ax}}{\sqrt{1+ax}} + \frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 85 leaves):

$$2 \operatorname{ArcTan}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[ax]}\right] + \operatorname{Log}\left[1 - e^{-\frac{1}{2} \operatorname{ArcTanh}[ax]}\right] - \operatorname{Log}\left[1 + e^{-\frac{1}{2} \operatorname{ArcTanh}[ax]}\right] +$$

$$\frac{1}{2} \operatorname{RootSum}\left[1 + \#1^4, \frac{\operatorname{ArcTanh}[ax] + 2 \operatorname{Log}\left[e^{-\frac{1}{2} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1}\right] \&$$

Problem 110: Unable to integrate problem.

$$\int e^{-\frac{5}{2} \operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, -\frac{5}{4}, \frac{5}{4}, 2+m, ax, -ax\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{-\frac{5}{2} \operatorname{ArcTanh}[ax]} x^m dx$$

Problem 111: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} \text{ArcTanh}[a x]} x^3 dx$$

Optimal (type 3, 317 leaves, 16 steps):

$$\begin{aligned} & -\frac{4 x^3 (1-a x)^{5/4}}{a (1+a x)^{1/4}} + \frac{475 (1-a x)^{1/4} (1+a x)^{3/4}}{64 a^4} + \frac{17 x^2 (1-a x)^{5/4} (1+a x)^{3/4}}{4 a^2} + \\ & \frac{(521-452 a x) (1-a x)^{5/4} (1+a x)^{3/4}}{96 a^4} + \frac{475 \text{ArcTan}\left[1-\frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{64 \sqrt{2} a^4} - \\ & \frac{475 \text{ArcTan}\left[1+\frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{64 \sqrt{2} a^4} + \frac{475 \text{Log}\left[1+\frac{\sqrt{1-a x}}{\sqrt{1+a x}}-\frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{128 \sqrt{2} a^4} - \frac{475 \text{Log}\left[1+\frac{\sqrt{1-a x}}{\sqrt{1+a x}}+\frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{128 \sqrt{2} a^4} \end{aligned}$$

Result (type 7, 114 leaves):

$$\begin{aligned} & \frac{1}{a^4} \\ & \left(\left(e^{-\frac{1}{2} \text{ArcTanh}[a x]} (768 + 4645 e^{2 \text{ArcTanh}[a x]} + 7483 e^{4 \text{ArcTanh}[a x]} + 5415 e^{6 \text{ArcTanh}[a x]} + 1425 e^{8 \text{ArcTanh}[a x]}) \right) / \right. \\ & \left. \left(96 (1 + e^{2 \text{ArcTanh}[a x]})^4 \right) - \frac{475}{256} \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] + 2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1^3} \&\right] \right) \end{aligned}$$

Problem 112: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} \text{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 305 leaves, 16 steps):

$$\begin{aligned} & -\frac{2 (1-a x)^{9/4}}{a^3 (1+a x)^{1/4}} - \frac{55 (1-a x)^{1/4} (1+a x)^{3/4}}{8 a^3} - \frac{11 (1-a x)^{5/4} (1+a x)^{3/4}}{4 a^3} - \\ & \frac{(1-a x)^{9/4} (1+a x)^{3/4}}{3 a^3} - \frac{55 \text{ArcTan}\left[1-\frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{8 \sqrt{2} a^3} + \frac{55 \text{ArcTan}\left[1+\frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{8 \sqrt{2} a^3} - \\ & \frac{55 \text{Log}\left[1+\frac{\sqrt{1-a x}}{\sqrt{1+a x}}-\frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{16 \sqrt{2} a^3} + \frac{55 \text{Log}\left[1+\frac{\sqrt{1-a x}}{\sqrt{1+a x}}+\frac{\sqrt{2} (1-a x)^{1/4}}{(1+a x)^{1/4}}\right]}{16 \sqrt{2} a^3} \end{aligned}$$

Result (type 7, 104 leaves):

$$\frac{1}{a^3} \left(- \left(\left(e^{-\frac{1}{2} \text{ArcTanh}[a x]} \left(96 + 425 e^{2 \text{ArcTanh}[a x]} + 462 e^{4 \text{ArcTanh}[a x]} + 165 e^{6 \text{ArcTanh}[a x]} \right) \right) \right) / \left(12 \left(1 + e^{2 \text{ArcTanh}[a x]} \right)^3 \right) + \frac{55}{32} \text{RootSum} \left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] + 2 \text{Log} \left[e^{-\frac{1}{2} \text{ArcTanh}[a x]} - \#1 \right]}{\#1^3} \& \right] \right)$$

Problem 113: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} \text{ArcTanh}[a x]} x \, dx$$

Optimal (type 3, 279 leaves, 15 steps):

$$\frac{2 (1 - a x)^{9/4}}{a^2 (1 + a x)^{1/4}} + \frac{25 (1 - a x)^{1/4} (1 + a x)^{3/4}}{4 a^2} + \frac{5 (1 - a x)^{5/4} (1 + a x)^{3/4}}{2 a^2} + \frac{25 \text{ArcTan} \left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{4 \sqrt{2} a^2} - \frac{25 \text{ArcTan} \left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{4 \sqrt{2} a^2} + \frac{25 \text{Log} \left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{8 \sqrt{2} a^2} - \frac{25 \text{Log} \left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{8 \sqrt{2} a^2}$$

Result (type 7, 94 leaves):

$$\frac{1}{a^2} \left(\frac{e^{-\frac{1}{2} \text{ArcTanh}[a x]} \left(16 + 45 e^{2 \text{ArcTanh}[a x]} + 25 e^{4 \text{ArcTanh}[a x]} \right)}{2 \left(1 + e^{2 \text{ArcTanh}[a x]} \right)^2} - \frac{25}{16} \text{RootSum} \left[1 + \#1^4 \&, \frac{\text{ArcTanh}[a x] + 2 \text{Log} \left[e^{-\frac{1}{2} \text{ArcTanh}[a x]} - \#1 \right]}{\#1^3} \& \right] \right)$$

Problem 114: Result is not expressed in closed-form.

$$\int e^{-\frac{5}{2} \text{ArcTanh}[a x]} \, dx$$

Optimal (type 3, 247 leaves, 14 steps):

$$-\frac{4 (1 - a x)^{5/4}}{a (1 + a x)^{1/4}} - \frac{5 (1 - a x)^{1/4} (1 + a x)^{3/4}}{a} - \frac{5 \text{ArcTan} \left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{\sqrt{2} a} + \frac{5 \text{ArcTan} \left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{\sqrt{2} a} - \frac{5 \text{Log} \left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{2 \sqrt{2} a} + \frac{5 \text{Log} \left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}} \right]}{2 \sqrt{2} a}$$

Result (type 7, 83 leaves):

$$\frac{1}{4a} \left(-\frac{8 e^{-\frac{1}{2} \text{ArcTanh}[ax]} (4 + 5 e^{2 \text{ArcTanh}[ax]})}{1 + e^{2 \text{ArcTanh}[ax]}} + \right. \\ \left. 5 \text{RootSum}\left[1 + \#1^4 \&, \frac{\text{ArcTanh}[ax] + 2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right] \right)$$

Problem 115: Result is not expressed in closed-form.

$$\int \frac{e^{-\frac{5}{2} \text{ArcTanh}[ax]}}{x} dx$$

Optimal (type 3, 248 leaves, 19 steps):

$$\frac{8 (1 - ax)^{1/4}}{(1 + ax)^{1/4}} + 2 \text{ArcTan}\left[\frac{(1 + ax)^{1/4}}{(1 - ax)^{1/4}}\right] + \sqrt{2} \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - ax)^{1/4}}{(1 + ax)^{1/4}}\right] - \\ \sqrt{2} \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - ax)^{1/4}}{(1 + ax)^{1/4}}\right] - 2 \text{ArcTanh}\left[\frac{(1 + ax)^{1/4}}{(1 - ax)^{1/4}}\right] + \\ \frac{\text{Log}\left[1 + \frac{\sqrt{1 - ax}}{\sqrt{1 + ax}} - \frac{\sqrt{2} (1 - ax)^{1/4}}{(1 + ax)^{1/4}}\right]}{\sqrt{2}} - \frac{\text{Log}\left[1 + \frac{\sqrt{1 - ax}}{\sqrt{1 + ax}} + \frac{\sqrt{2} (1 - ax)^{1/4}}{(1 + ax)^{1/4}}\right]}{\sqrt{2}}$$

Result (type 7, 99 leaves):

$$8 e^{-\frac{1}{2} \text{ArcTanh}[ax]} - 2 \text{ArcTan}\left[e^{-\frac{1}{2} \text{ArcTanh}[ax]}\right] + \text{Log}\left[1 - e^{-\frac{1}{2} \text{ArcTanh}[ax]}\right] - \\ \text{Log}\left[1 + e^{-\frac{1}{2} \text{ArcTanh}[ax]}\right] + \frac{1}{2} \text{RootSum}\left[1 + \#1^4 \&, \frac{-\text{ArcTanh}[ax] - 2 \text{Log}\left[e^{-\frac{1}{2} \text{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right]$$

Problem 120: Unable to integrate problem.

$$\int e^{\frac{\text{ArcTanh}[x]}{3}} x^m dx$$

Optimal (type 6, 28 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1 + m, \frac{1}{6}, -\frac{1}{6}, 2 + m, x, -x\right]}{1 + m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{\text{ArcTanh}[x]}{3}} x^m dx$$

Problem 121: Result is not expressed in closed-form.

$$\int e^{\frac{\text{ArcTanh}[x]}{3}} x^2 dx$$

Optimal (type 3, 245 leaves, 16 steps):

$$\begin{aligned}
 & -\frac{19}{54} (1-x)^{5/6} (1+x)^{1/6} - \frac{1}{18} (1-x)^{5/6} (1+x)^{7/6} - \frac{1}{3} (1-x)^{5/6} x (1+x)^{7/6} - \\
 & \frac{19}{81} \operatorname{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \frac{19}{162} \operatorname{ArcTan}\left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \frac{19}{162} \operatorname{ArcTan}\left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \\
 & \frac{19 \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{108\sqrt{3}} + \frac{19 \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{108\sqrt{3}}
 \end{aligned}$$

Result (type 7, 133 leaves):

$$\begin{aligned}
 & \frac{1}{486} \left(-\frac{18 e^{\frac{\operatorname{ArcTanh}[x]}{3}} (19 + 8 e^{2 \operatorname{ArcTanh}[x]} + 61 e^{4 \operatorname{ArcTanh}[x]})}{(1 + e^{2 \operatorname{ArcTanh}[x]})^3} + \right. \\
 & \left. 114 \operatorname{ArcTan}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] + 19 \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \frac{1}{-\#1 + 2 \#1^3}\right] \right. \\
 & \left. \left(-2 \operatorname{ArcTanh}[x] + 6 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2\right) \& \right)
 \end{aligned}$$

Problem 122: Result is not expressed in closed-form.

$$\int e^{\frac{\operatorname{ArcTanh}[x]}{3}} x \, dx$$

Optimal (type 3, 224 leaves, 15 steps):

$$\begin{aligned}
 & -\frac{1}{6} (1-x)^{5/6} (1+x)^{1/6} - \frac{1}{2} (1-x)^{5/6} (1+x)^{7/6} - \frac{1}{9} \operatorname{ArcTan}\left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}}\right] + \\
 & \frac{1}{18} \operatorname{ArcTan}\left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \frac{1}{18} \operatorname{ArcTan}\left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}}\right] - \\
 & \frac{\operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}} + \frac{\operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}}\right]}{12\sqrt{3}}
 \end{aligned}$$

Result (type 7, 127 leaves):

$$\begin{aligned}
 & \frac{1}{9} \left(-\frac{3 e^{\frac{\operatorname{ArcTanh}[x]}{3}} (1 + 7 e^{2 \operatorname{ArcTanh}[x]})}{(1 + e^{2 \operatorname{ArcTanh}[x]})^2} + \operatorname{ArcTan}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] \right) - \frac{1}{54} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \right. \\
 & \left. \frac{1}{-\#1 + 2 \#1^3} \left(2 \operatorname{ArcTanh}[x] - 6 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] - \operatorname{ArcTanh}[x] \#1^2 + 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2\right) \& \right)
 \end{aligned}$$

Problem 123: Result is not expressed in closed-form.

$$\int e^{\frac{\operatorname{ArcTanh}[x]}{3}} \, dx$$

Optimal (type 3, 202 leaves, 14 steps):

$$\begin{aligned}
 & - (1-x)^{5/6} (1+x)^{1/6} - \frac{2}{3} \operatorname{ArcTan} \left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}} \right] + \frac{1}{3} \operatorname{ArcTan} \left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}} \right] - \\
 & \frac{1}{3} \operatorname{ArcTan} \left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}} \right] - \frac{\operatorname{Log} \left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}} \right]}{2\sqrt{3}} + \frac{\operatorname{Log} \left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}} \right]}{2\sqrt{3}}
 \end{aligned}$$

Result (type 7, 116 leaves):

$$\begin{aligned}
 & - \frac{2 e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{1 + e^{2 \operatorname{ArcTanh}[x]}} + \frac{2}{3} \operatorname{ArcTan} \left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} \right] - \frac{1}{9} \operatorname{RootSum} \left[1 - \#1^2 + \#1^4 \& , \right. \\
 & \left. \frac{1}{-\#1 + 2 \#1^3} \left(2 \operatorname{ArcTanh}[x] - 6 \operatorname{Log} \left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1 \right] - \operatorname{ArcTanh}[x] \#1^2 + 3 \operatorname{Log} \left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1 \right] \#1^2 \right) \& \right]
 \end{aligned}$$

Problem 124: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{x} dx$$

Optimal (type 3, 346 leaves, 25 steps):

$$\begin{aligned}
 & -2 \operatorname{ArcTan} \left[\frac{(1-x)^{1/6}}{(1+x)^{1/6}} \right] + \operatorname{ArcTan} \left[\sqrt{3} - \frac{2(1-x)^{1/6}}{(1+x)^{1/6}} \right] - \operatorname{ArcTan} \left[\sqrt{3} + \frac{2(1-x)^{1/6}}{(1+x)^{1/6}} \right] + \\
 & \sqrt{3} \operatorname{ArcTan} \left[\frac{1 - \frac{2(1+x)^{1/6}}{(1-x)^{1/6}}}{\sqrt{3}} \right] - \sqrt{3} \operatorname{ArcTan} \left[\frac{1 + \frac{2(1+x)^{1/6}}{(1-x)^{1/6}}}{\sqrt{3}} \right] - 2 \operatorname{ArcTanh} \left[\frac{(1+x)^{1/6}}{(1-x)^{1/6}} \right] - \\
 & \frac{1}{2} \sqrt{3} \operatorname{Log} \left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} - \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}} \right] + \frac{1}{2} \sqrt{3} \operatorname{Log} \left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}} + \frac{\sqrt{3}(1-x)^{1/6}}{(1+x)^{1/6}} \right] + \\
 & \frac{1}{2} \operatorname{Log} \left[1 - \frac{(1+x)^{1/6}}{(1-x)^{1/6}} + \frac{(1+x)^{1/3}}{(1-x)^{1/3}} \right] - \frac{1}{2} \operatorname{Log} \left[1 + \frac{(1+x)^{1/6}}{(1-x)^{1/6}} + \frac{(1+x)^{1/3}}{(1-x)^{1/3}} \right]
 \end{aligned}$$

Result (type 7, 220 leaves):

$$\begin{aligned}
 & 2 \operatorname{ArcTan} \left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} \right] - \sqrt{3} \operatorname{ArcTan} \left[\frac{-1 + 2 e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{\sqrt{3}} \right] - \sqrt{3} \operatorname{ArcTan} \left[\frac{1 + 2 e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{\sqrt{3}} \right] + \\
 & \operatorname{Log} \left[1 - e^{\frac{\operatorname{ArcTanh}[x]}{3}} \right] - \operatorname{Log} \left[1 + e^{\frac{\operatorname{ArcTanh}[x]}{3}} \right] + \frac{1}{2} \operatorname{Log} \left[1 - e^{\frac{\operatorname{ArcTanh}[x]}{3}} + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} \right] - \\
 & \frac{1}{2} \operatorname{Log} \left[1 + e^{\frac{\operatorname{ArcTanh}[x]}{3}} + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} \right] - \frac{1}{3} \operatorname{RootSum} \left[1 - \#1^2 + \#1^4 \& , \right. \\
 & \left. \frac{1}{-\#1 + 2 \#1^3} \left(2 \operatorname{ArcTanh}[x] - 6 \operatorname{Log} \left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1 \right] - \operatorname{ArcTanh}[x] \#1^2 + 3 \operatorname{Log} \left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1 \right] \#1^2 \right) \& \right]
 \end{aligned}$$

Problem 127: Unable to integrate problem.

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} x^m dx$$

Optimal (type 6, 28 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{1}{3}, -\frac{1}{3}, 2+m, x, -x\right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} x^m dx$$

Problem 128: Result is not expressed in closed-form.

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} x^2 dx$$

Optimal (type 3, 133 leaves, 5 steps):

$$-\frac{11}{27} (1-x)^{2/3} (1+x)^{1/3} - \frac{1}{9} (1-x)^{2/3} (1+x)^{4/3} - \frac{1}{3} (1-x)^{2/3} x (1+x)^{4/3} +$$

$$\frac{22 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{27\sqrt{3}} + \frac{11}{81} \operatorname{Log}[1+x] + \frac{11}{27} \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]$$

Result (type 7, 154 leaves):

$$\frac{2}{243} \left(-\frac{324 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{(1 + e^{2 \operatorname{ArcTanh}[x]})^3} + \frac{540 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{(1 + e^{2 \operatorname{ArcTanh}[x]})^2} - \frac{315 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{1 + e^{2 \operatorname{ArcTanh}[x]}} - \right.$$

$$22 \operatorname{ArcTanh}[x] + 33 \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - 11 \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \right.$$

$$\left. \frac{1}{-2 + \#1^2} \left(\operatorname{ArcTanh}[x] - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2 \right) \& \right]$$

Problem 129: Result is not expressed in closed-form.

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} x dx$$

Optimal (type 3, 112 leaves, 4 steps):

$$-\frac{1}{3} (1-x)^{2/3} (1+x)^{1/3} - \frac{1}{2} (1-x)^{2/3} (1+x)^{4/3} +$$

$$\frac{2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{3\sqrt{3}} + \frac{1}{9} \operatorname{Log}[1+x] + \frac{1}{3} \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]$$

Result (type 7, 124 leaves):

$$\frac{2}{27} \left(-\frac{9 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} (1 + 4 e^{2 \operatorname{ArcTanh}[x]})}{(1 + e^{2 \operatorname{ArcTanh}[x]})^2} - 2 \operatorname{ArcTanh}[x] + 3 \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \right. \right. \\ \left. \left. -\frac{1}{-2 + \#1^2} \left(\operatorname{ArcTanh}[x] - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2\right) \& \right] \right)$$

Problem 130: Result is not expressed in closed-form.

$$\int e^{\frac{2 \operatorname{ArcTanh}[x]}{3}} dx$$

Optimal (type 3, 84 leaves, 3 steps):

$$-(1-x)^{2/3} (1+x)^{1/3} + \frac{2 \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right]}{\sqrt{3}} + \frac{1}{3} \operatorname{Log}[1+x] + \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right]$$

Result (type 7, 116 leaves):

$$-\frac{2 e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{1 + e^{2 \operatorname{ArcTanh}[x]}} - \frac{4 \operatorname{ArcTanh}[x]}{9} + \frac{2}{3} \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \frac{2}{9} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \right. \\ \left. \frac{1}{-2 + \#1^2} \left(\operatorname{ArcTanh}[x] - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2\right) \& \right]$$

Problem 131: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}}{x} dx$$

Optimal (type 3, 135 leaves, 4 steps):

$$\sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} - \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right] + \sqrt{3} \operatorname{ArcTan}\left[\frac{1}{\sqrt{3}} + \frac{2(1-x)^{1/3}}{\sqrt{3}(1+x)^{1/3}}\right] - \\ \frac{\operatorname{Log}[x]}{2} + \frac{1}{2} \operatorname{Log}[1+x] + \frac{3}{2} \operatorname{Log}\left[1 + \frac{(1-x)^{1/3}}{(1+x)^{1/3}}\right] + \frac{3}{2} \operatorname{Log}\left[(1-x)^{1/3} - (1+x)^{1/3}\right]$$

Result (type 7, 215 leaves):

$$-\sqrt{3} \operatorname{ArcTan}\left[\frac{-1 + 2 e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{\sqrt{3}}\right] + \sqrt{3} \operatorname{ArcTan}\left[\frac{1 + 2 e^{\frac{\operatorname{ArcTanh}[x]}{3}}}{\sqrt{3}}\right] - \\ \frac{2 \operatorname{ArcTanh}[x]}{3} + \operatorname{Log}\left[1 - e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] + \operatorname{Log}\left[1 + e^{\frac{\operatorname{ArcTanh}[x]}{3}}\right] + \operatorname{Log}\left[1 + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \\ \frac{1}{2} \operatorname{Log}\left[1 - e^{\frac{\operatorname{ArcTanh}[x]}{3}} + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \frac{1}{2} \operatorname{Log}\left[1 + e^{\frac{\operatorname{ArcTanh}[x]}{3}} + e^{\frac{2 \operatorname{ArcTanh}[x]}{3}}\right] - \frac{1}{3} \operatorname{RootSum}\left[1 - \#1^2 + \#1^4 \&, \right. \\ \left. \frac{1}{-2 + \#1^2} \left(\operatorname{ArcTanh}[x] - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] + \operatorname{ArcTanh}[x] \#1^2 - 3 \operatorname{Log}\left[e^{\frac{\operatorname{ArcTanh}[x]}{3}} - \#1\right] \#1^2\right) \& \right]$$

Problem 134: Unable to integrate problem.

$$\int e^{\frac{1}{4} \text{ArcTanh}[a x]} x^m dx$$

Optimal (type 6, 31 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, \frac{1}{8}, -\frac{1}{8}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 16 leaves):

$$\int e^{\frac{1}{4} \text{ArcTanh}[a x]} x^m dx$$

Problem 135: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} \text{ArcTanh}[a x]} x^2 dx$$

Optimal (type 3, 646 leaves, 27 steps):

$$\begin{aligned} & -\frac{11(1-a x)^{7/8}(1+a x)^{1/8}}{32 a^3} - \frac{(1-a x)^{7/8}(1+a x)^{9/8}}{24 a^3} - \frac{x(1-a x)^{7/8}(1+a x)^{9/8}}{3 a^2} + \\ & \frac{11 \sqrt{2+\sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}-\frac{2(1-a x)^{1/8}}{(1+a x)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{128 a^3} + \frac{11 \sqrt{2-\sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}-\frac{2(1-a x)^{1/8}}{(1+a x)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{128 a^3} - \\ & \frac{11 \sqrt{2+\sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}}+\frac{2(1-a x)^{1/8}}{(1+a x)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{128 a^3} - \frac{11 \sqrt{2-\sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}}+\frac{2(1-a x)^{1/8}}{(1+a x)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{128 a^3} - \\ & \frac{11 \sqrt{2-\sqrt{2}} \text{Log}\left[1+\frac{(1-a x)^{1/4}}{(1+a x)^{1/4}}-\frac{\sqrt{2-\sqrt{2}}(1-a x)^{1/8}}{(1+a x)^{1/8}}\right]}{256 a^3} + \frac{11 \sqrt{2-\sqrt{2}} \text{Log}\left[1+\frac{(1-a x)^{1/4}}{(1+a x)^{1/4}}+\frac{\sqrt{2-\sqrt{2}}(1-a x)^{1/8}}{(1+a x)^{1/8}}\right]}{256 a^3} - \\ & \frac{11 \sqrt{2+\sqrt{2}} \text{Log}\left[1+\frac{(1-a x)^{1/4}}{(1+a x)^{1/4}}-\frac{\sqrt{2+\sqrt{2}}(1-a x)^{1/8}}{(1+a x)^{1/8}}\right]}{256 a^3} + \frac{11 \sqrt{2+\sqrt{2}} \text{Log}\left[1+\frac{(1-a x)^{1/4}}{(1+a x)^{1/4}}+\frac{\sqrt{2+\sqrt{2}}(1-a x)^{1/8}}{(1+a x)^{1/8}}\right]}{256 a^3} \end{aligned}$$

Result (type 7, 94 leaves):

$$\frac{1}{a^3} \left(\frac{e^{\frac{1}{4} \text{ArcTanh}[a x]} (33 + 10 e^{2 \text{ArcTanh}[a x]} + 105 e^{4 \text{ArcTanh}[a x]})}{48 (1 + e^{2 \text{ArcTanh}[a x]})^3} - \frac{11}{512} \text{RootSum}\left[1 + \#1^8 \&, \frac{\text{ArcTanh}[a x] - 4 \text{Log}\left[e^{\frac{1}{4} \text{ArcTanh}[a x]} - \#1\right]}{\#1^7} \&\right] \right)$$

Problem 136: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} \text{ArcTanh}[a x]} x \, dx$$

Optimal (type 3, 619 leaves, 26 steps):

$$\begin{aligned} & -\frac{(1-ax)^{7/8} (1+ax)^{1/8}}{8a^2} - \frac{(1-ax)^{7/8} (1+ax)^{9/8}}{2a^2} + \\ & \frac{\sqrt{2+\sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{32a^2} + \frac{\sqrt{2-\sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{32a^2} - \\ & \frac{\sqrt{2+\sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{32a^2} - \frac{\sqrt{2-\sqrt{2}} \text{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{32a^2} - \\ & \frac{\sqrt{2-\sqrt{2}} \text{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2-\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{64a^2} + \frac{\sqrt{2-\sqrt{2}} \text{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2-\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{64a^2} - \\ & \frac{\sqrt{2+\sqrt{2}} \text{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2+\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{64a^2} + \frac{\sqrt{2+\sqrt{2}} \text{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2+\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{64a^2} \end{aligned}$$

Result (type 7, 83 leaves):

$$\begin{aligned} & \frac{1}{128a^2} \left(-\frac{32 e^{\frac{1}{4} \text{ArcTanh}[a x]} (1 + 9 e^{2 \text{ArcTanh}[a x]})}{(1 + e^{2 \text{ArcTanh}[a x]})^2} + \right. \\ & \left. \text{RootSum}\left[1 + \#1^8 \&, \frac{-\text{ArcTanh}[a x] + 4 \text{Log}\left[e^{\frac{1}{4} \text{ArcTanh}[a x]} - \#1\right]}{\#1^7} \&\right] \right) \end{aligned}$$

Problem 137: Result is not expressed in closed-form.

$$\int e^{\frac{1}{4} \text{ArcTanh}[a x]} \, dx$$

Optimal (type 3, 591 leaves, 25 steps):

$$\begin{aligned}
 & - \frac{(1-ax)^{7/8} (1+ax)^{1/8}}{a} + \frac{\sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{4a} + \frac{\sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{4a} \\
 & - \frac{\sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right]}{4a} - \frac{\sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right]}{4a} \\
 & - \frac{\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2-\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{8a} + \frac{\sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2-\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{8a} \\
 & - \frac{\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2+\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{8a} + \frac{\sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2+\sqrt{2}} (1-ax)^{1/8}}{(1+ax)^{1/8}}\right]}{8a}
 \end{aligned}$$

Result (type 7, 71 leaves):

$$\frac{-\frac{32 e^{\frac{1}{4} \operatorname{ArcTanh}[a x]}}{1+e^{2 \operatorname{ArcTanh}[a x]}} + \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{-\operatorname{ArcTanh}[a x] + 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1^7} \&\right]}{16 a}$$

Problem 138: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4} \operatorname{ArcTanh}[a x]}}{x} dx$$

Optimal (type 3, 759 leaves, 39 steps):

$$\begin{aligned}
 & -2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] + \sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right] + \\
 & \sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} - \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right] - \sqrt{2+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2-\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2+\sqrt{2}}}\right] - \\
 & \sqrt{2-\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2+\sqrt{2}} + \frac{2(1-ax)^{1/8}}{(1+ax)^{1/8}}}{\sqrt{2-\sqrt{2}}}\right] + \sqrt{2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] - \\
 & \sqrt{2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] - 2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] - \\
 & \frac{1}{2} \sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2-\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right] + \\
 & \frac{1}{2} \sqrt{2-\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2-\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right] - \\
 & \frac{1}{2} \sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} - \frac{\sqrt{2+\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right] + \\
 & \frac{1}{2} \sqrt{2+\sqrt{2}} \operatorname{Log}\left[1 + \frac{(1-ax)^{1/4}}{(1+ax)^{1/4}} + \frac{\sqrt{2+\sqrt{2}}(1-ax)^{1/8}}{(1+ax)^{1/8}}\right] + \\
 & \frac{\operatorname{Log}\left[1 - \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right]}{\sqrt{2}} - \frac{\operatorname{Log}\left[1 + \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right]}{\sqrt{2}}
 \end{aligned}$$

Result (type 7, 128 leaves):

$$\begin{aligned}
 & -2 \operatorname{ArcTan}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] - \operatorname{Log}\left[1 + e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] + \\
 & \frac{1}{4} \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right] + \\
 & \frac{1}{4} \operatorname{RootSum}\left[1 + \#1^8 \&, \frac{-\operatorname{ArcTanh}[ax] + 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^7} \&\right]
 \end{aligned}$$

Problem 139: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{x^2} dx$$

Optimal (type 3, 271 leaves, 16 steps):

$$\begin{aligned}
 & -\frac{(1-ax)^{7/8}(1+ax)^{1/8}}{x} - \frac{1}{2} a \operatorname{ArcTan}\left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] + \\
 & \frac{a \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}}\right]}{2\sqrt{2}} - \frac{a \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}}\right]}{2\sqrt{2}} - \frac{1}{2} a \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] + \\
 & \frac{a \operatorname{Log}\left[1 - \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right]}{4\sqrt{2}} - \frac{a \operatorname{Log}\left[1 + \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right]}{4\sqrt{2}}
 \end{aligned}$$

Result (type 7, 113 leaves):

$$\begin{aligned}
 & \frac{1}{16} a \\
 & \left(4 \left(-\frac{8 e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{-1 + e^{2 \operatorname{ArcTanh}[ax]}} - 2 \operatorname{ArcTan}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] - \operatorname{Log}\left[1 + e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] \right) + \right. \\
 & \left. \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right] \right)
 \end{aligned}$$

Problem 140: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{x^3} dx$$

Optimal (type 3, 312 leaves, 17 steps):

$$\begin{aligned}
 & -\frac{a(1-ax)^{7/8}(1+ax)^{1/8}}{8x} - \frac{(1-ax)^{7/8}(1+ax)^{9/8}}{2x^2} - \frac{1}{16} a^2 \operatorname{ArcTan}\left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] + \\
 & \frac{a^2 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}}\right]}{16\sqrt{2}} - \frac{a^2 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}}\right]}{16\sqrt{2}} - \frac{1}{16} a^2 \operatorname{ArcTanh}\left[\frac{(1+ax)^{1/8}}{(1-ax)^{1/8}}\right] + \\
 & \frac{a^2 \operatorname{Log}\left[1 - \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right]}{32\sqrt{2}} - \frac{a^2 \operatorname{Log}\left[1 + \frac{\sqrt{2}(1+ax)^{1/8}}{(1-ax)^{1/8}} + \frac{(1+ax)^{1/4}}{(1-ax)^{1/4}}\right]}{32\sqrt{2}}
 \end{aligned}$$

Result (type 7, 139 leaves):

$$\begin{aligned}
 & \frac{1}{128} a^2 \\
 & \left(4 \left(-\frac{64 e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{(-1 + e^{2 \operatorname{ArcTanh}[ax]})^2} - \frac{72 e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}}{-1 + e^{2 \operatorname{ArcTanh}[ax]}} - 2 \operatorname{ArcTan}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] + \operatorname{Log}\left[1 - e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] - \right. \right. \\
 & \left. \left. \operatorname{Log}\left[1 + e^{\frac{1}{4} \operatorname{ArcTanh}[ax]}\right] \right) + \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[ax] - 4 \operatorname{Log}\left[e^{\frac{1}{4} \operatorname{ArcTanh}[ax]} - \#1\right]}{\#1^3} \&\right] \right)
 \end{aligned}$$

Problem 142: Result unnecessarily involves higher level functions.

$$\int e^{3 \operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 5, 151 leaves, 9 steps):

$$\frac{3 x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{1+m} - \frac{a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{2+m} + \frac{4 x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{1+m} + \frac{4 a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{2+m}$$

Result (type 6, 265 leaves):

$$\left(2(2+m) x^{1+m} \sqrt{-1-ax} \left(2 \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right]\right) / \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] + ax \left(3 \operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, -ax, ax\right] + \operatorname{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right]\right)\right) - \left(\sqrt{1-ax} \sqrt{1-a^2x^2} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right]\right) / \left(\sqrt{1+ax} \left(2(2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] + ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right]\right)\right)\right) / \left((1+m) (-1+ax)^{3/2}\right)$$

Problem 144: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int e^{\operatorname{ArcTanh}[a x]} x^m dx$$

Optimal (type 5, 74 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{1+m} + \frac{a x^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{2+m}$$

Result (type 6, 166 leaves):

$$\left(2 (2+m) x^{1+m} \sqrt{-1-ax} \sqrt{1-ax} \sqrt{1-a^2x^2} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] \right) /$$

$$\left((1+m) (-1+ax)^{3/2} \sqrt{1+ax} \right.$$

$$\left. \left(2 (2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] + ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right]\right) \right) \right)$$

Problem 145: Result unnecessarily involves higher level functions.

$$\int e^{-\operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 5, 75 leaves, 4 steps):

$$\frac{x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right]}{1+m} - \frac{ax^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2x^2\right]}{2+m}$$

Result (type 6, 134 leaves):

$$\left(2 (2+m) x^{1+m} \sqrt{1-ax} \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] \right) /$$

$$\left((1+m) \sqrt{1+ax} \left(2 (2+m) \operatorname{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] - ax \left(\operatorname{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2x^2\right]\right) \right) \right)$$

Problem 147: Result unnecessarily involves higher level functions.

$$\int e^{-3 \operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 5, 150 leaves, 9 steps):

$$-\frac{3x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right]}{1+m} + \frac{ax^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2x^2\right]}{2+m} +$$

$$\frac{4x^{1+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2x^2\right]}{1+m} - \frac{4ax^{2+m} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2x^2\right]}{2+m}$$

Result (type 6, 237 leaves):

$$\begin{aligned}
 & \left(2 (2+m) x^{1+m} \sqrt{1-ax} \left(\left(2 \operatorname{AppellF1} \left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, ax, -ax \right] \right) / \right. \right. \\
 & \quad \left(2 (2+m) \operatorname{AppellF1} \left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, ax, -ax \right] - ax \right. \\
 & \quad \left. \left. \left(3 \operatorname{AppellF1} \left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, ax, -ax \right] + \operatorname{AppellF1} \left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax \right] \right) \right) \right) + \\
 & \quad \left((1+ax) \operatorname{AppellF1} \left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax \right] \right) / \\
 & \quad \left(-2 (2+m) \operatorname{AppellF1} \left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax \right] + \right. \\
 & \quad \left. ax \left(\operatorname{AppellF1} \left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{HypergeometricPFQ} \left[\left\{ \frac{1}{2}, 1+\frac{m}{2} \right\}, \left\{ 2+\frac{m}{2} \right\}, a^2 x^2 \right] \right) \right) \right) / \left((1+m) (1+ax)^{3/2} \right)
 \end{aligned}$$

Problem 148: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[ax]} x^m dx$$

Optimal (type 6, 35 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1} \left[1+m, \frac{n}{2}, -\frac{n}{2}, 2+m, ax, -ax \right]}{1+m}$$

Result (type 8, 14 leaves):

$$\int e^{n \operatorname{ArcTanh}[ax]} x^m dx$$

Problem 211: Unable to integrate problem.

$$\int \frac{e^{-2 \operatorname{ArcTanh}[ax]}}{c - acx} dx$$

Optimal (type 3, 13 leaves, 2 steps):

$$\frac{\operatorname{Log}[1+ax]}{ac}$$

Result (type 8, 20 leaves):

$$\int \frac{e^{-2 \operatorname{ArcTanh}[ax]}}{c - acx} dx$$

Problem 212: Unable to integrate problem.

$$\int \frac{e^{-2 \operatorname{ArcTanh}[ax]}}{(c - acx)^2} dx$$

Optimal (type 3, 11 leaves, 3 steps):

$$\frac{\text{ArcTanh}[a x]}{a c^2}$$

Result (type 8, 20 leaves):

$$\int \frac{e^{-2 \text{ArcTanh}[a x]}}{(c - a c x)^2} dx$$

Problem 278: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{7/2} dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{1}{a c (9-n)} 2^{1+\frac{n}{2}} (1-a x)^{-n/2} (c-a c x)^{9/2} \text{Hypergeometric2F1}\left[\frac{9-n}{2}, -\frac{n}{2}, \frac{11-n}{2}, \frac{1}{2} (1-a x)\right]$$

Result (type 8, 22 leaves):

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{7/2} dx$$

Problem 279: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{5/2} dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{1}{a c (7-n)} 2^{1+\frac{n}{2}} (1-a x)^{-n/2} (c-a c x)^{7/2} \text{Hypergeometric2F1}\left[\frac{7-n}{2}, -\frac{n}{2}, \frac{9-n}{2}, \frac{1}{2} (1-a x)\right]$$

Result (type 8, 22 leaves):

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{5/2} dx$$

Problem 280: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{3/2} dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{1}{a c (5-n)} 2^{1+\frac{n}{2}} (1-a x)^{-n/2} (c-a c x)^{5/2} \text{Hypergeometric2F1}\left[\frac{5-n}{2}, -\frac{n}{2}, \frac{7-n}{2}, \frac{1}{2} (1-a x)\right]$$

Result (type 8, 22 leaves):

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^{3/2} dx$$

Problem 281: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[a x]} \sqrt{c - a c x} \, dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{1}{a c (3-n)} 2^{1+\frac{n}{2}} (1-a x)^{-n/2} (c-a c x)^{3/2} \operatorname{Hypergeometric2F1}\left[\frac{3-n}{2}, -\frac{n}{2}, \frac{5-n}{2}, \frac{1}{2} (1-a x)\right]$$

Result (type 8, 22 leaves):

$$\int e^{n \operatorname{ArcTanh}[a x]} \sqrt{c - a c x} \, dx$$

Problem 282: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{\sqrt{c - a c x}} \, dx$$

Optimal (type 5, 81 leaves, 3 steps):

$$-\frac{1}{a c (1-n)} 2^{1+\frac{n}{2}} (1-a x)^{-n/2} \sqrt{c - a c x} \operatorname{Hypergeometric2F1}\left[\frac{1-n}{2}, -\frac{n}{2}, \frac{3-n}{2}, \frac{1}{2} (1-a x)\right]$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{\sqrt{c - a c x}} \, dx$$

Problem 283: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{(c - a c x)^{3/2}} \, dx$$

Optimal (type 5, 78 leaves, 3 steps):

$$\left(2^{1+\frac{n}{2}} (1-a x)^{-n/2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2} (-1-n), -\frac{n}{2}, \frac{1-n}{2}, \frac{1}{2} (1-a x)\right] \right) / (a c (1+n) \sqrt{c - a c x})$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{(c - a c x)^{3/2}} \, dx$$

Problem 284: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{(c - a c x)^{5/2}} \, dx$$

Optimal (type 5, 78 leaves, 3 steps):

$$\left(2^{1+\frac{n}{2}} (1-ax)^{-n/2} \text{Hypergeometric2F1}\left[\frac{1}{2}(-3-n), -\frac{n}{2}, \frac{1}{2}(-1-n), \frac{1}{2}(1-ax)\right] \right) / (ac(3+n)(c-acx)^{3/2})$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-acx)^{5/2}} dx$$

Problem 285: Unable to integrate problem.

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-acx)^{7/2}} dx$$

Optimal (type 5, 78 leaves, 3 steps):

$$\left(2^{1+\frac{n}{2}} (1-ax)^{-n/2} \text{Hypergeometric2F1}\left[\frac{1}{2}(-5-n), -\frac{n}{2}, \frac{1}{2}(-3-n), \frac{1}{2}(1-ax)\right] \right) / (ac(5+n)(c-acx)^{5/2})$$

Result (type 8, 22 leaves):

$$\int \frac{e^{n \text{ArcTanh}[ax]}}{(c-acx)^{7/2}} dx$$

Problem 378: Result more than twice size of optimal antiderivative.

$$\int e^{\text{ArcTanh}[x]} \sqrt{1-x} dx$$

Optimal (type 2, 11 leaves, 3 steps):

$$\frac{2}{3} (1+x)^{3/2}$$

Result (type 2, 27 leaves):

$$\frac{2(1+x)\sqrt{1-x^2}}{3\sqrt{1-x}}$$

Problem 387: Unable to integrate problem.

$$\int e^{\text{ArcTanh}[ax]} x^m \sqrt{c-acx} dx$$

Optimal (type 5, 64 leaves, 4 steps):

$$\frac{1}{3a\sqrt{c-acx}} 2cx^m (-ax)^{-m} (1+ax) \sqrt{1-a^2x^2} \text{Hypergeometric2F1}\left[\frac{3}{2}, -m, \frac{5}{2}, 1+ax\right]$$

Result (type 8, 23 leaves):

$$\int e^{\text{ArcTanh}[ax]} x^m \sqrt{c-acx} dx$$

Problem 411: Unable to integrate problem.

$$\int e^{-\text{ArcTanh}[a x]} x^m \sqrt{c - a c x} dx$$

Optimal (type 5, 114 leaves, 5 steps):

$$-\frac{2 c x^{1+m} \sqrt{1 - a^2 x^2}}{(3 + 2 m) \sqrt{c - a c x}} + \left(\frac{2 (5 + 4 m) x^m (-a x)^{-m} (1 + a x) \sqrt{c - a c x} \text{Hypergeometric2F1}\left[\frac{1}{2}, -m, \frac{3}{2}, 1 + a x\right]}{a (3 + 2 m) \sqrt{1 - a^2 x^2}} \right) /$$

Result (type 8, 25 leaves):

$$\int e^{-\text{ArcTanh}[a x]} x^m \sqrt{c - a c x} dx$$

Problem 437: Unable to integrate problem.

$$\int e^{-2 p \text{ArcTanh}[a x]} (c - a c x)^p dx$$

Optimal (type 5, 61 leaves, 3 steps):

$$-\frac{1}{a c (1 + 2 p)} 2^{-p} (1 - a x)^p (c - a c x)^{1+p} \text{Hypergeometric2F1}\left[p, 1 + 2 p, 2 (1 + p), \frac{1}{2} (1 - a x)\right]$$

Result (type 8, 21 leaves):

$$\int e^{-2 p \text{ArcTanh}[a x]} (c - a c x)^p dx$$

Problem 439: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^p dx$$

Optimal (type 5, 82 leaves, 3 steps):

$$-\frac{1}{a c (2 - n + 2 p)} 2^{1 + \frac{n}{2}} (1 - a x)^{-n/2} (c - a c x)^{1+p} \text{Hypergeometric2F1}\left[-\frac{n}{2}, 1 - \frac{n}{2} + p, 2 - \frac{n}{2} + p, \frac{1}{2} (1 - a x)\right]$$

Result (type 8, 20 leaves):

$$\int e^{n \text{ArcTanh}[a x]} (c - a c x)^p dx$$

Problem 440: Result more than twice size of optimal antiderivative.

$$\int e^{n \operatorname{ArcTanh}[a x]} (c - a c x)^3 dx$$

Optimal (type 5, 68 leaves, 2 steps):

$$\frac{2^{1+\frac{n}{2}} c^3 (1 - a x)^{4-\frac{n}{2}} \operatorname{Hypergeometric2F1}\left[4 - \frac{n}{2}, -\frac{n}{2}, 5 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (8 - n)}$$

Result (type 5, 195 leaves):

$$\begin{aligned} & -\frac{1}{24 a (2 + n)} c^3 e^{n \operatorname{ArcTanh}[a x]} \\ & \left(-e^{2 \operatorname{ArcTanh}[a x]} n (-48 + 44 n - 12 n^2 + n^3) \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, -e^{2 \operatorname{ArcTanh}[a x]}\right] + \right. \\ & (2 + n) \left(a n^3 x + n^2 (-1 - 12 a x + a^2 x^2) + \right. \\ & 2 n (6 + 21 a x - 6 a^2 x^2 + a^3 x^3) + 6 (-7 - 4 a x + 6 a^2 x^2 - 4 a^3 x^3 + a^4 x^4) + \\ & \left. \left. (-48 + 44 n - 12 n^2 + n^3) \operatorname{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, -e^{2 \operatorname{ArcTanh}[a x]}\right] \right) \right) \end{aligned}$$

Problem 441: Result more than twice size of optimal antiderivative.

$$\int e^{n \operatorname{ArcTanh}[a x]} (c - a c x)^2 dx$$

Optimal (type 5, 68 leaves, 2 steps):

$$\frac{2^{1+\frac{n}{2}} c^2 (1 - a x)^{3-\frac{n}{2}} \operatorname{Hypergeometric2F1}\left[3 - \frac{n}{2}, -\frac{n}{2}, 4 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]}{a (6 - n)}$$

Result (type 5, 149 leaves):

$$\begin{aligned} & \frac{1}{6 a (2 + n)} \\ & c^2 e^{n \operatorname{ArcTanh}[a x]} \left(-e^{2 \operatorname{ArcTanh}[a x]} n (8 - 6 n + n^2) \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, -e^{2 \operatorname{ArcTanh}[a x]}\right] + \right. \\ & (2 + n) \left(6 + 6 a x + a n^2 x - 6 a^2 x^2 + 2 a^3 x^3 + n (-1 - 6 a x + a^2 x^2) + \right. \\ & \left. \left. (8 - 6 n + n^2) \operatorname{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, -e^{2 \operatorname{ArcTanh}[a x]}\right] \right) \right) \end{aligned}$$

Problem 447: Unable to integrate problem.

$$\int e^{\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Optimal (type 6, 60 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{ax}\right)^p x (1 - ax)^{-p} \text{AppellF1}\left[1 - p, \frac{1}{2} - p, -\frac{1}{2}, 2 - p, ax, -ax\right]}{1 - p}$$

Result (type 8, 22 leaves):

$$\int e^{\text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 456: Unable to integrate problem.

$$\int e^{2 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 5, 59 leaves, 6 steps):

$$-\left(c - \frac{c}{ax}\right)^p x - \frac{(2 - p) \left(c - \frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left[1, p, 1 + p, 1 - \frac{1}{ax}\right]}{ap}$$

Result (type 8, 24 leaves):

$$\int e^{2 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 474: Unable to integrate problem.

$$\int e^{4 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 5, 93 leaves, 7 steps):

$$-\frac{c(5 - p) \left(c - \frac{c}{ax}\right)^{-1+p}}{a(1 - p)} + c \left(c - \frac{c}{ax}\right)^{-1+p} x + \frac{(4 - p) \left(c - \frac{c}{ax}\right)^p \text{Hypergeometric2F1}\left[1, p, 1 + p, 1 - \frac{1}{ax}\right]}{ap}$$

Result (type 8, 24 leaves):

$$\int e^{4 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 484: Unable to integrate problem.

$$\int e^{-\text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 6, 60 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{ax}\right)^p x (1 - ax)^{-p} \text{AppellF1}\left[1 - p, -\frac{1}{2} - p, \frac{1}{2}, 2 - p, ax, -ax\right]}{1 - p}$$

Result (type 8, 24 leaves):

$$\int e^{-\text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 493: Unable to integrate problem.

$$\int e^{-2 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Optimal (type 5, 114 leaves, 8 steps):

$$-\frac{\left(c - \frac{c}{a x} \right)^{2+p} x}{c^2} - \frac{\left(c - \frac{c}{a x} \right)^{2+p} \operatorname{Hypergeometric2F1}\left[1, 2+p, 3+p, \frac{a-x}{2a} \right]}{2 a c^2 (2+p)} +$$

$$\frac{\left(c - \frac{c}{a x} \right)^{2+p} \operatorname{Hypergeometric2F1}\left[1, 2+p, 3+p, 1 - \frac{1}{a x} \right]}{a c^2}$$

Result (type 8, 24 leaves):

$$\int e^{-2 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^p dx$$

Problem 499: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-2 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x} \right)^2} dx$$

Optimal (type 3, 18 leaves, 5 steps):

$$-\frac{x}{c^2} + \frac{\operatorname{ArcTanh}[a x]}{a c^2}$$

Result (type 3, 40 leaves):

$$-\frac{x}{c^2} - \frac{\operatorname{Log}[1 - a x]}{2 a c^2} + \frac{\operatorname{Log}[1 + a x]}{2 a c^2}$$

Problem 510: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{9/2} dx$$

Optimal (type 3, 225 leaves, 8 steps):

$$-\frac{a^3 \left(c - \frac{c}{a x} \right)^{9/2} x^4 (54 - 227 a x) \sqrt{1 + a x}}{105 (1 - a x)^{9/2}} - \frac{10 a^2 \left(c - \frac{c}{a x} \right)^{9/2} x^3 \sqrt{1 + a x}}{21 (1 - a x)^{5/2}} +$$

$$\frac{2 a \left(c - \frac{c}{a x} \right)^{9/2} x^2 \sqrt{1 + a x}}{5 (1 - a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{9/2} x \sqrt{1 + a x}}{7 \sqrt{1 - a x}} - \frac{7 a^{7/2} \left(c - \frac{c}{a x} \right)^{9/2} x^{9/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{9/2}}$$

Result (type 3, 151 leaves):

$$\begin{aligned}
 & - \left(\left(c^4 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2} (-30 + 162 ax - 356 a^2 x^2 + 292 a^3 x^3 + 105 a^4 x^4) \right) / \right. \\
 & \left. (105 a^4 x^3 (-1 + ax)) \right) - \frac{7 i c^{9/2} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right]}{2 a}
 \end{aligned}$$

Problem 511: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax} \right)^{7/2} dx$$

Optimal (type 3, 181 leaves, 7 steps):

$$\begin{aligned}
 & \frac{2 a \left(c - \frac{c}{ax} \right)^{7/2} x^2 \sqrt{1 + ax}}{3 (1 - ax)^{3/2}} - \frac{2 \left(c - \frac{c}{ax} \right)^{7/2} x \sqrt{1 + ax}}{5 \sqrt{1 - ax}} - \\
 & \frac{a^2 \left(c - \frac{c}{ax} \right)^{7/2} x^3 \sqrt{1 + ax} (18 + 31 ax)}{15 (1 - ax)^{7/2}} + \frac{5 a^{5/2} \left(c - \frac{c}{ax} \right)^{7/2} x^{7/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - ax)^{7/2}}
 \end{aligned}$$

Result (type 3, 143 leaves):

$$\begin{aligned}
 & - \frac{c^3 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2} (6 - 28 ax + 56 a^2 x^2 + 15 a^3 x^3)}{15 a^3 x^2 (-1 + ax)} - \\
 & \frac{5 i c^{7/2} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 ax) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right]}{2 a}
 \end{aligned}$$

Problem 512: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax} \right)^{5/2} dx$$

Optimal (type 3, 171 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{3 a^2 \left(c - \frac{c}{ax} \right)^{5/2} x^3 \sqrt{1 + ax}}{(1 - ax)^{5/2}} - \frac{2 \left(c - \frac{c}{ax} \right)^{5/2} x (1 + ax)^{3/2}}{3 (1 - ax)^{5/2}} + \\
 & \frac{4 a \left(c - \frac{c}{ax} \right)^{5/2} x^2 (1 + ax)^{3/2}}{(1 - ax)^{5/2}} - \frac{3 a^{3/2} \left(c - \frac{c}{ax} \right)^{5/2} x^{5/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - ax)^{5/2}}
 \end{aligned}$$

Result (type 3, 133 leaves):

$$\frac{1}{6 a^2} c^2 \left(- \frac{2 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2} (-2 + 10 a x + 3 a^2 x^2)}{x (-1 + a x)} - \right. \\ \left. 9 i a \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}] \right)$$

Problem 513: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{3/2} dx$$

Optimal (type 3, 128 leaves, 7 steps):

$$\frac{a \left(c - \frac{c}{a x} \right)^{3/2} x^2 \sqrt{1 + a x}}{(1 - a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{3/2} x (1 - a^2 x^2)^{3/2}}{(1 - a x)^3} + \frac{\sqrt{a} \left(c - \frac{c}{a x} \right)^{3/2} x^{3/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{3/2}}$$

Result (type 3, 119 leaves):

$$- \frac{c \sqrt{c - \frac{c}{a x}} (2 + a x) \sqrt{1 - a^2 x^2}}{a (-1 + a x)} - \frac{i c^{3/2} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a}$$

Problem 514: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} dx$$

Optimal (type 3, 84 leaves, 6 steps):

$$- \frac{c \sqrt{1 - a^2 x^2}}{a \sqrt{c - \frac{c}{a x}}} + \frac{\sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}}$$

Result (type 3, 111 leaves):

$$- \frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} + \frac{i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a}$$

Problem 515: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{\sqrt{c - \frac{c}{a x}}} dx$$

Optimal (type 3, 157 leaves, 8 steps):

$$\frac{\sqrt{1 - a x} \sqrt{1 + a x}}{a \sqrt{c - \frac{c}{a x}}} - \frac{3 \sqrt{1 - a x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{3/2} \sqrt{c - \frac{c}{a x}} \sqrt{x}} + \frac{2 \sqrt{2} \sqrt{1 - a x} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{a^{3/2} \sqrt{c - \frac{c}{a x}} \sqrt{x}}$$

Result (type 3, 203 leaves):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{c - a c x} + \frac{3 i \text{Log}\left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}\right]}{2 a \sqrt{c}} - \frac{i \sqrt{2} \text{Log}\left[\frac{4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} - i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{4 (-1 + a x)^2}\right]}{a \sqrt{c}}$$

Problem 516: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{3/2}} dx$$

Optimal (type 3, 198 leaves, 9 steps):

$$\frac{\sqrt{1 - a x} \sqrt{1 + a x}}{a \left(c - \frac{c}{a x}\right)^{3/2}} + \frac{2 (1 - a x)^{3/2} \sqrt{1 + a x}}{a^2 \left(c - \frac{c}{a x}\right)^{3/2} x} + \frac{5 (1 - a x)^{3/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{5/2} \left(c - \frac{c}{a x}\right)^{3/2} x^{3/2}} - \frac{7 (1 - a x)^{3/2} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{\sqrt{2} a^{5/2} \left(c - \frac{c}{a x}\right)^{3/2} x^{3/2}}$$

Result (type 3, 211 leaves):

$$\frac{1}{4a} \left(-\frac{4a \sqrt{c - \frac{c}{ax}} x (-2 + ax) \sqrt{1 - a^2 x^2}}{c^2 (-1 + ax)^2} + \frac{10i \operatorname{Log} \left[-i \sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right]}{c^{3/2}} - \frac{7i \sqrt{2} \operatorname{Log} \left[\frac{4ac \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} - i \sqrt{2} c^{3/2} (-1 - 2ax + 3a^2 x^2)}{7(-1 + ax)^2} \right]}{c^{3/2}} \right)$$

Problem 517: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\operatorname{ArcTanh}[ax]}}{\left(c - \frac{c}{ax}\right)^{5/2}} dx$$

Optimal (type 3, 249 leaves, 10 steps):

$$\frac{\sqrt{1 - ax} \sqrt{1 + ax}}{2a \left(c - \frac{c}{ax}\right)^{5/2}} - \frac{11(1 - ax)^{3/2} \sqrt{1 + ax}}{8a^2 \left(c - \frac{c}{ax}\right)^{5/2} x} - \frac{23(1 - ax)^{5/2} \sqrt{1 + ax}}{8a^3 \left(c - \frac{c}{ax}\right)^{5/2} x^2} + \frac{7(1 - ax)^{5/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{7/2} \left(c - \frac{c}{ax}\right)^{5/2} x^{5/2}} + \frac{79(1 - ax)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}}\right]}{8\sqrt{2} a^{7/2} \left(c - \frac{c}{ax}\right)^{5/2} x^{5/2}}$$

Result (type 3, 222 leaves):

$$\frac{1}{32a} \left(-\frac{4a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} (23 - 35ax + 8a^2 x^2)}{c^3 (-1 + ax)^3} + \frac{112i \operatorname{Log} \left[-i \sqrt{c} (1 + 2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right]}{c^{5/2}} - \frac{79i \sqrt{2} \operatorname{Log} \left[\frac{32ac^2 \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} - 8i \sqrt{2} c^{5/2} (-1 - 2ax + 3a^2 x^2)}{79(-1 + ax)^2} \right]}{c^{5/2}} \right)$$

Problem 527: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{9/2} dx$$

Optimal (type 3, 223 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{3 a^3 \left(c - \frac{c}{a x}\right)^{9/2} x^4 \sqrt{1+a x}}{(1-a x)^{9/2}} + \frac{3 a^2 \left(c - \frac{c}{a x}\right)^{9/2} x^3 (6-17 a x) (1+a x)^{3/2}}{35 (1-a x)^{9/2}} + \\
 & \frac{6 a \left(c - \frac{c}{a x}\right)^{9/2} x^2 (1+a x)^{3/2}}{35 (1-a x)^{5/2}} - \frac{2 \left(c - \frac{c}{a x}\right)^{9/2} x (1+a x)^{3/2}}{7 (1-a x)^{3/2}} + \frac{3 a^{7/2} \left(c - \frac{c}{a x}\right)^{9/2} x^{9/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-a x)^{9/2}}
 \end{aligned}$$

Result (type 3, 151 leaves):

$$\begin{aligned}
 & \frac{c^4 \sqrt{c - \frac{c}{a x}} \sqrt{1-a^2 x^2} (10-26 a x-12 a^2 x^2+164 a^3 x^3+35 a^4 x^4)}{35 a^4 x^3 (-1+a x)} + \\
 & \frac{3 i c^{9/2} \text{Log}\left[-i \sqrt{c} (1+2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1-a^2 x^2}}{-1+a x}\right]}{2 a}
 \end{aligned}$$

Problem 528: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \text{ArcTanh}[a x]} \left(c - \frac{c}{a x}\right)^{7/2} dx$$

Optimal (type 3, 217 leaves, 8 steps):

$$\begin{aligned}
 & -\frac{a^3 \left(c - \frac{c}{a x}\right)^{7/2} x^4 \sqrt{1+a x}}{(1-a x)^{7/2}} + \frac{2 a^2 \left(c - \frac{c}{a x}\right)^{7/2} x^3 (1+a x)^{3/2}}{3 (1-a x)^{7/2}} - \frac{2 \left(c - \frac{c}{a x}\right)^{7/2} x (1+a x)^{5/2}}{5 (1-a x)^{7/2}} + \\
 & \frac{4 a \left(c - \frac{c}{a x}\right)^{7/2} x^2 (1+a x)^{5/2}}{3 (1-a x)^{7/2}} - \frac{a^{5/2} \left(c - \frac{c}{a x}\right)^{7/2} x^{7/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-a x)^{7/2}}
 \end{aligned}$$

Result (type 3, 143 leaves):

$$\begin{aligned}
 & \frac{c^3 \sqrt{c - \frac{c}{a x}} \sqrt{1-a^2 x^2} (-6+8 a x+44 a^2 x^2+15 a^3 x^3)}{15 a^3 x^2 (-1+a x)} + \\
 & \frac{i c^{7/2} \text{Log}\left[-i \sqrt{c} (1+2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1-a^2 x^2}}{-1+a x}\right]}{2 a}
 \end{aligned}$$

Problem 529: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \text{ArcTanh}[a x]} \left(c - \frac{c}{a x}\right)^{5/2} dx$$

Optimal (type 3, 176 leaves, 8 steps):

$$-\frac{a^2 \left(c - \frac{c}{ax}\right)^{5/2} x^3 \sqrt{1+ax}}{(1-ax)^{5/2}} + \frac{2a \left(c - \frac{c}{ax}\right)^{5/2} x^2 (1+ax)^{3/2}}{3(1-ax)^{5/2}} - \frac{2 \left(c - \frac{c}{ax}\right)^{5/2} x (1-a^2 x^2)^{5/2}}{3(1-ax)^5} - \frac{a^{3/2} \left(c - \frac{c}{ax}\right)^{5/2} x^{5/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-ax)^{5/2}}$$

Result (type 3, 133 leaves):

$$\frac{1}{6a^2} c^2 \left(\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1-a^2 x^2} (2+2ax+3a^2 x^2)}{x(-1+ax)} - \left[3i a \sqrt{c} \text{Log}[-i \sqrt{c} (1+2ax)] + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2}}{-1+ax} \right] \right)$$

Problem 530: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{3/2} dx$$

Optimal (type 3, 127 leaves, 7 steps):

$$\frac{3a \left(c - \frac{c}{ax}\right)^{3/2} x^2 \sqrt{1+ax}}{(1-ax)^{3/2}} - \frac{2 \left(c - \frac{c}{ax}\right)^{3/2} x (1+ax)^{3/2}}{(1-ax)^{3/2}} + \frac{3\sqrt{a} \left(c - \frac{c}{ax}\right)^{3/2} x^{3/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-ax)^{3/2}}$$

Result (type 3, 118 leaves):

$$\frac{1}{2a} \left(\frac{2c \sqrt{c - \frac{c}{ax}} (-2+ax) \sqrt{1-a^2 x^2}}{-1+ax} - 3i c^{3/2} \text{Log}[-i \sqrt{c} (1+2ax)] + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2}}{-1+ax} \right)$$

Problem 531: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \text{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} dx$$

Optimal (type 3, 155 leaves, 8 steps):

$$-\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1+ax}}{\sqrt{1-ax}} - \frac{5 \sqrt{c - \frac{c}{ax}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1-ax}} + \frac{4\sqrt{2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1+ax}}\right]}{\sqrt{a} \sqrt{1-ax}}$$

Result (type 3, 204 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} - \frac{5 i \sqrt{c} \operatorname{Log}\left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}\right]}{2 a} +$$

$$\frac{2 i \sqrt{2} \sqrt{c} \operatorname{Log}\left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{8 c (-1 + ax)^2}\right]}{a}$$

Problem 532: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]}}{\sqrt{c - \frac{c}{ax}}} dx$$

Optimal (type 3, 195 leaves, 9 steps):

$$\frac{2 \sqrt{1 - ax} \sqrt{1 + ax}}{a \sqrt{c - \frac{c}{ax}}} + \frac{(1 + ax)^{3/2}}{a \sqrt{c - \frac{c}{ax}} \sqrt{1 - ax}} +$$

$$\frac{7 \sqrt{1 - ax} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x}} - \frac{5 \sqrt{2} \sqrt{1 - ax} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}}\right]}{a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x}}$$

Result (type 3, 210 leaves):

$$\frac{1}{2 a} \left(\frac{2 a \sqrt{c - \frac{c}{ax}} x (-3 + ax) \sqrt{1 - a^2 x^2}}{c (-1 + ax)^2} - \frac{7 i \operatorname{Log}\left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}\right]}{\sqrt{c}} + \right.$$

$$\left. \frac{5 i \sqrt{2} \operatorname{Log}\left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{10 (-1 + ax)^2}\right]}{\sqrt{c}} \right)$$

Problem 533: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{ax}\right)^{3/2}} dx$$

Optimal (type 3, 249 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{21 (1 - a x)^{3/2} \sqrt{1 + a x}}{8 a^2 \left(c - \frac{c}{a x}\right)^{3/2} x} + \frac{(1 + a x)^{3/2}}{2 a \left(c - \frac{c}{a x}\right)^{3/2} \sqrt{1 - a x}} - \frac{9 \sqrt{1 - a x} (1 + a x)^{3/2}}{8 a^2 \left(c - \frac{c}{a x}\right)^{3/2} x} \\
 & \frac{9 (1 - a x)^{3/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{5/2} \left(c - \frac{c}{a x}\right)^{3/2} x^{3/2}} + \frac{51 (1 - a x)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{4 \sqrt{2} a^{5/2} \left(c - \frac{c}{a x}\right)^{3/2} x^{3/2}}
 \end{aligned}$$

Result (type 3, 220 leaves):

$$\begin{aligned}
 & \frac{1}{16 a} \\
 & \left(\frac{4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (15 - 23 a x + 4 a^2 x^2)}{c^2 (-1 + a x)^3} - \frac{72 i \operatorname{Log}\left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}\right]}{c^{3/2}} + \right. \\
 & \left. \frac{51 i \sqrt{2} \operatorname{Log}\left[\frac{-16 a c \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} + 4 i \sqrt{2} c^{3/2} (-1 - 2 a x + 3 a^2 x^2)}{51 (-1 + a x)^2}\right]}{c^{3/2}} \right)
 \end{aligned}$$

Problem 534: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{5/2}} dx$$

Optimal (type 3, 293 leaves, 11 steps):

$$\begin{aligned}
 & \frac{103 (1 - a x)^{5/2} \sqrt{1 + a x}}{32 a^3 \left(c - \frac{c}{a x}\right)^{5/2} x^2} + \frac{(1 + a x)^{3/2}}{3 a \left(c - \frac{c}{a x}\right)^{5/2} \sqrt{1 - a x}} - \frac{13 \sqrt{1 - a x} (1 + a x)^{3/2}}{24 a^2 \left(c - \frac{c}{a x}\right)^{5/2} x} + \\
 & \frac{43 (1 - a x)^{3/2} (1 + a x)^{3/2}}{32 a^3 \left(c - \frac{c}{a x}\right)^{5/2} x^2} + \frac{11 (1 - a x)^{5/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{7/2} \left(c - \frac{c}{a x}\right)^{5/2} x^{5/2}} - \frac{249 (1 - a x)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{16 \sqrt{2} a^{7/2} \left(c - \frac{c}{a x}\right)^{5/2} x^{5/2}}
 \end{aligned}$$

Result (type 3, 232 leaves):

$$\frac{1}{64 a} \left(\frac{4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (-219 + 554 a x - 415 a^2 x^2 + 48 a^3 x^3)}{3 c^3 (-1 + a x)^4} - \frac{352 i \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right]}{c^{5/2}} + \frac{249 i \sqrt{2} \operatorname{Log} \left[\frac{-64 a c^2 \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} + 16 i \sqrt{2} c^{5/2} (-1 - 2 a x + 3 a^2 x^2)}{249 (-1 + a x)^2} \right]}{c^{5/2}} \right)$$

Problem 535: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{9/2} dx$$

Optimal (type 3, 225 leaves, 8 steps):

$$-\frac{94 a^2 \left(c - \frac{c}{a x} \right)^{9/2} x^3 \sqrt{1 + a x}}{21 (1 - a x)^{5/2}} + \frac{6 a \left(c - \frac{c}{a x} \right)^{9/2} x^2 \sqrt{1 + a x}}{5 (1 - a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{9/2} x \sqrt{1 + a x}}{7 \sqrt{1 - a x}} + \frac{a^3 \left(c - \frac{c}{a x} \right)^{9/2} x^4 \sqrt{1 + a x} (2718 + 521 a x)}{105 (1 - a x)^{9/2}} + \frac{11 a^{7/2} \left(c - \frac{c}{a x} \right)^{9/2} x^{9/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{9/2}}$$

Result (type 3, 151 leaves):

$$\left(c^4 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2} (30 - 246 a x + 1028 a^2 x^2 - 4156 a^3 x^3 + 105 a^4 x^4) \right) / (105 a^4 x^3 (-1 + a x)) + \frac{11 i c^{9/2} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right]}{2 a}$$

Problem 536: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{7/2} dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\frac{2 a \left(c - \frac{c}{a x} \right)^{7/2} x^2 \sqrt{1+a x}}{(1-a x)^{3/2}} - \frac{2 \left(c - \frac{c}{a x} \right)^{7/2} x \sqrt{1+a x}}{5 \sqrt{1-a x}} - \frac{a^2 \left(c - \frac{c}{a x} \right)^{7/2} x^3 \sqrt{1+a x} (66+7 a x)}{5 (1-a x)^{7/2}} - \frac{9 a^{5/2} \left(c - \frac{c}{a x} \right)^{7/2} x^{7/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-a x)^{7/2}}$$

Result (type 3, 143 leaves):

$$\frac{c^3 \sqrt{c - \frac{c}{a x}} \sqrt{1-a^2 x^2} (-2 + 16 a x - 92 a^2 x^2 + 5 a^3 x^3)}{5 a^3 x^2 (-1+a x)} + \frac{9 i c^{7/2} \text{Log}[-i \sqrt{c} (1+2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1-a^2 x^2}}{-1+a x}]}{2 a}$$

Problem 537: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{5/2} dx$$

Optimal (type 3, 137 leaves, 6 steps):

$$-\frac{2 \left(c - \frac{c}{a x} \right)^{5/2} x \sqrt{1+a x}}{3 \sqrt{1-a x}} + \frac{a \left(c - \frac{c}{a x} \right)^{5/2} x^2 (18-a x) \sqrt{1+a x}}{3 (1-a x)^{5/2}} + \frac{7 a^{3/2} \left(c - \frac{c}{a x} \right)^{5/2} x^{5/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-a x)^{5/2}}$$

Result (type 3, 135 leaves):

$$\frac{c^2 \sqrt{c - \frac{c}{a x}} \sqrt{1-a^2 x^2} (2 - 22 a x + 3 a^2 x^2)}{3 a^2 x (-1+a x)} + \frac{7 i c^{5/2} \text{Log}[-i \sqrt{c} (1+2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1-a^2 x^2}}{-1+a x}]}{2 a}$$

Problem 538: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{3/2} dx$$

Optimal (type 3, 126 leaves, 6 steps):

$$-\frac{2 \left(c - \frac{c}{a x} \right)^{3/2} x \sqrt{1+a x}}{(1-a x)^{3/2}} + \frac{a \left(c - \frac{c}{a x} \right)^{3/2} x^2 \sqrt{1+a x}}{(1-a x)^{3/2}} - \frac{5 \sqrt{a} \left(c - \frac{c}{a x} \right)^{3/2} x^{3/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-a x)^{3/2}}$$

Result (type 3, 118 leaves):

$$\frac{1}{2a} \left(\frac{2c \sqrt{c - \frac{c}{ax}} (-2 + ax) \sqrt{1 - a^2 x^2}}{-1 + ax} + 5i c^{3/2} \text{Log}[-i \sqrt{c} (1 + 2ax)] + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right)$$

Problem 539: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} dx$$

Optimal (type 3, 90 leaves, 6 steps):

$$-\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{1 - ax} + \frac{3 \sqrt{c - \frac{c}{ax}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - ax}}$$

Result (type 3, 110 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} + \frac{3i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2ax)] + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}}{2a}$$

Problem 540: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\text{ArcTanh}[ax]}}{\sqrt{c - \frac{c}{ax}}} dx$$

Optimal (type 3, 88 leaves, 6 steps):

$$\frac{\sqrt{1 - ax} \sqrt{1 + ax}}{a \sqrt{c - \frac{c}{ax}}} - \frac{\sqrt{1 - ax} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x}}$$

Result (type 3, 113 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{c (-1 + ax)} + \frac{i \text{Log}[-i \sqrt{c} (1 + 2ax)] + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax}}{2a \sqrt{c}}$$

Problem 541: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\text{ArcTanh}[ax]}}{\left(c - \frac{c}{ax}\right)^{3/2}} dx$$

Optimal (type 3, 159 leaves, 9 steps):

$$-\frac{(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c-\frac{c}{ax}\right)^{3/2}x} - \frac{(1-ax)^{3/2}\text{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}} + \frac{\sqrt{2}(1-ax)^{3/2}\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{a^{5/2}\left(c-\frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Result (type 3, 205 leaves):

$$\frac{\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{c^2(-1+ax)} - \frac{i\text{Log}\left[-i\sqrt{c}(1+2ax) + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax}\right]}{2ac^{3/2}} + \frac{i\text{Log}\left[\frac{-4ac\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2} + i\sqrt{2}c^{3/2}(-1-2ax+3a^2x^2)}{2(-1+ax)^2}\right]}{\sqrt{2}ac^{3/2}}$$

Problem 542: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\text{ArcTanh}[ax]}}{\left(c-\frac{c}{ax}\right)^{5/2}} dx$$

Optimal (type 3, 208 leaves, 9 steps):

$$\frac{(1-ax)^{3/2}\sqrt{1+ax}}{2a^2\left(c-\frac{c}{ax}\right)^{5/2}x} + \frac{3(1-ax)^{5/2}\sqrt{1+ax}}{2a^3\left(c-\frac{c}{ax}\right)^{5/2}x^2} + \frac{3(1-ax)^{5/2}\text{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}} - \frac{9(1-ax)^{5/2}\text{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{2\sqrt{2}a^{7/2}\left(c-\frac{c}{ax}\right)^{5/2}x^{5/2}}$$

Result (type 3, 214 leaves):

$$\frac{1}{8a} \left(\frac{4a\sqrt{c-\frac{c}{ax}}x(-3+2ax)\sqrt{1-a^2x^2}}{c^3(-1+ax)^2} - \frac{12i\text{Log}\left[-i\sqrt{c}(1+2ax) + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax}\right]}{c^{5/2}} + \frac{9i\sqrt{2}\text{Log}\left[\frac{-8ac^2\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2} + 2i\sqrt{2}c^{5/2}(-1-2ax+3a^2x^2)}{9(-1+ax)^2}\right]}{c^{5/2}} \right)$$

Problem 543: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\text{ArcTanh}[ax]}}{\left(c-\frac{c}{ax}\right)^{7/2}} dx$$

Optimal (type 3, 252 leaves, 10 steps):

$$\frac{(1-ax)^{3/2} \sqrt{1+ax}}{4a^2 \left(c - \frac{c}{ax}\right)^{7/2} x} - \frac{15(1-ax)^{5/2} \sqrt{1+ax}}{16a^3 \left(c - \frac{c}{ax}\right)^{7/2} x^2} - \frac{35(1-ax)^{7/2} \sqrt{1+ax}}{16a^4 \left(c - \frac{c}{ax}\right)^{7/2} x^3} -$$

$$\frac{5(1-ax)^{7/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{9/2} \left(c - \frac{c}{ax}\right)^{7/2} x^{7/2}} + \frac{115(1-ax)^{7/2} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1+ax}}\right]}{16\sqrt{2} a^{9/2} \left(c - \frac{c}{ax}\right)^{7/2} x^{7/2}}$$

Result (type 3, 222 leaves):

$$\frac{1}{64a} \left(\frac{4a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2} (35 - 55ax + 16a^2x^2)}{c^4 (-1+ax)^3} - \frac{160i \text{Log}\left[-i\sqrt{c} (1+2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2}}{-1+ax}\right]}{c^{7/2}} + \frac{115i\sqrt{2} \text{Log}\left[\frac{-64ac^3 \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2} + 16i\sqrt{2} c^{7/2} (-1-2ax+3a^2x^2)}{115(-1+ax)^2}\right]}{c^{7/2}} \right)$$

Problem 554: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \text{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{9/2} dx$$

Optimal (type 3, 267 leaves, 9 steps):

$$\frac{5a^4 \left(c - \frac{c}{ax}\right)^{9/2} x^5 (587 - 109ax)}{7(1-ax)^{9/2} \sqrt{1+ax}} + \frac{70a^3 \left(c - \frac{c}{ax}\right)^{9/2} x^4}{(1-ax)^{5/2} \sqrt{1+ax}} - \frac{50a^2 \left(c - \frac{c}{ax}\right)^{9/2} x^3}{7(1-ax)^{3/2} \sqrt{1+ax}} +$$

$$\frac{10a \left(c - \frac{c}{ax}\right)^{9/2} x^2}{7\sqrt{1-ax} \sqrt{1+ax}} - \frac{2 \left(c - \frac{c}{ax}\right)^{9/2} x \sqrt{1-ax}}{7\sqrt{1+ax}} - \frac{15a^{7/2} \left(c - \frac{c}{ax}\right)^{9/2} x^{9/2} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1-ax)^{9/2}}$$

Result (type 3, 152 leaves):

$$c^4 \frac{\sqrt{c - \frac{c}{ax}} (-2 + 20ax - 110a^2x^2 + 720a^3x^3 + 1755a^4x^4 + 7a^5x^5)}{7a^4x^3 \sqrt{1-a^2x^2}} -$$

$$\frac{15i c^{9/2} \text{Log}\left[-i\sqrt{c} (1+2ax) + \frac{2a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2}}{-1+ax}\right]}{2a}$$

Problem 555: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{7/2} dx$$

Optimal (type 3, 225 leaves, 8 steps):

$$\begin{aligned} & -\frac{a^3 \left(c - \frac{c}{a x} \right)^{7/2} x^4 (2525 - 427 a x)}{15 (1 - a x)^{7/2} \sqrt{1 + a x}} - \frac{398 a^2 \left(c - \frac{c}{a x} \right)^{7/2} x^3}{15 (1 - a x)^{3/2} \sqrt{1 + a x}} + \frac{38 a \left(c - \frac{c}{a x} \right)^{7/2} x^2}{15 \sqrt{1 - a x} \sqrt{1 + a x}} - \\ & \frac{2 \left(c - \frac{c}{a x} \right)^{7/2} x \sqrt{1 - a x}}{5 \sqrt{1 + a x}} + \frac{13 a^{5/2} \left(c - \frac{c}{a x} \right)^{7/2} x^{7/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{7/2}} \end{aligned}$$

Result (type 3, 144 leaves):

$$\begin{aligned} & \frac{c^3 \sqrt{c - \frac{c}{a x}} (6 - 62 a x + 548 a^2 x^2 + 1591 a^3 x^3 + 15 a^4 x^4)}{15 a^3 x^2 \sqrt{1 - a^2 x^2}} - \\ & \frac{13 i c^{7/2} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}]}{2 a} \end{aligned}$$

Problem 556: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{5/2} dx$$

Optimal (type 3, 181 leaves, 7 steps):

$$\begin{aligned} & \frac{a^2 \left(c - \frac{c}{a x} \right)^{5/2} x^3 (191 - 25 a x)}{3 (1 - a x)^{5/2} \sqrt{1 + a x}} + \frac{26 a \left(c - \frac{c}{a x} \right)^{5/2} x^2}{3 \sqrt{1 - a x} \sqrt{1 + a x}} - \\ & \frac{2 \left(c - \frac{c}{a x} \right)^{5/2} x \sqrt{1 - a x}}{3 \sqrt{1 + a x}} - \frac{11 a^{3/2} \left(c - \frac{c}{a x} \right)^{5/2} x^{5/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{5/2}} \end{aligned}$$

Result (type 3, 134 leaves):

$$\begin{aligned} & \frac{1}{6 a^2} c^2 \left(\frac{2 \sqrt{c - \frac{c}{a x}} (-2 + 32 a x + 133 a^2 x^2 + 3 a^3 x^3)}{x \sqrt{1 - a^2 x^2}} - \right. \\ & \left. 33 i a \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}] \right) \end{aligned}$$

Problem 557: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a x} \right)^{3/2} dx$$

Optimal (type 3, 133 leaves, 6 steps):

$$-\frac{2 \left(c - \frac{c}{a x} \right)^{3/2} x \sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{a \left(c - \frac{c}{a x} \right)^{3/2} x^2 (23 - a x)}{(1 - a x)^{3/2} \sqrt{1 + a x}} + \frac{9 \sqrt{a} \left(c - \frac{c}{a x} \right)^{3/2} x^{3/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{(1 - a x)^{3/2}}$$

Result (type 3, 119 leaves):

$$\frac{1}{2 a} \left(\frac{2 c \sqrt{c - \frac{c}{a x}} (2 + 19 a x + a^2 x^2)}{\sqrt{1 - a^2 x^2}} - 9 i c^{3/2} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right)$$

Problem 558: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} dx$$

Optimal (type 3, 123 leaves, 6 steps):

$$\frac{8 \sqrt{c - \frac{c}{a x}} x}{\sqrt{1 - a x} \sqrt{1 + a x}} + \frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{\sqrt{1 - a x}} - \frac{7 \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}}$$

Result (type 3, 108 leaves):

$$\frac{\sqrt{c - \frac{c}{a x}} x (9 + a x)}{\sqrt{1 - a^2 x^2}} - \frac{7 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}}{2 a}$$

Problem 559: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[a x]}}{\sqrt{c - \frac{c}{a x}}} dx$$

Optimal (type 3, 127 leaves, 7 steps):

$$-\frac{5 \sqrt{1 - a x}}{a \sqrt{c - \frac{c}{a x}} \sqrt{1 + a x}} - \frac{x (1 - a x)}{\sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2}} + \frac{5 \sqrt{1 - a x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{3/2} \sqrt{c - \frac{c}{a x}} \sqrt{x}}$$

Result (type 3, 140 leaves):

$$\frac{\sqrt{\frac{c(-1+ax)}{ax}} \sqrt{1-a^2x^2} \left(-\frac{1}{c} - \frac{3}{c(-1+ax)} - \frac{2}{c(1+ax)}\right)}{a} - \frac{5i \operatorname{Log}\left[-\frac{i(c+2acx)}{\sqrt{c}} + \frac{2ax\sqrt{\frac{c(-1+ax)}{ax}}\sqrt{1-a^2x^2}}{-1+ax}\right]}{2a\sqrt{c}}$$

Problem 560: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[ax]}}{\left(c - \frac{c}{ax}\right)^{3/2}} dx$$

Optimal (type 3, 131 leaves, 7 steps):

$$-\frac{2(1-ax)^{3/2}}{a\left(c - \frac{c}{ax}\right)^{3/2}\sqrt{1+ax}} + \frac{3(1-ax)^{3/2}\sqrt{1+ax}}{a^2\left(c - \frac{c}{ax}\right)^{3/2}x} - \frac{3(1-ax)^{3/2}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{5/2}\left(c - \frac{c}{ax}\right)^{3/2}x^{3/2}}$$

Result (type 3, 111 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x (3+ax)}{c^2\sqrt{1-a^2x^2}} - \frac{3i \operatorname{Log}\left[-i\sqrt{c}(1+2ax) + \frac{2a\sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2}}{-1+ax}\right]}{2ac^{3/2}}$$

Problem 561: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[ax]}}{\left(c - \frac{c}{ax}\right)^{5/2}} dx$$

Optimal (type 3, 199 leaves, 9 steps):

$$\frac{(1-ax)^{5/2}}{a^2\left(c - \frac{c}{ax}\right)^{5/2}x\sqrt{1+ax}} - \frac{2(1-ax)^{5/2}\sqrt{1+ax}}{a^3\left(c - \frac{c}{ax}\right)^{5/2}x^2} + \frac{(1-ax)^{5/2}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{a^{7/2}\left(c - \frac{c}{ax}\right)^{5/2}x^{5/2}} + \frac{(1-ax)^{5/2}\operatorname{ArcTanh}\left[\frac{\sqrt{2}\sqrt{a}\sqrt{x}}{\sqrt{1+ax}}\right]}{\sqrt{2}a^{7/2}\left(c - \frac{c}{ax}\right)^{5/2}x^{5/2}}$$

Result (type 3, 205 leaves):

$$\frac{1}{4c^3} \left(\frac{4\sqrt{c - \frac{c}{ax}} x (2+ax)}{\sqrt{1-a^2x^2}} - \frac{2i\sqrt{c} \operatorname{Log}\left[-i\sqrt{c}(1+2ax) + \frac{2a\sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2}}{-1+ax}\right]}{a} - \frac{i\sqrt{2}\sqrt{c} \operatorname{Log}\left[\frac{4ac^2\sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2x^2} - i\sqrt{2}c^{5/2}(-1-2ax+3a^2x^2)}{(-1+ax)^2}\right]}{a} \right)$$

Problem 562: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{7/2}} dx$$

Optimal (type 3, 251 leaves, 10 steps):

$$\frac{(1 - a x)^{5/2}}{2 a^2 \left(c - \frac{c}{a x}\right)^{7/2} x \sqrt{1 + a x}} - \frac{(1 - a x)^{7/2}}{4 a^3 \left(c - \frac{c}{a x}\right)^{7/2} x^2 \sqrt{1 + a x}} + \frac{7 (1 - a x)^{7/2} \sqrt{1 + a x}}{4 a^4 \left(c - \frac{c}{a x}\right)^{7/2} x^3} +$$

$$\frac{(1 - a x)^{7/2} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{a^{9/2} \left(c - \frac{c}{a x}\right)^{7/2} x^{7/2}} - \frac{11 (1 - a x)^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{4 \sqrt{2} a^{9/2} \left(c - \frac{c}{a x}\right)^{7/2} x^{7/2}}$$

Result (type 3, 228 leaves):

$$\frac{1}{16 a}$$

$$\left(-\frac{4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (-7 + a x + 4 a^2 x^2)}{c^4 (-1 + a x)^2 (1 + a x)} + \frac{8 i \operatorname{Log}\left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}\right]}{c^{7/2}} - \frac{11 i \sqrt{2} \operatorname{Log}\left[\frac{16 a c^3 \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} - 4 i \sqrt{2} c^{7/2} (-1 - 2 a x + 3 a^2 x^2)}{11 (-1 + a x)^2}\right]}{c^{7/2}} \right)$$

Problem 565: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x^2 dx$$

Optimal (type 3, 179 leaves, 8 steps):

$$-\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{8 a^2 \sqrt{1 - a x}} + \frac{\sqrt{c - \frac{c}{a x}} x^2 \sqrt{1 + a x}}{12 a \sqrt{1 - a x}} +$$

$$\frac{\sqrt{c - \frac{c}{a x}} x^3 \sqrt{1 + a x}}{3 \sqrt{1 - a x}} + \frac{\sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{8 a^{5/2} \sqrt{1 - a x}}$$

Result (type 3, 128 leaves):

$$\frac{1}{48 a^3} \left(-\frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (-3 + 2 a x + 8 a^2 x^2)}{-1 + a x} + 3 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right)$$

Problem 566: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x \, dx$$

Optimal (type 3, 135 leaves, 7 steps):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{4 a \sqrt{1 - a x}} + \frac{\sqrt{c - \frac{c}{a x}} x^2 \sqrt{1 + a x}}{2 \sqrt{1 - a x}} - \frac{\sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{4 a^{3/2} \sqrt{1 - a x}}$$

Result (type 3, 120 leaves):

$$-\frac{1}{8 a^2} \left(\frac{2 a \sqrt{c - \frac{c}{a x}} x (1 + 2 a x) \sqrt{1 - a^2 x^2}}{-1 + a x} + i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right)$$

Problem 567: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} \, dx$$

Optimal (type 3, 85 leaves, 6 steps):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{\sqrt{1 - a x}} + \frac{\sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}}$$

Result (type 3, 111 leaves):

$$-\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} + \frac{i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}}{2 a}$$

Problem 568: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}}}{x} dx$$

Optimal (type 3, 86 leaves, 6 steps):

$$-\frac{2 \sqrt{c - \frac{c}{a x}} \sqrt{1 + a x}}{\sqrt{1 - a x}} + \frac{2 \sqrt{a} \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{1 - a x}}$$

Result (type 3, 105 leaves):

$$\frac{2 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2}}{-1 + a x} + i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}$$

Problem 582: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x^3 dx$$

Optimal (type 3, 292 leaves, 11 steps):

$$\begin{aligned} & -\frac{107 \sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{64 a^3 \sqrt{1 - a x}} - \frac{21 \sqrt{c - \frac{c}{a x}} x (1 + a x)^{3/2}}{32 a^3 \sqrt{1 - a x}} \\ & - \frac{11 \sqrt{c - \frac{c}{a x}} x^2 (1 + a x)^{3/2}}{24 a^2 \sqrt{1 - a x}} - \frac{\sqrt{c - \frac{c}{a x}} x^3 (1 + a x)^{3/2}}{4 a \sqrt{1 - a x}} \\ & + \frac{363 \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{64 a^{7/2} \sqrt{1 - a x}} + \frac{4 \sqrt{2} \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{a^{7/2} \sqrt{1 - a x}} \end{aligned}$$

Result (type 3, 231 leaves):

$$\frac{1}{384 a^4} \left(\frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (447 + 214 a x + 136 a^2 x^2 + 48 a^3 x^3)}{-1 + a x} - \right.$$

$$1089 i \sqrt{c} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right] +$$

$$\left. 768 i \sqrt{2} \sqrt{c} \operatorname{Log} \left[\frac{i a^4 \left(4 i a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} + \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2) \right)}{8 c (-1 + a x)^2} \right] \right)$$

Problem 583: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x^2 dx$$

Optimal (type 3, 248 leaves, 10 steps):

$$\frac{13 \sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{8 a^2 \sqrt{1 - a x}} - \frac{3 \sqrt{c - \frac{c}{a x}} x (1 + a x)^{3/2}}{4 a^2 \sqrt{1 - a x}} - \frac{\sqrt{c - \frac{c}{a x}} x^2 (1 + a x)^{3/2}}{3 a \sqrt{1 - a x}}$$

$$+ \frac{45 \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{8 a^{5/2} \sqrt{1 - a x}} + \frac{4 \sqrt{2} \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{a^{5/2} \sqrt{1 - a x}}$$

Result (type 3, 223 leaves):

$$\frac{1}{48 a^3} \left(\frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (57 + 26 a x + 8 a^2 x^2)}{-1 + a x} - \right.$$

$$135 i \sqrt{c} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right] +$$

$$\left. 96 i \sqrt{2} \sqrt{c} \operatorname{Log} \left[\frac{i a^3 \left(4 i a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} + \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2) \right)}{8 c (-1 + a x)^2} \right] \right)$$

Problem 584: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x \, dx$$

Optimal (type 3, 204 leaves, 9 steps):

$$\begin{aligned} & -\frac{7 \sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{4 a \sqrt{1 - a x}} - \frac{\sqrt{c - \frac{c}{a x}} x (1 + a x)^{3/2}}{2 a \sqrt{1 - a x}} \\ & + \frac{23 \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{4 a^{3/2} \sqrt{1 - a x}} + \frac{4 \sqrt{2} \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{a^{3/2} \sqrt{1 - a x}} \end{aligned}$$

Result (type 3, 211 leaves):

$$\begin{aligned} & \frac{1}{8 a^2} \\ & \left(\frac{2 a \sqrt{c - \frac{c}{a x}} x (9 + 2 a x) \sqrt{1 - a^2 x^2}}{-1 + a x} - 23 i \sqrt{c} \operatorname{Log}\left[-i \sqrt{c} (1 + 2 a x)\right] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right) + \\ & \left. 16 i \sqrt{2} \sqrt{c} \operatorname{Log}\left[\frac{-4 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{8 c (-1 + a x)^2}\right] \right) \end{aligned}$$

Problem 585: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} \, dx$$

Optimal (type 3, 155 leaves, 8 steps):

$$\begin{aligned} & -\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{\sqrt{1 - a x}} - \frac{5 \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}} + \frac{4 \sqrt{2} \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + a x}}\right]}{\sqrt{a} \sqrt{1 - a x}} \end{aligned}$$

Result (type 3, 204 leaves):

$$\frac{\sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} - \frac{5 i \sqrt{c} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right]}{2 a} +$$

$$\frac{2 i \sqrt{2} \sqrt{c} \operatorname{Log} \left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{8 c (-1 + ax)^2} \right]}{a}$$

Problem 586: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x} dx$$

Optimal (type 3, 154 leaves, 8 steps):

$$\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{\sqrt{1 - ax}} - \frac{2 \sqrt{a} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{1 - ax}} +$$

$$\frac{4 \sqrt{2} \sqrt{a} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}} \right]}{\sqrt{1 - ax}}$$

Result (type 3, 196 leaves):

$$\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2}}{-1 + ax} - i \sqrt{c} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + ax} \right] +$$

$$2 i \sqrt{2} \sqrt{c} \operatorname{Log} \left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{8 c (-1 + ax)^2} \right]$$

Problem 587: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x^2} dx$$

Optimal (type 3, 147 leaves, 6 steps):

$$\frac{4 a \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{\sqrt{1 - ax}} - \frac{2 \sqrt{c - \frac{c}{ax}} (1 + ax)^{3/2}}{3 x \sqrt{1 - ax}} + \frac{4 \sqrt{2} a^{3/2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh} \left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}} \right]}{\sqrt{1 - ax}}$$

Result (type 3, 145 leaves):

$$\frac{2 \sqrt{c - \frac{c}{ax}} (1 + 7ax) \sqrt{1 - a^2 x^2}}{3x(-1 + ax)} +$$

$$2i\sqrt{2} a \sqrt{c} \operatorname{Log}\left[\frac{-4a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i\sqrt{2} \sqrt{c} (-1 - 2ax + 3a^2 x^2)}{8ac(-1 + ax)^2}\right]$$

Problem 588: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x^3} dx$$

Optimal (type 3, 191 leaves, 7 steps):

$$\frac{4a^2 \sqrt{c - \frac{c}{ax}} \sqrt{1 + ax}}{\sqrt{1 - ax}} - \frac{2a \sqrt{c - \frac{c}{ax}} (1 + ax)^{3/2}}{3x \sqrt{1 - ax}} -$$

$$\frac{2 \sqrt{c - \frac{c}{ax}} (1 + ax)^{5/2}}{5x^2 \sqrt{1 - ax}} + \frac{4\sqrt{2} a^{5/2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1 + ax}}\right]}{\sqrt{1 - ax}}$$

Result (type 3, 155 leaves):

$$\frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1 - a^2 x^2} (3 + 11ax + 38a^2 x^2)}{15x^2(-1 + ax)} +$$

$$2i\sqrt{2} a^2 \sqrt{c} \operatorname{Log}\left[\frac{-4a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2} + i\sqrt{2} \sqrt{c} (-1 - 2ax + 3a^2 x^2)}{8a^2 c (-1 + ax)^2}\right]$$

Problem 589: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x^4} dx$$

Optimal (type 3, 237 leaves, 9 steps):

$$\begin{aligned}
& - \frac{104 a^3 \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{21 \sqrt{1-ax}} - \frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{7 x^3 \sqrt{1-ax}} - \frac{6 a \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{7 x^2 \sqrt{1-ax}} - \\
& \frac{32 a^2 \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{21 x \sqrt{1-ax}} + \frac{4 \sqrt{2} a^{7/2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1+ax}}\right]}{\sqrt{1-ax}}
\end{aligned}$$

Result (type 3, 163 leaves):

$$\begin{aligned}
& \frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1-a^2 x^2} (3 + 9 a x + 16 a^2 x^2 + 52 a^3 x^3)}{21 x^3 (-1 + a x)} + \\
& 2 i \sqrt{2} a^3 \sqrt{c} \operatorname{Log}\left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{8 a^3 c (-1 + a x)^2}\right]
\end{aligned}$$

Problem 590: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}}}{x^5} dx$$

Optimal (type 3, 281 leaves, 10 steps):

$$\begin{aligned}
& - \frac{1576 a^4 \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{315 \sqrt{1-ax}} - \frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{9 x^4 \sqrt{1-ax}} - \frac{38 a \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{63 x^3 \sqrt{1-ax}} - \\
& \frac{92 a^2 \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{105 x^2 \sqrt{1-ax}} - \frac{472 a^3 \sqrt{c - \frac{c}{ax}} \sqrt{1+ax}}{315 x \sqrt{1-ax}} + \frac{4 \sqrt{2} a^{9/2} \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{2} \sqrt{a} \sqrt{x}}{\sqrt{1+ax}}\right]}{\sqrt{1-ax}}
\end{aligned}$$

Result (type 3, 171 leaves):

$$\begin{aligned}
& \frac{2 \sqrt{c - \frac{c}{ax}} \sqrt{1-a^2 x^2} (35 + 95 a x + 138 a^2 x^2 + 236 a^3 x^3 + 788 a^4 x^4)}{315 x^4 (-1 + a x)} + \\
& 2 i \sqrt{2} a^4 \sqrt{c} \operatorname{Log}\left[\frac{-4 a \sqrt{c - \frac{c}{ax}} x \sqrt{1-a^2 x^2} + i \sqrt{2} \sqrt{c} (-1 - 2 a x + 3 a^2 x^2)}{8 a^4 c (-1 + a x)^2}\right]
\end{aligned}$$

Problem 592: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x^2 dx$$

Optimal (type 3, 182 leaves, 8 steps):

$$\frac{11 \sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{8 a^2 \sqrt{1 - a x}} + \frac{11 \sqrt{c - \frac{c}{a x}} x^2 \sqrt{1 + a x}}{12 a \sqrt{1 - a x}}$$

$$\frac{\sqrt{c - \frac{c}{a x}} x^3 \sqrt{1 - a^2 x^2}}{3 (1 - a x)} + \frac{11 \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{8 a^{5/2} \sqrt{1 - a x}}$$

Result (type 3, 128 leaves):

$$\frac{1}{48 a^3} \left(\frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2} (33 - 22 a x + 8 a^2 x^2)}{-1 + a x} + \right.$$

$$\left. 33 i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}] \right)$$

Problem 593: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x dx$$

Optimal (type 3, 138 leaves, 7 steps):

$$\frac{7 \sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{4 a \sqrt{1 - a x}} - \frac{\sqrt{c - \frac{c}{a x}} x^2 \sqrt{1 - a^2 x^2}}{2 (1 - a x)} - \frac{7 \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{4 a^{3/2} \sqrt{1 - a x}}$$

Result (type 3, 120 leaves):

$$\frac{1}{8 a^2} \left(\frac{2 a \sqrt{c - \frac{c}{a x}} x (-7 + 2 a x) \sqrt{1 - a^2 x^2}}{-1 + a x} - 7 i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}] \right)$$

Problem 594: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} dx$$

Optimal (type 3, 90 leaves, 6 steps):

$$-\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{1 - a x} + \frac{3 \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}}$$

Result (type 3, 110 leaves):

$$\frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} + \frac{3 i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}}{2 a}$$

Problem 595: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-\text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}}}{x} dx$$

Optimal (type 3, 89 leaves, 6 steps):

$$-\frac{2 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2}}{1 - a x} - \frac{2 \sqrt{a} \sqrt{c - \frac{c}{a x}} \sqrt{x} \text{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{1 - a x}}$$

Result (type 3, 105 leaves):

$$\frac{2 \sqrt{c - \frac{c}{a x}} \sqrt{1 - a^2 x^2}}{-1 + a x} - i \sqrt{c} \text{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}$$

Problem 608: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \text{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x^3 dx$$

Optimal (type 3, 262 leaves, 9 steps):

$$\frac{8 \sqrt{c - \frac{c}{ax}} x^4}{\sqrt{1-ax} \sqrt{1+ax}} - \frac{1115 \sqrt{c - \frac{c}{ax}} x \sqrt{1+ax}}{64 a^3 \sqrt{1-ax}} + \frac{1115 \sqrt{c - \frac{c}{ax}} x^2 \sqrt{1+ax}}{96 a^2 \sqrt{1-ax}} -$$

$$\frac{223 \sqrt{c - \frac{c}{ax}} x^3 \sqrt{1+ax}}{24 a \sqrt{1-ax}} + \frac{\sqrt{c - \frac{c}{ax}} x^4 \sqrt{1+ax}}{4 \sqrt{1-ax}} + \frac{1115 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{64 a^{7/2} \sqrt{1-ax}}$$

Result (type 3, 137 leaves):

$$\frac{1}{384 a^4} \left(\frac{2 a \sqrt{c - \frac{c}{ax}} x (-3345 - 1115 a x + 446 a^2 x^2 - 200 a^3 x^3 + 48 a^4 x^4)}{\sqrt{1 - a^2 x^2}} + \right.$$

$$\left. 3345 i \sqrt{c} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right] \right)$$

Problem 609: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} x^2 dx$$

Optimal (type 3, 218 leaves, 8 steps):

$$\frac{8 \sqrt{c - \frac{c}{ax}} x^3}{\sqrt{1-ax} \sqrt{1+ax}} + \frac{119 \sqrt{c - \frac{c}{ax}} x \sqrt{1+ax}}{8 a^2 \sqrt{1-ax}} - \frac{119 \sqrt{c - \frac{c}{ax}} x^2 \sqrt{1+ax}}{12 a \sqrt{1-ax}} +$$

$$\frac{\sqrt{c - \frac{c}{ax}} x^3 \sqrt{1+ax}}{3 \sqrt{1-ax}} - \frac{119 \sqrt{c - \frac{c}{ax}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{8 a^{5/2} \sqrt{1-ax}}$$

Result (type 3, 129 leaves):

$$\frac{1}{48 a^3} \left(\frac{2 a \sqrt{c - \frac{c}{ax}} x (357 + 119 a x - 38 a^2 x^2 + 8 a^3 x^3)}{\sqrt{1 - a^2 x^2}} - \right.$$

$$\left. 357 i \sqrt{c} \operatorname{Log} \left[-i \sqrt{c} (1 + 2 a x) + \frac{2 a \sqrt{c - \frac{c}{ax}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right] \right)$$

Problem 610: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} x dx$$

Optimal (type 3, 174 leaves, 7 steps):

$$\frac{8 \sqrt{c - \frac{c}{a x}} x^2}{\sqrt{1 - a x} \sqrt{1 + a x}} - \frac{47 \sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{4 a \sqrt{1 - a x}} + \frac{\sqrt{c - \frac{c}{a x}} x^2 \sqrt{1 + a x}}{2 \sqrt{1 - a x}} + \frac{47 \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{4 a^{3/2} \sqrt{1 - a x}}$$

Result (type 3, 121 leaves):

$$\frac{1}{8 a^2} \left(\frac{2 a \sqrt{c - \frac{c}{a x}} x (-47 - 13 a x + 2 a^2 x^2)}{\sqrt{1 - a^2 x^2}} + 47 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x} \right)$$

Problem 611: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}} dx$$

Optimal (type 3, 123 leaves, 6 steps):

$$\frac{8 \sqrt{c - \frac{c}{a x}} x}{\sqrt{1 - a x} \sqrt{1 + a x}} + \frac{\sqrt{c - \frac{c}{a x}} x \sqrt{1 + a x}}{\sqrt{1 - a x}} - \frac{7 \sqrt{c - \frac{c}{a x}} \sqrt{x} \operatorname{ArcSinh}[\sqrt{a} \sqrt{x}]}{\sqrt{a} \sqrt{1 - a x}}$$

Result (type 3, 108 leaves):

$$\frac{\sqrt{c - \frac{c}{a x}} x (9 + a x)}{\sqrt{1 - a^2 x^2}} - \frac{7 i \sqrt{c} \operatorname{Log}[-i \sqrt{c} (1 + 2 a x)] + \frac{2 a \sqrt{c - \frac{c}{a x}} x \sqrt{1 - a^2 x^2}}{-1 + a x}}{2 a}$$

Problem 612: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[a x]} \sqrt{c - \frac{c}{a x}}}{x} dx$$

Optimal (type 3, 124 leaves, 6 steps):

$$-\frac{2\sqrt{c-\frac{c}{ax}}}{\sqrt{1-ax}\sqrt{1+ax}} - \frac{10a\sqrt{c-\frac{c}{ax}}x}{\sqrt{1-ax}\sqrt{1+ax}} + \frac{2\sqrt{a}\sqrt{c-\frac{c}{ax}}\sqrt{x}\operatorname{ArcSinh}[\sqrt{a}\sqrt{x}]}{\sqrt{1-ax}}$$

Result (type 3, 104 leaves):

$$-\frac{2\sqrt{c-\frac{c}{ax}}(1+5ax)}{\sqrt{1-a^2x^2}} + i\sqrt{c}\operatorname{Log}[-i\sqrt{c}(1+2ax)] + \frac{2a\sqrt{c-\frac{c}{ax}}x\sqrt{1-a^2x^2}}{-1+ax}$$

Problem 617: Unable to integrate problem.

$$\int e^{n\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 6, 64 leaves, 3 steps):

$$\frac{1}{1-p} \left(c - \frac{c}{ax}\right)^p x (1-ax)^{-p} \operatorname{AppellF1}\left[1-p, \frac{1}{2}(n-2p), -\frac{n}{2}, 2-p, ax, -ax\right]$$

Result (type 8, 24 leaves):

$$\int e^{n\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 618: Unable to integrate problem.

$$\int e^{-2p\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 6, 54 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{ax}\right)^p x (1-ax)^{-p} \operatorname{AppellF1}\left[1-p, -2p, p, 2-p, ax, -ax\right]}{1-p}$$

Result (type 8, 25 leaves):

$$\int e^{-2p\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 619: Unable to integrate problem.

$$\int e^{2p\operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Optimal (type 5, 50 leaves, 3 steps):

$$\frac{\left(c - \frac{c}{ax}\right)^p x (1-ax)^{-p} \operatorname{Hypergeometric2F1}\left[1-p, -p, 2-p, -ax\right]}{1-p}$$

Result (type 8, 25 leaves):

$$\int e^{2p \operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^p dx$$

Problem 624: Attempted integration timed out after 120 seconds.

$$\int e^{n \operatorname{ArcTanh}[ax]} \left(c - \frac{c}{ax}\right)^{3/2} dx$$

Optimal (type 6, 54 leaves, 3 steps):

$$\frac{2 \left(c - \frac{c}{ax}\right)^{3/2} x \operatorname{AppellF1}\left[-\frac{1}{2}, \frac{1}{2}(-3+n), -\frac{n}{2}, \frac{1}{2}, ax, -ax\right]}{(1-ax)^{3/2}}$$

Result (type 1, 1 leaves):

???

Problem 625: Attempted integration timed out after 120 seconds.

$$\int e^{n \operatorname{ArcTanh}[ax]} \sqrt{c - \frac{c}{ax}} dx$$

Optimal (type 6, 54 leaves, 3 steps):

$$\frac{2 \sqrt{c - \frac{c}{ax}} x \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}(-1+n), -\frac{n}{2}, \frac{3}{2}, ax, -ax\right]}{\sqrt{1-ax}}$$

Result (type 1, 1 leaves):

???

Problem 626: Attempted integration timed out after 120 seconds.

$$\int \frac{e^{n \operatorname{ArcTanh}[ax]}}{\sqrt{c - \frac{c}{ax}}} dx$$

Optimal (type 6, 56 leaves, 3 steps):

$$\frac{2x \sqrt{1-ax} \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1+n}{2}, -\frac{n}{2}, \frac{5}{2}, ax, -ax\right]}{3 \sqrt{c - \frac{c}{ax}}}$$

Result (type 1, 1 leaves):

???

Problem 627: Attempted integration timed out after 120 seconds.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]}}{\left(c - \frac{c}{a x}\right)^{3/2}} dx$$

Optimal (type 6, 56 leaves, 3 steps):

$$\frac{2 x (1 - a x)^{3/2} \operatorname{AppellF1}\left[\frac{5}{2}, \frac{3+n}{2}, -\frac{n}{2}, \frac{7}{2}, a x, -a x\right]}{5 \left(c - \frac{c}{a x}\right)^{3/2}}$$

Result (type 1, 1 leaves):

???

Problem 789: Unable to integrate problem.

$$\int e^{-2 p \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 5, 53 leaves, 3 steps):

$$\frac{1}{1 - 2 p} \left(c - \frac{c}{a^2 x^2}\right)^p x (1 - a^2 x^2)^{-p} \operatorname{Hypergeometric2F1}[1 - 2 p, -2 p, 2 - 2 p, a x]$$

Result (type 8, 25 leaves):

$$\int e^{-2 p \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Problem 790: Unable to integrate problem.

$$\int e^{2 p \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 5, 54 leaves, 3 steps):

$$\frac{1}{1 - 2 p} \left(c - \frac{c}{a^2 x^2}\right)^p x (1 - a^2 x^2)^{-p} \operatorname{Hypergeometric2F1}[1 - 2 p, -2 p, 2 - 2 p, -a x]$$

Result (type 8, 25 leaves):

$$\int e^{2 p \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Problem 800: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 6, 72 leaves, 3 steps):

$$\frac{1}{1-2p} \left(c - \frac{c}{a^2 x^2} \right)^p x (1-a^2 x^2)^{-p} \text{AppellF1} \left[1-2p, \frac{1}{2} (n-2p), -\frac{n}{2}-p, 2-2p, ax, -ax \right]$$

Result (type 8, 24 leaves):

$$\int e^{n \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Problem 801: Result unnecessarily involves higher level functions.

$$\int e^{4 \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 339 leaves, 13 steps):

$$\begin{aligned} & \frac{2a \left(c - \frac{c}{a^2 x^2} \right)^p x^2}{(1-p)(1-ax)(1+ax)} + \frac{1}{1-2p} \\ & \left(c - \frac{c}{a^2 x^2} \right)^p x (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (1-2p), 2-p, \frac{1}{2} (3-2p), a^2 x^2 \right] + \\ & \frac{1}{3-2p} 6a^2 \left(c - \frac{c}{a^2 x^2} \right)^p x^3 (1-ax)^{-p} (1+ax)^{-p} \\ & \text{Hypergeometric2F1} \left[\frac{1}{2} (3-2p), 2-p, \frac{1}{2} (5-2p), a^2 x^2 \right] + \frac{1}{5-2p} \\ & a^4 \left(c - \frac{c}{a^2 x^2} \right)^p x^5 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (5-2p), 2-p, \frac{1}{2} (7-2p), a^2 x^2 \right] + \\ & \frac{1}{2-p} 2a^3 \left(c - \frac{c}{a^2 x^2} \right)^p x^4 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1} [2-p, 2-p, 3-p, a^2 x^2] \end{aligned}$$

Result (type 6, 319 leaves):

$$\begin{aligned} & \left(c - \frac{c}{a^2 x^2} \right)^p x \\ & \left(\frac{1}{1-2p} \left(4(-1+ax)^p \left(\frac{1-ax}{1+ax} \right)^{-p} (1+ax)^{-1+p} (-1+a^2 x^2)^{-p} \text{Hypergeometric2F1} [1-2p, 2-p, \right. \right. \\ & \quad \left. \left. 2-2p, \frac{2ax}{1+ax}] + (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2}-p, -p, \frac{3}{2}-p, a^2 x^2 \right] \right) - \right. \\ & \quad \left(8(-1+p)(1-ax)^{-p} (-1+ax)^{-1+p} (1-a^2 x^2)^p (-1+a^2 x^2)^{-p} \right. \\ & \quad \left. \text{AppellF1} [1-2p, 1-p, -p, 2-2p, ax, -ax] \right) / \\ & \quad \left((-1+2p)(2(-1+p) \text{AppellF1} [1-2p, 1-p, -p, 2-2p, ax, -ax] + \right. \\ & \quad \left. ax((-1+p) \text{AppellF1} [2-2p, 2-p, -p, 3-2p, ax, -ax] - \right. \\ & \quad \left. \left. p \text{HypergeometricPFQ} [\{1-p, 1-p\}, \{2-p\}, a^2 x^2] \right) \right) \end{aligned}$$

Problem 802: Unable to integrate problem.

$$\int e^{3 \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 217 leaves, 7 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2}\right)^p x}{(1-2p)\sqrt{1-a^2 x^2}} - \frac{a\left(c - \frac{c}{a^2 x^2}\right)^p x^2}{\sqrt{1-a^2 x^2}} + \frac{1}{3-2p}$$

$$3a^2\left(c - \frac{c}{a^2 x^2}\right)^p x^3 (1-a^2 x^2)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}(3-2p), \frac{3}{2}-p, \frac{1}{2}(5-2p), a^2 x^2\right] +$$

$$\frac{1}{2(1-p)} a(5-2p)\left(c - \frac{c}{a^2 x^2}\right)^p x^2 (1-a^2 x^2)^{-p} \text{Hypergeometric2F1}\left[1-p, \frac{3}{2}-p, 2-p, a^2 x^2\right]$$

Result (type 8, 24 leaves):

$$\int e^{3 \text{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Problem 803: Result unnecessarily involves higher level functions.

$$\int e^{2 \text{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 5, 217 leaves, 10 steps):

$$\frac{1}{1-2p}\left(c - \frac{c}{a^2 x^2}\right)^p x (1-ax)^{-p} (1+ax)^{-p}$$

$$\text{Hypergeometric2F1}\left[\frac{1}{2}(1-2p), 1-p, \frac{1}{2}(3-2p), a^2 x^2\right] + \frac{1}{3-2p}$$

$$a^2\left(c - \frac{c}{a^2 x^2}\right)^p x^3 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}(3-2p), 1-p, \frac{1}{2}(5-2p), a^2 x^2\right] +$$

$$\frac{1}{1-p} a\left(c - \frac{c}{a^2 x^2}\right)^p x^2 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1}\left[1-p, 1-p, 2-p, a^2 x^2\right]$$

Result (type 6, 235 leaves):

$$\frac{1}{-1+2p}\left(c - \frac{c}{a^2 x^2}\right)^p x (1-a^2 x^2)^{-p}$$

$$\left(\text{Hypergeometric2F1}\left[\frac{1}{2}-p, -p, \frac{3}{2}-p, a^2 x^2\right] + \left(4(-1+p)(1-ax)^{-p}(-1+ax)^{-1+p}\right.\right.$$

$$\left.\left.(1-a^2 x^2)^{2p}(-1+a^2 x^2)^{-p} \text{AppellF1}\left[1-2p, 1-p, -p, 2-2p, ax, -ax\right]\right) /$$

$$\left(2(-1+p) \text{AppellF1}\left[1-2p, 1-p, -p, 2-2p, ax, -ax\right] + ax\left((-1+p) \text{AppellF1}\left[2-2p, 2-p, -p, 3-2p, ax, -ax\right] - p \text{HypergeometricPFQ}\left[\{1-p, 1-p\}, \{2-p\}, a^2 x^2\right]\right)\right)$$

Problem 804: Unable to integrate problem.

$$\int e^{\text{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2}\right)^p dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{1}{1-2p} \left(c - \frac{c}{a^2 x^2} \right)^p x (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (1-2p), \frac{1}{2} - p, \frac{1}{2} (3-2p), a^2 x^2 \right] +$$

$$\frac{1}{2(1-p)} a \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} - p, 1-p, 2-p, a^2 x^2 \right]$$

Result (type 8, 22 leaves):

$$\int e^{\text{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Problem 805: Unable to integrate problem.

$$\int e^{-\text{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\frac{1}{1-2p} \left(c - \frac{c}{a^2 x^2} \right)^p x (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (1-2p), \frac{1}{2} - p, \frac{1}{2} (3-2p), a^2 x^2 \right] -$$

$$\frac{1}{2(1-p)} a \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1-a^2 x^2)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} - p, 1-p, 2-p, a^2 x^2 \right]$$

Result (type 8, 24 leaves):

$$\int e^{-\text{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Problem 806: Result unnecessarily involves higher level functions.

$$\int e^{-2 \text{ArcTanh}[a x]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 218 leaves, 10 steps):

$$\frac{1}{1-2p} \left(c - \frac{c}{a^2 x^2} \right)^p x (1-ax)^{-p} (1+ax)^{-p}$$

$$\text{Hypergeometric2F1} \left[\frac{1}{2} (1-2p), 1-p, \frac{1}{2} (3-2p), a^2 x^2 \right] + \frac{1}{3-2p}$$

$$a^2 \left(c - \frac{c}{a^2 x^2} \right)^p x^3 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1} \left[\frac{1}{2} (3-2p), 1-p, \frac{1}{2} (5-2p), a^2 x^2 \right] -$$

$$\frac{1}{1-p} a \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1-ax)^{-p} (1+ax)^{-p} \text{Hypergeometric2F1} [1-p, 1-p, 2-p, a^2 x^2]$$

Result (type 6, 226 leaves):

$$\left(c - \frac{c}{a^2 x^2} \right)^p x \left(\frac{(1 - a^2 x^2)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2} - p, -p, \frac{3}{2} - p, a^2 x^2\right]}{-1 + 2p} + (4(-1 + p)(-1 + ax)^p(1 + ax)^{-1+p} \right. \\ \left. (-1 + a^2 x^2)^{-p} \text{AppellF1}[1 - 2p, -p, 1 - p, 2 - 2p, ax, -ax] \right) / \\ \left((1 - 2p)(2(-1 + p) \text{AppellF1}[1 - 2p, -p, 1 - p, 2 - 2p, ax, -ax] + ax(-(-1 + p) \text{AppellF1}[2 - 2p, -p, 2 - p, 3 - 2p, ax, -ax] + \right. \\ \left. p \text{HypergeometricPFQ}[\{1 - p, 1 - p\}, \{2 - p\}, a^2 x^2]) \right)$$

Problem 807: Unable to integrate problem.

$$\int e^{-3 \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Optimal (type 5, 216 leaves, 7 steps):

$$\frac{\left(c - \frac{c}{a^2 x^2} \right)^p x}{(1 - 2p) \sqrt{1 - a^2 x^2}} + \frac{a \left(c - \frac{c}{a^2 x^2} \right)^p x^2}{\sqrt{1 - a^2 x^2}} + \frac{1}{3 - 2p} \\ 3a^2 \left(c - \frac{c}{a^2 x^2} \right)^p x^3 (1 - a^2 x^2)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}(3 - 2p), \frac{3}{2} - p, \frac{1}{2}(5 - 2p), a^2 x^2\right] - \\ \frac{1}{2(1 - p)} a (5 - 2p) \left(c - \frac{c}{a^2 x^2} \right)^p x^2 (1 - a^2 x^2)^{-p} \text{Hypergeometric2F1}\left[1 - p, \frac{3}{2} - p, 2 - p, a^2 x^2\right]$$

Result (type 8, 24 leaves):

$$\int e^{-3 \text{ArcTanh}[ax]} \left(c - \frac{c}{a^2 x^2} \right)^p dx$$

Problem 808: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[x]} x \sqrt{1 + x} \text{Sin}[x] dx$$

Optimal (type 4, 240 leaves, 16 steps):

$$\begin{aligned}
& 3\sqrt{1-x} \cos[x] - (1-x)^{3/2} \cos[x] - 3\sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] - \\
& \frac{3}{2}\sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] + 2\sqrt{2\pi} \cos[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] + \\
& \frac{3}{2}\sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] \sin[1] - 2\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] \sin[1] - \\
& 3\sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] \sin[1] - \frac{3}{2}\sqrt{1-x} \sin[x]
\end{aligned}$$

Result (type 4, 185 leaves):

$$\begin{aligned}
& \frac{1}{8\sqrt{1-x^2}} \operatorname{Im} \sqrt{1+x} \left((-11-i) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \operatorname{Erfi}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\cos[1] + i \sin[1]) + \right. \\
& \left((-4-3i) + (2+3i)x + 2x^2 \right) (2i \cos[x] - 2 \sin[x]) + \\
& \left. \left(2((-3-4i) + (3+2i)x + 2ix^2) (\cos[1] + i \sin[1]) - (1+11i) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \right. \right. \\
& \left. \left. \operatorname{Erf}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\cos[x] + i \sin[x]) \right) (\cos[1+x] - i \sin[1+x]) \right)
\end{aligned}$$

Problem 809: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} \sqrt{1+x} \sin[x] dx$$

Optimal (type 4, 141 leaves, 11 steps):

$$\begin{aligned}
& \sqrt{1-x} \cos[x] - \sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] + 2\sqrt{2\pi} \cos[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] - \\
& 2\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] \sin[1] - \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}}\sqrt{1-x}\right] \sin[1]
\end{aligned}$$

Result (type 4, 129 leaves):

$$\begin{aligned}
& \frac{1}{4} \left((1+4i) (-1)^{3/4} e^{-i\sqrt{\pi}} \operatorname{Erfi}\left[(-1)^{1/4}\sqrt{1-x}\right] + \frac{1}{\sqrt{-1+x}\sqrt{1+x}} \right. \\
& \left. e^{-ix}\sqrt{1-x^2} \left(2(1+e^{2ix})\sqrt{-1+x} + (1-4i) (-1)^{3/4} e^{i(1+x)} \sqrt{\pi} \operatorname{Erfi}\left[(-1)^{1/4}\sqrt{-1+x}\right] \right) \right)
\end{aligned}$$

Problem 810: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[x]} \sqrt{1-x} x \text{Sin}[x] dx$$

Optimal (type 4, 163 leaves, 13 steps):

$$\begin{aligned} & \sqrt{1+x} \text{Cos}[x] - (1+x)^{3/2} \text{Cos}[x] - \sqrt{\frac{\pi}{2}} \text{Cos}[1] \text{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] - \\ & \frac{3}{2} \sqrt{\frac{\pi}{2}} \text{Cos}[1] \text{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] + \frac{3}{2} \sqrt{\frac{\pi}{2}} \text{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] \text{Sin}[1] - \\ & \sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] \text{Sin}[1] + \frac{3}{2} \sqrt{1+x} \text{Sin}[x] \end{aligned}$$

Result (type 4, 168 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{1-x^2}} \left(\frac{1}{16} + \frac{i}{16} \right) e^{-i(1+x)} \sqrt{1-x} \\ & \left((-3-2i) e^{ix} \sqrt{2\pi} \sqrt{-1-x} \text{Erf}\left[\frac{(1+i)\sqrt{-1-x}}{\sqrt{2}}\right] + e^i \left((2+2i) (3+e^{2ix}(-3+2ix) + 2ix) \right. \right. \\ & \left. \left. (1+x) + (3-2i) e^{i(1+x)} \sqrt{2\pi} \sqrt{-1-x} \text{Erfi}\left[\frac{(1+i)\sqrt{-1-x}}{\sqrt{2}}\right] \right) \right) \end{aligned}$$

Problem 811: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[x]} \sqrt{1-x} \text{Sin}[x] dx$$

Optimal (type 4, 72 leaves, 7 steps):

$$-\sqrt{1+x} \text{Cos}[x] + \sqrt{\frac{\pi}{2}} \text{Cos}[1] \text{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] + \sqrt{\frac{\pi}{2}} \text{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] \text{Sin}[1]$$

Result (type 4, 138 leaves):

$$\begin{aligned} & - \left(\left(e^{-i(1+x)} \sqrt{1-x^2} \left(2 e^i (1+e^{2ix}) \sqrt{-1-x} + (-1)^{3/4} e^{i(2+x)} \sqrt{\pi} \text{Erfi}\left[(-1)^{1/4} \sqrt{-1-x}\right] + \right. \right. \right. \\ & \left. \left. (-1)^{1/4} e^{ix} \sqrt{\pi} \text{Erfi}\left[(-1)^{3/4} \sqrt{-1-x}\right] \right) \right) / \left(4 \sqrt{-1-x} \sqrt{1-x} \right) \end{aligned}$$

Problem 812: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\text{ArcTanh}[x]} x (1+x)^{3/2} \text{Sin}[x] dx$$

Optimal (type 4, 335 leaves, 22 steps):

$$\begin{aligned} & \frac{17}{4} \sqrt{1-x} \operatorname{Cos}[x] - 5 (1-x)^{3/2} \operatorname{Cos}[x] + \\ & (1-x)^{5/2} \operatorname{Cos}[x] + \frac{15}{4} \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \\ & 4 \sqrt{2\pi} \operatorname{Cos}[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \frac{15}{2} \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + \\ & 4 \sqrt{2\pi} \operatorname{Cos}[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + \frac{15}{2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - \\ & 4 \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] + \frac{15}{4} \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - \\ & 4 \sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - \frac{15}{2} \sqrt{1-x} \operatorname{Sin}[x] + \frac{5}{2} (1-x)^{3/2} \operatorname{Sin}[x] \end{aligned}$$

Result (type 4, 201 leaves):

$$\begin{aligned} & \frac{1}{\sqrt{1-x^2}} \left(\frac{1}{32} + \frac{i}{32} \right) \sqrt{1+x} \left((-2-17i) \sqrt{2\pi} \sqrt{-1+x} \operatorname{Erfi}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\operatorname{Cos}[1] + i \operatorname{Sin}[1]) - \right. \\ & (2-2i) \left((-1-20i) - (11-10i)x + (8+10i)x^2 + 4x^3 \right) (\operatorname{Cos}[x] + i \operatorname{Sin}[x]) - \\ & \left. (1+i) \left(2 \left((-1+20i) - (11+10i)x + (8-10i)x^2 + 4x^3 \right) (-i \operatorname{Cos}[1] + \operatorname{Sin}[1]) + (15+19i) \right. \right. \\ & \left. \left. \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \operatorname{Erf}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\operatorname{Cos}[x] + i \operatorname{Sin}[x]) \right) (\operatorname{Cos}[1+x] - i \operatorname{Sin}[1+x]) \right) \end{aligned}$$

Problem 813: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} (1+x)^{3/2} \operatorname{Sin}[x] dx$$

Optimal (type 4, 236 leaves, 16 steps):

$$\begin{aligned}
 & 4\sqrt{1-x} \cos[x] - (1-x)^{3/2} \cos[x] - 2\sqrt{2\pi} \cos[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \\
 & \frac{3}{2} \sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + 4\sqrt{2\pi} \cos[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + \\
 & \frac{3}{2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \sin[1] - 4\sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \sin[1] - \\
 & 2\sqrt{2\pi} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \sin[1] - \frac{3}{2} \sqrt{1-x} \sin[x]
 \end{aligned}$$

Result (type 4, 178 leaves):

$$\begin{aligned}
 & \frac{1}{8\sqrt{-1+x}\sqrt{1+x}} \\
 & \sqrt{1-x^2} \left((5+21i) \sqrt{\frac{\pi}{2}} \operatorname{Erfi}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\cos[1] + i \sin[1]) + 2\sqrt{-1+x} ((6+3i) + 2x) \right. \\
 & \quad \left. (\cos[x] + i \sin[x]) - i \left(2((3+6i) + 2ix) \sqrt{-1+x} (\cos[1] + i \sin[1]) + \right. \right. \\
 & \quad \left. \left. (21+5i) \sqrt{\frac{\pi}{2}} \operatorname{Erf}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (-i \cos[x] + \sin[x]) \right) (\cos[1+x] - i \sin[1+x]) \right)
 \end{aligned}$$

Problem 814: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} (1-x)^{3/2} x \sin[x] dx$$

Optimal (type 4, 193 leaves, 19 steps):

$$\begin{aligned}
 & -\frac{7}{4} \sqrt{1+x} \cos[x] - 3(1+x)^{3/2} \cos[x] + (1+x)^{5/2} \cos[x] + \frac{7}{4} \sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] - \\
 & \frac{9}{2} \sqrt{\frac{\pi}{2}} \cos[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] + \frac{9}{2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] \sin[1] + \\
 & \frac{7}{4} \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1+x}\right] \sin[1] + \frac{9}{2} \sqrt{1+x} \sin[x] - \frac{5}{2} (1+x)^{3/2} \sin[x]
 \end{aligned}$$

Result (type 4, 215 leaves):

$$\frac{1}{16 \sqrt{1-x^2}} \left(e^{-i} \left((18-7i) \sqrt{\pi} \sqrt{-i(1+x)} + 2 e^{i(1+x)} \left((-15-8i) - (19-2i)x + 10i x^2 + 4x^3 \right) - (18-7i) \sqrt{\pi} \sqrt{-i(1+x)} \operatorname{Erf} \left[\sqrt{-i(1+x)} \right] \right) + e^{-ix} \left((-30+16i) - (38+4i)x - 20i x^2 + 8x^3 + (18+7i) e^{i(1+x)} \sqrt{\pi} \sqrt{i(1+x)} - (18+7i) e^{i(1+x)} \sqrt{\pi} \sqrt{i(1+x)} \operatorname{Erf} \left[\sqrt{i(1+x)} \right] \right) \right)$$

Problem 815: Result unnecessarily involves imaginary or complex numbers.

$$\int e^{\operatorname{ArcTanh}[x]} (1-x)^{3/2} \operatorname{Sin}[x] dx$$

Optimal (type 4, 157 leaves, 13 steps):

$$\begin{aligned} & -2 \sqrt{1+x} \operatorname{Cos}[x] + (1+x)^{3/2} \operatorname{Cos}[x] + \sqrt{2\pi} \operatorname{Cos}[1] \operatorname{FresnelC} \left[\sqrt{\frac{2}{\pi}} \sqrt{1+x} \right] + \\ & \frac{3}{2} \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelS} \left[\sqrt{\frac{2}{\pi}} \sqrt{1+x} \right] - \frac{3}{2} \sqrt{\frac{\pi}{2}} \operatorname{FresnelC} \left[\sqrt{\frac{2}{\pi}} \sqrt{1+x} \right] \operatorname{Sin}[1] + \\ & \sqrt{2\pi} \operatorname{FresnelS} \left[\sqrt{\frac{2}{\pi}} \sqrt{1+x} \right] \operatorname{Sin}[1] - \frac{3}{2} \sqrt{1+x} \operatorname{Sin}[x] \end{aligned}$$

Result (type 4, 176 leaves):

$$\frac{1}{\sqrt{-1-x} \sqrt{1-x}} \left(\left(\frac{1}{16} + \frac{i}{16} \right) e^{-ix} \sqrt{1-x^2} \left((2+2i) \sqrt{-1-x} \left((-3+2i) + e^{2ix} \left((3+2i) - 2ix \right) - 2ix \right) - (3+4i) e^{ix} \sqrt{2\pi} \operatorname{Erf} \left[\frac{(1+i) \sqrt{-1-x}}{\sqrt{2}} \right] (\operatorname{Cos}[1] - i \operatorname{Sin}[1]) + (4+3i) e^{ix} \sqrt{2\pi} \operatorname{Erfi} \left[\frac{(1+i) \sqrt{-1-x}}{\sqrt{2}} \right] (-i \operatorname{Cos}[1] + \operatorname{Sin}[1]) \right)$$

Problem 816: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\operatorname{ArcTanh}[x]} x \operatorname{Sin}[x]}{\sqrt{1+x}} dx$$

Optimal (type 4, 140 leaves, 11 steps):

$$\sqrt{1-x} \operatorname{Cos}[x] - \sqrt{\frac{\pi}{2}} \operatorname{Cos}[1] \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] + \sqrt{2\pi} \operatorname{Cos}[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1] - \sqrt{\frac{\pi}{2}} \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1]$$

Result (type 4, 165 leaves):

$$\frac{1}{\sqrt{1-x^2}} \left(\frac{1}{8} + \frac{i}{8}\right) \sqrt{1+x} \left((-2-i) \sqrt{2\pi} \sqrt{-1+x} \operatorname{Erfi}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\operatorname{Cos}[1] + i \operatorname{Sin}[1]) - (2-2i) (-1+x) (\operatorname{Cos}[x] + i \operatorname{Sin}[x]) - (1-i) \left(2(-1+x) (\operatorname{Cos}[1] + i \operatorname{Sin}[1]) - (3+i) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \operatorname{Erf}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\operatorname{Cos}[x] + i \operatorname{Sin}[x]) \right) (\operatorname{Cos}[1+x] - i \operatorname{Sin}[1+x]) \right)$$

Problem 817: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\operatorname{ArcTanh}[x]} \operatorname{Sin}[x]}{\sqrt{1+x}} dx$$

Optimal (type 4, 62 leaves, 6 steps):

$$\sqrt{2\pi} \operatorname{Cos}[1] \operatorname{FresnelS}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] - \sqrt{2\pi} \operatorname{FresnelC}\left[\sqrt{\frac{2}{\pi}} \sqrt{1-x}\right] \operatorname{Sin}[1]$$

Result (type 4, 98 leaves):

$$\frac{1}{\sqrt{1-x^2}} \left(\frac{1}{2} + \frac{i}{2}\right) \sqrt{\frac{\pi}{2}} \sqrt{-1+x} \sqrt{1+x} \left(\operatorname{Erf}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\operatorname{Cos}[1] - i \operatorname{Sin}[1]) - \operatorname{Erfi}\left[\frac{(1+i)\sqrt{-1+x}}{\sqrt{2}}\right] (\operatorname{Cos}[1] + i \operatorname{Sin}[1]) \right)$$

Problem 872: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a+bx]}}{1-a^2-2abx-b^2x^2} dx$$

Optimal (type 2, 27 leaves, 2 steps):

$$\frac{\sqrt{1+ax+bx^2}}{b\sqrt{1-a-bx}}$$

Result (type 3, 12 leaves):

$$\frac{e^{\text{ArcTanh}[a+bx]}}{b}$$

Problem 875: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[a+bx]} x^m dx$$

Optimal (type 6, 109 leaves, 4 steps):

$$\frac{1}{1+m} x^{1+m} (1-a-bx)^{-n/2} (1+a+bx)^{n/2} \left(1 - \frac{bx}{1-a}\right)^{n/2} \left(1 + \frac{bx}{1+a}\right)^{-n/2} \text{AppellF1}\left[1+m, \frac{n}{2}, -\frac{n}{2}, 2+m, \frac{bx}{1-a}, -\frac{bx}{1+a}\right]$$

Result (type 8, 16 leaves):

$$\int e^{n \text{ArcTanh}[a+bx]} x^m dx$$

Problem 880: Unable to integrate problem.

$$\int \frac{e^{n \text{ArcTanh}[a+bx]}}{x} dx$$

Optimal (type 5, 135 leaves, 5 steps):

$$\frac{1}{n} 2 (1-a-bx)^{-n/2} (1+a+bx)^{n/2} \text{Hypergeometric2F1}\left[1, -\frac{n}{2}, 1-\frac{n}{2}, \frac{(1+a)(1-a-bx)}{(1-a)(1+a+bx)}\right] - \frac{1}{n} 2^{1+\frac{n}{2}} (1-a-bx)^{-n/2} \text{Hypergeometric2F1}\left[-\frac{n}{2}, -\frac{n}{2}, 1-\frac{n}{2}, \frac{1}{2}(1-a-bx)\right]$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \text{ArcTanh}[a+bx]}}{x} dx$$

Problem 881: Unable to integrate problem.

$$\int \frac{e^{n \text{ArcTanh}[a+bx]}}{x^2} dx$$

Optimal (type 5, 92 leaves, 2 steps):

$$-\frac{1}{(1-a)^2 (2-n)} 4b (1-a-bx)^{1-\frac{n}{2}} (1+a+bx)^{\frac{1}{2}(-2+n)} \text{Hypergeometric2F1}\left[2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{(1+a)(1-a-bx)}{(1-a)(1+a+bx)}\right]$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \operatorname{ArcTanh}[a+bx]}}{x^2} dx$$

Problem 882: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcTanh}[a+bx]}}{x^3} dx$$

Optimal (type 5, 152 leaves, 3 steps):

$$-\frac{(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{2+n}{2}}}{2(1-a^2)x^2} - \left(2b^2(2a+n)(1-a-bx)^{1-\frac{n}{2}}(1+a+bx)^{\frac{1}{2}(-2+n)} \right. \\ \left. \operatorname{Hypergeometric2F1}\left[2, 1-\frac{n}{2}, 2-\frac{n}{2}, \frac{(1+a)(1-a-bx)}{(1-a)(1+a+bx)}\right] \right) / \left((1-a)^3(1+a)(2-n) \right)$$

Result (type 8, 16 leaves):

$$\int \frac{e^{n \operatorname{ArcTanh}[a+bx]}}{x^3} dx$$

Problem 924: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{\operatorname{ArcTanh}[ax]}}{\sqrt{1-a^2x^2}} dx$$

Optimal (type 3, 12 leaves, 2 steps):

$$\frac{\operatorname{Log}[1-ax]}{a}$$

Result (type 4, 52 leaves):

$$\frac{2i\sqrt{-a^2} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2}x\right], 1\right] - a \operatorname{Log}\left[-1+a^2x^2\right]}{2a^2}$$

Problem 961: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{\operatorname{ArcTanh}[ax]}}{\sqrt{c-a^2cx^2}} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{\sqrt{1-a^2x^2} \operatorname{Log}[1-ax]}{a\sqrt{c-a^2cx^2}}$$

Result (type 4, 87 leaves):

$$\frac{\left(a \sqrt{1 - a^2 x^2} \left(2 \operatorname{EllipticF} \left[\operatorname{ArcSinh} \left[\sqrt{-a^2} x \right], 1 \right] + \sqrt{-a^2} \operatorname{Log} \left[-1 + a^2 x^2 \right] \right) \right)}{\left(2 (-a^2)^{3/2} \sqrt{c - a^2 c x^2} \right)}$$

Problem 970: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} x}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{2 a^2 c (1 - a x) \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2} \operatorname{ArcTanh}[a x]}{2 a^2 c \sqrt{c - a^2 c x^2}}$$

Result (type 4, 93 leaves):

$$-\left(\left(\operatorname{EllipticF} \left[\operatorname{ArcSinh} \left[\sqrt{-a^2} x \right], 1 \right] + \sqrt{-a^2} \operatorname{Log} \left[-1 + a^2 x^2 \right] \right) \right) / \left(2 (-a^2)^{3/2} c (-1 + a x) \sqrt{c - a^2 c x^2} \right)$$

Problem 971: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]}}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 91 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{2 a c (1 - a x) \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2} \operatorname{ArcTanh}[a x]}{2 a c \sqrt{c - a^2 c x^2}}$$

Result (type 4, 91 leaves):

$$\left(a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} + \operatorname{EllipticF} \left[\operatorname{ArcSinh} \left[\sqrt{-a^2} x \right], 1 \right] \right) \right) / \left(2 (-a^2)^{3/2} c (-1 + a x) \sqrt{c - a^2 c x^2} \right)$$

Problem 972: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]}}{x (c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 165 leaves, 4 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{2 c (1 - a x) \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2} \operatorname{Log}[x]}{c \sqrt{c - a^2 c x^2}} - \frac{3 \sqrt{1 - a^2 x^2} \operatorname{Log}[1 - a x]}{4 c \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2} \operatorname{Log}[1 + a x]}{4 c \sqrt{c - a^2 c x^2}}$$

Result (type 4, 121 leaves):

$$\left(\sqrt{c - a^2 c x^2} \left(-i a (-1 + a x) \text{EllipticF}[i \text{ArcSinh}[\sqrt{-a^2} x], 1] + \sqrt{-a^2} \right. \right. \\ \left. \left. (-1 + (-1 + a x) \text{Log}[x^2] + (1 - a x) \text{Log}[1 - a^2 x^2]) \right) \right) / \left(2 \sqrt{-a^2} c^2 (-1 + a x) \sqrt{1 - a^2 x^2} \right)$$

Problem 973: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x^2 (c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 206 leaves, 4 steps):

$$-\frac{\sqrt{1 - a^2 x^2}}{c x \sqrt{c - a^2 c x^2}} + \frac{a \sqrt{1 - a^2 x^2}}{2 c (1 - a x) \sqrt{c - a^2 c x^2}} + \\ \frac{a \sqrt{1 - a^2 x^2} \text{Log}[x]}{c \sqrt{c - a^2 c x^2}} - \frac{5 a \sqrt{1 - a^2 x^2} \text{Log}[1 - a x]}{4 c \sqrt{c - a^2 c x^2}} + \frac{a \sqrt{1 - a^2 x^2} \text{Log}[1 + a x]}{4 c \sqrt{c - a^2 c x^2}}$$

Result (type 4, 135 leaves):

$$\left(\sqrt{c - a^2 c x^2} \left(-3 i a^2 x (-1 + a x) \text{EllipticF}[i \text{ArcSinh}[\sqrt{-a^2} x], 1] + \right. \right. \\ \left. \left. \sqrt{-a^2} (2 - 3 a x + a x (-1 + a x) \text{Log}[x^2] + a x (1 - a x) \text{Log}[1 - a^2 x^2]) \right) \right) / \\ \left(2 \sqrt{-a^2} c^2 x (-1 + a x) \sqrt{1 - a^2 x^2} \right)$$

Problem 974: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x^3 (c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 255 leaves, 4 steps):

$$-\frac{\sqrt{1 - a^2 x^2}}{2 c x^2 \sqrt{c - a^2 c x^2}} - \frac{a \sqrt{1 - a^2 x^2}}{c x \sqrt{c - a^2 c x^2}} + \frac{a^2 \sqrt{1 - a^2 x^2}}{2 c (1 - a x) \sqrt{c - a^2 c x^2}} + \\ \frac{2 a^2 \sqrt{1 - a^2 x^2} \text{Log}[x]}{c \sqrt{c - a^2 c x^2}} - \frac{7 a^2 \sqrt{1 - a^2 x^2} \text{Log}[1 - a x]}{4 c \sqrt{c - a^2 c x^2}} - \frac{a^2 \sqrt{1 - a^2 x^2} \text{Log}[1 + a x]}{4 c \sqrt{c - a^2 c x^2}}$$

Result (type 4, 153 leaves):

$$\left(\sqrt{c - a^2 c x^2} \left(-3 i a^3 x^2 (-1 + a x) \text{EllipticF}[i \text{ArcSinh}[\sqrt{-a^2} x], 1] + \right. \right. \\ \left. \left. \sqrt{-a^2} (1 + a x - 3 a^2 x^2 + 2 a^2 x^2 (-1 + a x) \text{Log}[x^2] - 2 a^2 x^2 (-1 + a x) \text{Log}[1 - a^2 x^2]) \right) \right) / \\ \left(2 \sqrt{-a^2} c^2 x^2 (-1 + a x) \sqrt{1 - a^2 x^2} \right)$$

Problem 975: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x^4 (c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 297 leaves, 4 steps):

$$\begin{aligned} & -\frac{\sqrt{1-a^2 x^2}}{3 c x^3 \sqrt{c-a^2 c x^2}} - \frac{a \sqrt{1-a^2 x^2}}{2 c x^2 \sqrt{c-a^2 c x^2}} - \frac{2 a^2 \sqrt{1-a^2 x^2}}{c x \sqrt{c-a^2 c x^2}} + \frac{a^3 \sqrt{1-a^2 x^2}}{2 c (1-a x) \sqrt{c-a^2 c x^2}} + \\ & \frac{2 a^3 \sqrt{1-a^2 x^2} \text{Log}[x]}{c \sqrt{c-a^2 c x^2}} - \frac{9 a^3 \sqrt{1-a^2 x^2} \text{Log}[1-a x]}{4 c \sqrt{c-a^2 c x^2}} + \frac{a^3 \sqrt{1-a^2 x^2} \text{Log}[1+a x]}{4 c \sqrt{c-a^2 c x^2}} \end{aligned}$$

Result (type 4, 161 leaves):

$$\begin{aligned} & \left(\sqrt{c-a^2 c x^2} \left(-15 i a^4 x^3 (-1+a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] + \sqrt{-a^2} \right. \right. \\ & \quad \left. \left. (2+a x+9 a^2 x^2-15 a^3 x^3+6 a^3 x^3 (-1+a x) \text{Log}[x^2]-6 a^3 x^3 (-1+a x) \text{Log}[1-a^2 x^2]) \right] \right) \Big/ \\ & \left(6 \sqrt{-a^2} c^2 x^3 (-1+a x) \sqrt{1-a^2 x^2} \right) \end{aligned}$$

Problem 979: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^3}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 184 leaves, 5 steps):

$$\begin{aligned} & \frac{\sqrt{1-a^2 x^2}}{8 a^4 c^2 (1-a x)^2 \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{2 a^4 c^2 (1-a x) \sqrt{c-a^2 c x^2}} + \\ & \frac{\sqrt{1-a^2 x^2}}{8 a^4 c^2 (1+a x) \sqrt{c-a^2 c x^2}} + \frac{3 \sqrt{1-a^2 x^2} \text{ArcTanh}[a x]}{8 a^4 c^2 \sqrt{c-a^2 c x^2}} \end{aligned}$$

Result (type 4, 122 leaves):

$$\begin{aligned} & \left(\sqrt{1-a^2 x^2} \right. \\ & \quad \left. \left(\sqrt{-a^2} (-2-a x+5 a^2 x^2) - 3 i a (-1+a x)^2 (1+a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right) \right) \Big/ \\ & \left(8 a^4 \sqrt{-a^2} c^2 (-1+a x)^2 (1+a x) \sqrt{c-a^2 c x^2} \right) \end{aligned}$$

Problem 980: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^2}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 184 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{8 a^3 c^2 (1-a x)^2 \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{4 a^3 c^2 (1-a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{8 a^3 c^2 (1+a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a^3 c^2 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 119 leaves):

$$\left(a \sqrt{1-a^2 x^2} \left(\sqrt{-a^2} (-2+3 a x+a^2 x^2) + i a (-1+a x)^2 (1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x \right], 1 \right] \right) \right) / \left(8 (-a^2)^{5/2} c^2 (-1+a x)^2 (1+a x) \sqrt{c-a^2 c x^2} \right)$$

Problem 981: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} x}{(c-a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 137 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{8 a^2 c^2 (1-a x)^2 \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2}}{8 a^2 c^2 (1+a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a^2 c^2 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 118 leaves):

$$- \left(\left(\sqrt{1-a^2 x^2} \left(\sqrt{-a^2} (2-a x+a^2 x^2) + i a (-1+a x)^2 (1+a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x \right], 1 \right] \right) \right) \right) / \left(8 (-a^2)^{3/2} c^2 (-1+a x)^2 (1+a x) \sqrt{c-a^2 c x^2} \right)$$

Problem 982: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\operatorname{ArcTanh}[a x]}}{(c-a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 184 leaves, 5 steps):

$$\frac{\sqrt{1-a^2 x^2}}{8 a c^2 (1-a x)^2 \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2}}{4 a c^2 (1-a x) \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{8 a c^2 (1+a x) \sqrt{c-a^2 c x^2}} + \frac{3 \sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a c^2 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 120 leaves):

$$- \left(\left(a \sqrt{1 - a^2 x^2} \right. \right. \\ \left. \left. \left(\sqrt{-a^2} (2 + 3 a x - 3 a^2 x^2) - 3 i a (-1 + a x)^2 (1 + a x) \text{EllipticF} [i \text{ArcSinh} [\sqrt{-a^2} x], 1] \right) \right) \right) / \\ \left(8 (-a^2)^{3/2} c^2 (-1 + a x)^2 (1 + a x) \sqrt{c - a^2 c x^2} \right)$$

Problem 983: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x (c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 252 leaves, 4 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{8 c^2 (1 - a x)^2 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2}}{2 c^2 (1 - a x) \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2}}{8 c^2 (1 + a x) \sqrt{c - a^2 c x^2}} + \\ \frac{\sqrt{1 - a^2 x^2} \text{Log}[x]}{c^2 \sqrt{c - a^2 c x^2}} - \frac{11 \sqrt{1 - a^2 x^2} \text{Log}[1 - a x]}{16 c^2 \sqrt{c - a^2 c x^2}} - \frac{5 \sqrt{1 - a^2 x^2} \text{Log}[1 + a x]}{16 c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 162 leaves):

$$\left(\sqrt{c - a^2 c x^2} \left(-3 i a (-1 + a x)^2 (1 + a x) \text{EllipticF} [i \text{ArcSinh} [\sqrt{-a^2} x], 1] + \sqrt{-a^2} \right. \right. \\ \left. \left. \left(6 - a x - 3 a^2 x^2 + 4 (-1 + a x)^2 (1 + a x) \text{Log}[x^2] - 4 (-1 + a x)^2 (1 + a x) \text{Log}[1 - a^2 x^2] \right) \right) \right) / \\ \left(8 \sqrt{-a^2} c^3 (-1 + a x)^2 (1 + a x) \sqrt{1 - a^2 x^2} \right)$$

Problem 984: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x^2 (c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 295 leaves, 4 steps):

$$- \frac{\sqrt{1 - a^2 x^2}}{c^2 x \sqrt{c - a^2 c x^2}} + \frac{a \sqrt{1 - a^2 x^2}}{8 c^2 (1 - a x)^2 \sqrt{c - a^2 c x^2}} + \\ \frac{3 a \sqrt{1 - a^2 x^2}}{4 c^2 (1 - a x) \sqrt{c - a^2 c x^2}} - \frac{a \sqrt{1 - a^2 x^2}}{8 c^2 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{a \sqrt{1 - a^2 x^2} \text{Log}[x]}{c^2 \sqrt{c - a^2 c x^2}} - \\ \frac{23 a \sqrt{1 - a^2 x^2} \text{Log}[1 - a x]}{16 c^2 \sqrt{c - a^2 c x^2}} + \frac{7 a \sqrt{1 - a^2 x^2} \text{Log}[1 + a x]}{16 c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 180 leaves):

$$\left(\sqrt{c - a^2 c x^2} \left(-15 i a^2 x (-1 + a x)^2 (1 + a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] + \right. \right. \\ \left. \sqrt{-a^2} \left(-8 + 14 a x + 11 a^2 x^2 - 15 a^3 x^3 + 4 a x (-1 + a x)^2 (1 + a x) \text{Log}[x^2] - \right. \right. \\ \left. \left. 4 a x (-1 + a x)^2 (1 + a x) \text{Log}[1 - a^2 x^2] \right) \right) \Big/ \left(8 \sqrt{-a^2} c^3 x (-1 + a x)^2 (1 + a x) \sqrt{1 - a^2 x^2} \right)$$

Problem 985: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{x^3 (c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 345 leaves, 4 steps):

$$-\frac{\sqrt{1 - a^2 x^2}}{2 c^2 x^2 \sqrt{c - a^2 c x^2}} - \frac{a \sqrt{1 - a^2 x^2}}{c^2 x \sqrt{c - a^2 c x^2}} + \frac{a^2 \sqrt{1 - a^2 x^2}}{8 c^2 (1 - a x)^2 \sqrt{c - a^2 c x^2}} + \\ \frac{a^2 \sqrt{1 - a^2 x^2}}{c^2 (1 - a x) \sqrt{c - a^2 c x^2}} + \frac{a^2 \sqrt{1 - a^2 x^2}}{8 c^2 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{3 a^2 \sqrt{1 - a^2 x^2} \text{Log}[x]}{c^2 \sqrt{c - a^2 c x^2}} - \\ \frac{39 a^2 \sqrt{1 - a^2 x^2} \text{Log}[1 - a x]}{16 c^2 \sqrt{c - a^2 c x^2}} - \frac{9 a^2 \sqrt{1 - a^2 x^2} \text{Log}[1 + a x]}{16 c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 198 leaves):

$$\left(\sqrt{c - a^2 c x^2} \left(-15 i a^3 x^2 (-1 + a x)^2 (1 + a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] + \right. \right. \\ \left. \sqrt{-a^2} \left(-4 - 4 a x + 22 a^2 x^2 + 3 a^3 x^3 - 15 a^4 x^4 + 12 a^2 x^2 (-1 + a x)^2 (1 + a x) \text{Log}[x^2] - \right. \right. \\ \left. \left. 12 a^2 x^2 (-1 + a x)^2 (1 + a x) \text{Log}[1 - a^2 x^2] \right) \right) \Big/ \left(8 \sqrt{-a^2} c^3 x^2 (-1 + a x)^2 (1 + a x) \sqrt{1 - a^2 x^2} \right)$$

Problem 986: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcTanh}[a x]}}{(c - a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 277 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{24 a c^3 (1 - a x)^3 \sqrt{c - a^2 c x^2}} + \frac{3 \sqrt{1 - a^2 x^2}}{32 a c^3 (1 - a x)^2 \sqrt{c - a^2 c x^2}} + \frac{3 \sqrt{1 - a^2 x^2}}{16 a c^3 (1 - a x) \sqrt{c - a^2 c x^2}} - \\ \frac{\sqrt{1 - a^2 x^2}}{32 a c^3 (1 + a x)^2 \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{8 a c^3 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{5 \sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{16 a c^3 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 138 leaves):

$$- \left(\left(a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (-8 - 25 a x + 25 a^2 x^2 + 15 a^3 x^3 - 15 a^4 x^4) - \right. \right. \right. \\ \left. \left. \left. 15 i a (-1 + a x)^3 (1 + a x)^2 \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right]\right) \right) \right) / \\ \left(48 (-a^2)^{3/2} c^3 (-1 + a x)^3 (1 + a x)^2 \sqrt{c - a^2 c x^2} \right)$$

Problem 989: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{c - a^2 c x^2} dx$$

Optimal (type 5, 80 leaves, 4 steps):

$$\frac{x^{1+m} \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c (1+m)} + \frac{a x^{2+m} \text{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c (2+m)}$$

Result (type 6, 391 leaves):

$$\frac{1}{2 c (1+m)} (2+m) x^{1+m} \left(\left(2 \sqrt{-1 - a x} \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -a x, a x\right] \right) / \right. \\ \left. \left((-1 + a x)^{3/2} \left(2 (2+m) \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -a x, a x\right] + a x \left(3 \text{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, -a x, a x\right] + \text{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, -a x, a x\right] \right) \right) \right) \right) + \frac{1}{\sqrt{1 + a x}} \\ \sqrt{1 - a x} \left(\left(\sqrt{-1 - a x} \sqrt{1 - a^2 x^2} \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -a x, a x\right] \right) / \right. \\ \left. \left((-1 + a x)^{3/2} \left(2 (2+m) \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -a x, a x\right] + a x \left(\text{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, -a x, a x\right] + \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1 + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}\right\}, a^2 x^2\right]\right) \right) \right) \right) + \\ \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, a x, -a x\right] / \left(2 (2+m) \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, a x, -a x\right] - a x \left(\text{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, 3+m, a x, -a x\right] + \right. \right. \\ \left. \left. \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1 + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}\right\}, a^2 x^2\right]\right) \right) \right)$$

Problem 990: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^2} dx$$

Optimal (type 5, 80 leaves, 4 steps):

$$\frac{x^{1+m} \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c^2 (1+m)} + \frac{a x^{2+m} \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c^2 (2+m)}$$

Result (type 6, 711 leaves):

$$\begin{aligned} & \left((2+m) x^{1+m} \sqrt{-1-ax} \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] \right) / \\ & \left(2 c^2 (1+m) (-1+ax)^{3/2} \left(2 (2+m) \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, -ax, ax\right] + ax \right. \right. \\ & \quad \left. \left. \left(3 \text{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, -ax, ax\right] + \text{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, -ax, ax\right] \right) \right) \right) + \\ & \left((2+m) x^{1+m} \sqrt{1-ax} \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, ax, -ax\right] \right) / \\ & \left(4 c^2 (1+m) (1+ax)^{3/2} \left(2 (2+m) \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{3}{2}, 2+m, ax, -ax\right] - ax \right. \right. \\ & \quad \left. \left. \left(3 \text{AppellF1}\left[2+m, -\frac{1}{2}, \frac{5}{2}, 3+m, ax, -ax\right] + \text{AppellF1}\left[2+m, \frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] \right) \right) \right) - \\ & \left((2+m) x^{1+m} \sqrt{-1-ax} \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{5}{2}, 2+m, -ax, ax\right] \right) / \\ & \left(2 c^2 (1+m) (-1+ax)^{5/2} \left(2 (2+m) \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{5}{2}, 2+m, -ax, ax\right] + ax \right. \right. \\ & \quad \left. \left. \left(5 \text{AppellF1}\left[2+m, -\frac{1}{2}, \frac{7}{2}, 3+m, -ax, ax\right] + \text{AppellF1}\left[2+m, \frac{1}{2}, \frac{5}{2}, 3+m, -ax, ax\right] \right) \right) \right) + \\ & \left(3 (2+m) x^{1+m} \sqrt{-1-ax} \sqrt{1-ax} \sqrt{1-a^2 x^2} \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] \right) / \\ & \left(8 c^2 (1+m) (-1+ax)^{3/2} \sqrt{1+ax} \right. \\ & \quad \left. \left(2 (2+m) \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, -ax, ax\right] + ax \left(\text{AppellF1}\left[2+m, -\frac{1}{2}, \frac{3}{2}, \right. \right. \right. \right. \\ & \quad \quad \left. \left. \left. 3+m, -ax, ax\right] + \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right] \right) \right) \right) + \\ & \left(3 (2+m) x^{1+m} \sqrt{1-ax} \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] \right) / \\ & \left(8 c^2 (1+m) \sqrt{1+ax} \left(2 (2+m) \text{AppellF1}\left[1+m, -\frac{1}{2}, \frac{1}{2}, 2+m, ax, -ax\right] - ax \left(\text{AppellF1}\left[2+m, \right. \right. \right. \right. \\ & \quad \left. \left. \left. -\frac{1}{2}, \frac{3}{2}, 3+m, ax, -ax\right] + \text{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right] \right) \right) \right) \end{aligned}$$

Problem 991: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[ax]} x^m}{(c - a^2 c x^2)^3} dx$$

Optimal (type 5, 80 leaves, 4 steps):

$$\frac{x^{1+m} \text{Hypergeometric2F1}\left[\frac{7}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c^3 (1+m)} + \frac{a x^{2+m} \text{Hypergeometric2F1}\left[\frac{7}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c^3 (2+m)}$$

Result (type 8, 25 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^3} dx$$

Problem 1001: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 5, 51 leaves, 3 steps):

$$\frac{x^{1+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[1, 1 + m, 2 + m, a x\right]}{(1 + m) \sqrt{c - a^2 c x^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{\sqrt{c - a^2 c x^2}} dx$$

Problem 1002: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 5, 134 leaves, 7 steps):

$$\frac{x^{1+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[2, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c (1 + m) \sqrt{c - a^2 c x^2}} +$$

$$\frac{a x^{2+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[2, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c (2 + m) \sqrt{c - a^2 c x^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{3/2}} dx$$

Problem 1003: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 5, 134 leaves, 7 steps):

$$\frac{x^{1+m} \sqrt{1-a^2 x^2} \operatorname{Hypergeometric2F1}\left[3, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{c^2 (1+m) \sqrt{c-a^2 c x^2}} +$$

$$\frac{a x^{2+m} \sqrt{1-a^2 x^2} \operatorname{Hypergeometric2F1}\left[3, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{c^2 (2+m) \sqrt{c-a^2 c x^2}}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{\operatorname{ArcTanh}[a x]} x^m}{(c-a^2 c x^2)^{5/2}} dx$$

Problem 1004: Unable to integrate problem.

$$\int e^{\operatorname{ArcTanh}[a x]} x^m (c-a^2 c x^2)^p dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{1}{1+m} x^{1+m} (1-a^2 x^2)^{-p} (c-a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{1}{2}-p, \frac{3+m}{2}, a^2 x^2\right] +$$

$$\frac{1}{2+m} a x^{2+m} (1-a^2 x^2)^{-p} (c-a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{2+m}{2}, \frac{1}{2}-p, \frac{4+m}{2}, a^2 x^2\right]$$

Result (type 8, 25 leaves):

$$\int e^{\operatorname{ArcTanh}[a x]} x^m (c-a^2 c x^2)^p dx$$

Problem 1005: Result more than twice size of optimal antiderivative.

$$\int e^{\operatorname{ArcTanh}[a x]} x^3 (1-a^2 x^2)^p dx$$

Optimal (type 5, 85 leaves, 6 steps):

$$-\frac{(1-a^2 x^2)^{\frac{1}{2}+p}}{a^4 (1+2p)} + \frac{(1-a^2 x^2)^{\frac{3}{2}+p}}{a^4 (3+2p)} + \frac{1}{5} a x^5 \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1}{2}-p, \frac{7}{2}, a^2 x^2\right]$$

Result (type 5, 183 leaves):

$$\frac{1}{3 a^4} \left(-3 a x \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1}{2}-p, \frac{3}{2}, a^2 x^2\right] + \frac{1}{3+2p} \left(-3+3(1-a^2 x^2)^{\frac{1}{2}+p} - \right. \right.$$

$$3 a^2 x^2 (1-a^2 x^2)^{\frac{1}{2}+p} - a^3 (3+2p) x^3 \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, -\frac{1}{2}-p, \frac{5}{2}, a^2 x^2\right] +$$

$$\left. \left. 3(1-a x)^{-\frac{1}{2}-p} (1+a x) (2-2 a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}-p, \frac{3}{2}+p, \frac{5}{2}+p, \frac{1}{2}(1+a x)\right] \right) \right)$$

Problem 1009: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{\text{ArcTanh}[a x]} (1 - a^2 x^2)^p}{x} dx$$

Optimal (type 5, 72 leaves, 5 steps):

$$a x \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - p, \frac{3}{2}, a^2 x^2\right] - \frac{(1 - a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 5, 147 leaves):

$$(1 - a^2 x^2)^{\frac{1}{2}+p} \left(\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2} - p, -\frac{1}{2} - p, \frac{1}{2} - p, \frac{1}{a^2 x^2}\right]}{\left(1 - \frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p} + 2 p \left(1 - \frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p}} + \frac{1}{3 + 2 p} \right. \\ \left. 2^{\frac{1}{2}+p} (1 - a x)^{-\frac{1}{2}-p} (1 + a x) \text{Hypergeometric2F1}\left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} (1 + a x)\right] \right)$$

Problem 1010: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{\text{ArcTanh}[a x]} (1 - a^2 x^2)^p}{x^2} dx$$

Optimal (type 5, 75 leaves, 5 steps):

$$\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{a (1 - a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]}{1 + 2 p}$$

Result (type 5, 170 leaves):

$$-\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} + \frac{1}{1 + 2 p} \\ a \left(1 - \frac{1}{a^2 x^2}\right)^{-\frac{1}{2}-p} (1 - a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[-\frac{1}{2} - p, -\frac{1}{2} - p, \frac{1}{2} - p, \frac{1}{a^2 x^2}\right] + \frac{1}{3 + 2 p} \\ a (1 - a x)^{-\frac{1}{2}-p} (1 + a x) (2 - 2 a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[\frac{1}{2} - p, \frac{3}{2} + p, \frac{5}{2} + p, \frac{1}{2} (1 + a x)\right]$$

Problem 1011: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{\text{ArcTanh}[a x]} (1 - a^2 x^2)^p}{x^3} dx$$

Optimal (type 5, 78 leaves, 5 steps):

$$-\frac{a \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2}-p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{a^2 (1-a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[2, \frac{1}{2}+p, \frac{3}{2}+p, 1-a^2 x^2\right]}{1+2p}$$

Result (type 5, 262 leaves):

$$-\frac{a \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{1}{2}-p, \frac{1}{2}, a^2 x^2\right]}{x} + \frac{a^2 (1-a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}-p, -\frac{1}{2}-p, \frac{1}{2}-p, \frac{1}{a^2 x^2}\right]}{\left(1-\frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p} + 2p \left(1-\frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p}} + \frac{1}{(-1+2p)x^2}$$

$$\left(1-\frac{1}{a^2 x^2}\right)^{-\frac{1}{2}-p} (1-a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}-p, \frac{1}{2}-p, \frac{3}{2}-p, \frac{1}{a^2 x^2}\right] + \frac{1}{3+2p}$$

$$a^2 (1-ax)^{-\frac{1}{2}-p} (1+ax) (2-2a^2 x^2)^{\frac{1}{2}+p} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}-p, \frac{3}{2}+p, \frac{5}{2}+p, \frac{1}{2}(1+ax)\right]$$

Problem 1012: Result more than twice size of optimal antiderivative.

$$\int e^{\operatorname{ArcTanh}[ax]} x^3 (c-a^2 c x^2)^p dx$$

Optimal (type 5, 134 leaves, 7 steps):

$$-\frac{\sqrt{1-a^2 x^2} (c-a^2 c x^2)^p}{a^4 (1+2p)} + \frac{(1-a^2 x^2)^{3/2} (c-a^2 c x^2)^p}{a^4 (3+2p)} + \frac{1}{5} a x^5 (1-a^2 x^2)^{-p} (c-a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1}{2}-p, \frac{7}{2}, a^2 x^2\right]$$

Result (type 5, 295 leaves):

$$\frac{1}{a^4 (3+2p) (5+2p) (7+2p) (9+2p)} 4^{1+p} e^{3 \operatorname{ArcTanh}[ax]} \left(\frac{e^{\operatorname{ArcTanh}[ax]}}{1+e^{2 \operatorname{ArcTanh}[ax]}} \right)^{2p} \left(1+e^{2 \operatorname{ArcTanh}[ax]} \right)^{2p} (1-a^2 x^2)^{-p} (c(1-a^2 x^2))^p$$

$$\left(- (315+286p+84p^2+8p^3) \operatorname{Hypergeometric2F1}\left[\frac{3}{2}+p, 5+2p, \frac{5}{2}+p, -e^{2 \operatorname{ArcTanh}[ax]}\right] + e^{2 \operatorname{ArcTanh}[ax]} (3+2p) \right.$$

$$\left. \left(3 (63+32p+4p^2) \operatorname{Hypergeometric2F1}\left[\frac{5}{2}+p, 5+2p, \frac{7}{2}+p, -e^{2 \operatorname{ArcTanh}[ax]}\right] + e^{2 \operatorname{ArcTanh}[ax]} (5+2p) \left(-3 (9+2p) \operatorname{Hypergeometric2F1}\left[\frac{7}{2}+p, 5+2p, \frac{9}{2}+p, -e^{2 \operatorname{ArcTanh}[ax]}\right] + e^{2 \operatorname{ArcTanh}[ax]} (7+2p) \operatorname{Hypergeometric2F1}\left[\frac{9}{2}+p, 5+2p, \frac{11}{2}+p, -e^{2 \operatorname{ArcTanh}[ax]}\right] \right) \right) \right)$$

Problem 1016: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Optimal (type 5, 110 leaves, 6 steps):

$$a x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2} - p, \frac{3}{2}, a^2 x^2\right] - \frac{1}{1 + 2p} \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]$$

Result (type 8, 25 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Problem 1017: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Optimal (type 5, 113 leaves, 6 steps):

$$-\frac{(1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{1}{1 + 2p} a \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]$$

Result (type 8, 25 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Problem 1018: Unable to integrate problem.

$$\int \frac{e^{\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^3} dx$$

Optimal (type 5, 116 leaves, 6 steps):

$$-\frac{1}{x} a (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right] - \frac{1}{1 + 2p} a^2 \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[2, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]$$

Result (type 8, 25 leaves):

$$\int \frac{e^{\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^3} dx$$

Problem 1035: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^2}{x^3} dx$$

Optimal (type 1, 17 leaves, 2 steps):

$$-\frac{c^2 (1 + a x)^4}{2 x^2}$$

Result (type 1, 42 leaves):

$$-\frac{c^2}{2 x^2} - \frac{2 a c^2}{x} - 2 a^3 c^2 x - \frac{1}{2} a^4 c^2 x^2$$

Problem 1053: Result unnecessarily involves higher level functions.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]}}{c - a^2 c x^2} dx$$

Optimal (type 1, 15 leaves, 2 steps):

$$\frac{1}{a c (1 - a x)}$$

Result (type 3, 18 leaves):

$$\frac{e^{2 \operatorname{ArcTanh}[a x]}}{2 a c}$$

Problem 1130: Result unnecessarily involves higher level functions.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^3} dx$$

Optimal (type 5, 203 leaves, 8 steps):

$$\begin{aligned} & -\frac{(2-m)(4-m)x^{1+m}}{24c^3(1+ax)} + \frac{x^{1+m}}{6c^3(1-ax)^3(1+ax)} + \frac{(4-m)x^{1+m}}{12c^3(1-ax)^2(1+ax)} + \\ & \frac{(7-2m)(2-m)x^{1+m}}{24c^3(1-ax)(1+ax)} + \frac{(2-m)x^{1+m} \operatorname{Hypergeometric2F1}[1, 1+m, 2+m, -ax]}{16c^3(1+m)} + \\ & \frac{(2-m)(3-8m+2m^2)x^{1+m} \operatorname{Hypergeometric2F1}[1, 1+m, 2+m, ax]}{48c^3(1+m)} \end{aligned}$$

Result (type 6, 109 leaves):

$$\begin{aligned} & ((2+m)x^{1+m} \operatorname{AppellF1}[1+m, 4, 2, 2+m, ax, -ax]) / \\ & (c^3(1+m)(-1+ax)^4(1+ax)^2((2+m) \operatorname{AppellF1}[1+m, 4, 2, 2+m, ax, -ax] - \\ & 2ax(\operatorname{AppellF1}[2+m, 4, 3, 3+m, ax, -ax] - 2 \operatorname{AppellF1}[2+m, 5, 2, 3+m, ax, -ax]))) \end{aligned}$$

Problem 1133: Result unnecessarily involves higher level functions.

$$\int e^{2 \operatorname{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Optimal (type 5, 172 leaves, 7 steps):

$$-\frac{x^{1+m} \sqrt{c - a^2 c x^2}}{2 + m} + \frac{c (3 + 2 m) x^{1+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{(1 + m) (2 + m) \sqrt{c - a^2 c x^2}} + \frac{2 a c x^{2+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2 + m) \sqrt{c - a^2 c x^2}}$$

Result (type 6, 193 leaves):

$$\frac{1}{1 + m} x^{1+m} \left(-\frac{\sqrt{c - a^2 c x^2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{\sqrt{1 - a^2 x^2}} - \left(4 (2 + m) \sqrt{-c (1 + a x)} \operatorname{AppellF1}\left[1 + m, \frac{1}{2}, -\frac{1}{2}, 2 + m, a x, -a x\right] \right) / \left(\sqrt{-1 + a x} \left(2 (2 + m) \operatorname{AppellF1}\left[1 + m, \frac{1}{2}, -\frac{1}{2}, 2 + m, a x, -a x\right] + a x \left(\operatorname{AppellF1}\left[2 + m, \frac{3}{2}, -\frac{1}{2}, 3 + m, a x, -a x\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1 + \frac{m}{2}\right\}, \left\{2 + \frac{m}{2}\right\}, a^2 x^2\right]\right) \right) \right) \right)$$

Problem 1134: Result unnecessarily involves higher level functions.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]} x^m}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 5, 169 leaves, 7 steps):

$$\frac{2 x^{1+m} (1 + a x)}{\sqrt{c - a^2 c x^2}} - \frac{(1 + 2 m) x^{1+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{(1 + m) \sqrt{c - a^2 c x^2}} - \frac{2 a (1 + m) x^{2+m} \sqrt{1 - a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2 + m) \sqrt{c - a^2 c x^2}}$$

Result (type 6, 133 leaves):

$$\left(2 (2 + m) x^{1+m} \sqrt{-c (1 + a x)} \operatorname{AppellF1}\left[1 + m, \frac{3}{2}, -\frac{1}{2}, 2 + m, a x, -a x\right] \right) / \left(c (1 + m) (-1 + a x)^{3/2} \left(2 (2 + m) \operatorname{AppellF1}\left[1 + m, \frac{3}{2}, -\frac{1}{2}, 2 + m, a x, -a x\right] + a x \left(\operatorname{AppellF1}\left[2 + m, \frac{3}{2}, \frac{1}{2}, 3 + m, a x, -a x\right] + 3 \operatorname{AppellF1}\left[2 + m, \frac{5}{2}, -\frac{1}{2}, 3 + m, a x, -a x\right] \right) \right) \right)$$

Problem 1135: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{2 \operatorname{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 5, 183 leaves, 7 steps):

$$\frac{2 x^{1+m} (1+a x)}{3 (c-a^2 c x^2)^{3/2}} + \frac{(1-2 m) x^{1+m} \sqrt{1-a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{3 c (1+m) \sqrt{c-a^2 c x^2}} +$$

$$\frac{2 a (1-m) x^{2+m} \sqrt{1-a^2 x^2} \operatorname{Hypergeometric2F1}\left[\frac{3}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{3 c (2+m) \sqrt{c-a^2 c x^2}}$$

Result (type 6, 582 leaves):

$$\left((2+m) x^{1+m} \sqrt{-c(1+ax)} \operatorname{AppellF1}\left[1+m, \frac{3}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] \right) /$$

$$\left(2 c^2 (1+m) (-1+ax)^{3/2} \left(2 (2+m) \operatorname{AppellF1}\left[1+m, \frac{3}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] + ax \right. \right.$$

$$\left. \left. \left(\operatorname{AppellF1}\left[2+m, \frac{3}{2}, \frac{1}{2}, 3+m, ax, -ax\right] + 3 \operatorname{AppellF1}\left[2+m, \frac{5}{2}, -\frac{1}{2}, 3+m, ax, -ax\right] \right) \right) \right) -$$

$$\left((2+m) x^{1+m} \sqrt{-c(1+ax)} \operatorname{AppellF1}\left[1+m, \frac{5}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] \right) /$$

$$\left(c^2 (1+m) (-1+ax)^{5/2} \left(2 (2+m) \operatorname{AppellF1}\left[1+m, \frac{5}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] + ax \right. \right.$$

$$\left. \left. \left(\operatorname{AppellF1}\left[2+m, \frac{5}{2}, \frac{1}{2}, 3+m, ax, -ax\right] + 5 \operatorname{AppellF1}\left[2+m, \frac{7}{2}, -\frac{1}{2}, 3+m, ax, -ax\right] \right) \right) \right) +$$

$$\left((2+m) x^{1+m} \sqrt{c-acx} \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -ax, ax\right] \right) /$$

$$\left(4 c^2 (1+m) \sqrt{1+ax} \left(2 (2+m) \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -ax, ax\right] - ax \left(\operatorname{AppellF1}\left[2+m, \right. \right. \right.$$

$$\left. \left. \left. \frac{3}{2}, -\frac{1}{2}, 3+m, -ax, ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right] \right) \right) \right) +$$

$$\left((2+m) x^{1+m} \sqrt{1-ax} \sqrt{-c(1+ax)} \sqrt{1-a^2 x^2} \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] \right) /$$

$$\left(4 c^2 (1+m) (-1+ax)^{3/2} \sqrt{1+ax} \right.$$

$$\left. \left(2 (2+m) \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, ax, -ax\right] + ax \left(\operatorname{AppellF1}\left[2+m, \frac{3}{2}, \right. \right. \right.$$

$$\left. \left. \left. -\frac{1}{2}, 3+m, ax, -ax\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right] \right) \right) \right)$$

Problem 1136: Result more than twice size of optimal antiderivative.

$$\int e^{2 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p dx$$

Optimal (type 5, 55 leaves, 3 steps):

$$-\frac{1}{a^p} 2^{1+p} (1+ax)^{-p} (c-a^2cx^2)^p \text{Hypergeometric2F1}\left[-1-p, p, 1+p, \frac{1}{2}(1-ax)\right]$$

Result (type 5, 133 leaves):

$$\frac{1}{a(1+p)} \left(-(-1+ax)^2 \right)^{-p} (-2+2ax)^p (1-a^2x^2)^{-p} (c-a^2cx^2)^p \\ \left(-a(1+p)x \left(\frac{1}{2} - \frac{ax}{2} \right)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, a^2x^2\right] + \right. \\ \left. (1+ax)(1-a^2x^2)^p \text{Hypergeometric2F1}\left[1-p, 1+p, 2+p, \frac{1}{2}(1+ax)\right] \right)$$

Problem 1171: Result unnecessarily involves higher level functions.

$$\int \frac{e^{3 \text{ArcTanh}[ax]}}{(c-a^2cx^2)^{5/2}} dx$$

Optimal (type 3, 185 leaves, 5 steps):

$$\frac{\sqrt{1-a^2x^2}}{6ac^2(1-ax)^3\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)^2\sqrt{c-a^2cx^2}} + \\ \frac{\sqrt{1-a^2x^2}}{8ac^2(1-ax)\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2} \text{ArcTanh}[ax]}{8ac^2\sqrt{c-a^2cx^2}}$$

Result (type 4, 108 leaves):

$$-\left(\left(a\sqrt{1-a^2x^2} \right. \right. \\ \left. \left(\sqrt{-a^2}(-10+9ax-3a^2x^2) - 3ia(-1+ax)^3 \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2}x\right], 1\right] \right) \right) \right) / \\ \left(24(-a^2)^{3/2}c^2(-1+ax)^3\sqrt{c-a^2cx^2} \right)$$

Problem 1172: Result unnecessarily involves higher level functions.

$$\int \frac{e^{3 \text{ArcTanh}[ax]}}{(c-a^2cx^2)^{7/2}} dx$$

Optimal (type 3, 278 leaves, 5 steps):

$$\frac{\sqrt{1-a^2x^2}}{16ac^3(1-ax)^4\sqrt{c-a^2cx^2}} + \frac{\sqrt{1-a^2x^2}}{12ac^3(1-ax)^3\sqrt{c-a^2cx^2}} + \frac{3\sqrt{1-a^2x^2}}{32ac^3(1-ax)^2\sqrt{c-a^2cx^2}} + \\ \frac{\sqrt{1-a^2x^2}}{8ac^3(1-ax)\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}}{32ac^3(1+ax)\sqrt{c-a^2cx^2}} + \frac{5\sqrt{1-a^2x^2} \text{ArcTanh}[ax]}{32ac^3\sqrt{c-a^2cx^2}}$$

Result (type 4, 136 leaves):

$$\begin{aligned}
 & - \left(\left(a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (32 - 15 a x - 35 a^2 x^2 + 45 a^3 x^3 - 15 a^4 x^4) - \right. \right. \right. \\
 & \quad \left. \left. \left. 15 i a (-1 + a x)^4 (1 + a x) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right]\right) \right) \right) / \\
 & \left(96 (-a^2)^{3/2} c^3 (-1 + a x)^4 (1 + a x) \sqrt{c - a^2 c x^2} \right)
 \end{aligned}$$

Problem 1173: Unable to integrate problem.

$$\int e^{3 \operatorname{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$\frac{3 x^{1+m} \sqrt{c - a^2 c x^2}}{(1+m) \sqrt{1 - a^2 x^2}} - \frac{a x^{2+m} \sqrt{c - a^2 c x^2}}{(2+m) \sqrt{1 - a^2 x^2}} + \frac{4 x^{1+m} \sqrt{c - a^2 c x^2} \operatorname{Hypergeometric2F1}\left[1, 1+m, 2+m, a x\right]}{(1+m) \sqrt{1 - a^2 x^2}}$$

Result (type 8, 29 leaves):

$$\int e^{3 \operatorname{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Problem 1174: Unable to integrate problem.

$$\int e^{3 \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Optimal (type 5, 251 leaves, 7 steps):

$$\begin{aligned}
 & - \frac{3 x^{1+m} (c - a^2 c x^2)^p}{(m+2p) \sqrt{1 - a^2 x^2}} - \frac{a x^{2+m} (c - a^2 c x^2)^p}{(1+m+2p) \sqrt{1 - a^2 x^2}} + \frac{1}{(1+m)(m+2p)} \\
 & (3+4m+2p) x^{1+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{1+m}{2}, \frac{3}{2} - p, \frac{3+m}{2}, a^2 x^2\right] + \\
 & \left(a (5+4m+6p) x^{2+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{2+m}{2}, \frac{3}{2} - p, \frac{4+m}{2}, a^2 x^2\right] \right) / \\
 & ((2+m)(1+m+2p))
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int e^{3 \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Problem 1179: Unable to integrate problem.

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Optimal (type 5, 193 leaves, 8 steps):

$$\frac{4 (c - a^2 c x^2)^p}{(1 - 2p) \sqrt{1 - a^2 x^2}} - \frac{a x (c - a^2 c x^2)^p}{2 p \sqrt{1 - a^2 x^2}} + \frac{1}{2 p}$$

$$a (1 + 6 p) x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2} - p, \frac{3}{2}, a^2 x^2\right] -$$

$$\frac{1}{1 + 2 p} \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]$$

Result (type 8, 27 leaves):

$$\int \frac{e^{3 \text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Problem 1180: Unable to integrate problem.

$$\int \frac{e^{3 \text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Optimal (type 5, 187 leaves, 9 steps):

$$\frac{4 a (c - a^2 c x^2)^p}{(1 - 2 p) \sqrt{1 - a^2 x^2}} - \frac{(c - a^2 c x^2)^p}{x \sqrt{1 - a^2 x^2}} +$$

$$a^2 (5 - 2 p) x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2} - p, \frac{3}{2}, a^2 x^2\right] -$$

$$\frac{1}{1 + 2 p} 3 a \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]$$

Result (type 8, 27 leaves):

$$\int \frac{e^{3 \text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Problem 1181: Unable to integrate problem.

$$\int \frac{e^{3 \text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^3} dx$$

Optimal (type 5, 194 leaves, 8 steps):

$$-\frac{(c - a^2 c x^2)^p}{2 x^2 \sqrt{1 - a^2 x^2}} - \frac{3 a (c - a^2 c x^2)^p}{x \sqrt{1 - a^2 x^2}} +$$

$$a^3 (7 - 6 p) x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{2} - p, \frac{3}{2}, a^2 x^2\right] +$$

$$\left(a^2 (9 - 2 p) (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[1, -\frac{1}{2} + p, \frac{1}{2} + p, 1 - a^2 x^2\right] \right) /$$

$$\left(2 (1 - 2 p) \sqrt{1 - a^2 x^2} \right)$$

Result (type 8, 27 leaves):

$$\int \frac{e^{3 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^3} dx$$

Problem 1185: Result more than twice size of optimal antiderivative.

$$\int e^{4 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^2 dx$$

Optimal (type 1, 17 leaves, 2 steps):

$$\frac{c^2 (1 + a x)^5}{5 a}$$

Result (type 1, 49 leaves):

$$c^2 x + 2 a c^2 x^2 + 2 a^2 c^2 x^3 + a^3 c^2 x^4 + \frac{1}{5} a^4 c^2 x^5$$

Problem 1187: Result unnecessarily involves higher level functions.

$$\int \frac{e^{4 \operatorname{ArcTanh}[a x]}}{c - a^2 c x^2} dx$$

Optimal (type 1, 13 leaves, 2 steps):

$$\frac{x}{c (1 - a x)^2}$$

Result (type 3, 18 leaves):

$$\frac{e^{4 \operatorname{ArcTanh}[a x]}}{4 a c}$$

Problem 1191: Result more than twice size of optimal antiderivative.

$$\int e^{4 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p dx$$

Optimal (type 5, 63 leaves, 3 steps):

$$\frac{1}{a (1 - p)} 2^{2+p} c (1 + a x)^{1-p} (c - a^2 c x^2)^{-1+p} \operatorname{Hypergeometric2F1}\left[-2 - p, -1 + p, p, \frac{1}{2} (1 - a x)\right]$$

Result (type 5, 159 leaves):

$$\begin{aligned} & \frac{1}{a (1 + p)} \left(-(-1 + a x)^2 \right)^{-p} (-2 + 2 a x)^p (1 - a^2 x^2)^{-p} \\ & (c - a^2 c x^2)^p \left(a (1 + p) x \left(\frac{1}{2} - \frac{a x}{2} \right)^p \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, a^2 x^2\right] - \right. \\ & \left. (1 + a x) (1 - a^2 x^2)^p \left(2 \operatorname{Hypergeometric2F1}\left[1 - p, 1 + p, 2 + p, \frac{1}{2} (1 + a x)\right] - \right. \right. \\ & \left. \left. \operatorname{Hypergeometric2F1}\left[2 - p, 1 + p, 2 + p, \frac{1}{2} (1 + a x)\right] \right) \right) \end{aligned}$$

Problem 1211: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{e^{-\text{ArcTanh}[a x]}}{\sqrt{c - a^2 c x^2}} dx$$

Optimal (type 3, 39 leaves, 3 steps):

$$\frac{\sqrt{1 - a^2 x^2} \text{Log}[1 + a x]}{a \sqrt{c - a^2 c x^2}}$$

Result (type 4, 87 leaves):

$$-\left(\left(a \sqrt{1 - a^2 x^2} \left(-2 i a \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] + \sqrt{-a^2} \text{Log}\left[-1 + a^2 x^2\right] \right) \right) / \left(2 (-a^2)^{3/2} \sqrt{c - a^2 c x^2} \right) \right)$$

Problem 1212: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-\text{ArcTanh}[a x]}}{(c - a^2 c x^2)^{3/2}} dx$$

Optimal (type 3, 90 leaves, 5 steps):

$$-\frac{\sqrt{1 - a^2 x^2}}{2 a c (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{2 a c \sqrt{c - a^2 c x^2}}$$

Result (type 4, 89 leaves):

$$\left(a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} + i a (1 + a x) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right) \right) / \left(2 (-a^2)^{3/2} (c + a c x) \sqrt{c - a^2 c x^2} \right)$$

Problem 1213: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-\text{ArcTanh}[a x]}}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 183 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{8 a c^2 (1 - a x) \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{8 a c^2 (1 + a x)^2 \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{4 a c^2 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{3 \sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{8 a c^2 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 118 leaves):

$$\begin{aligned}
 & - \left(\left(a \sqrt{1 - a^2 x^2} \right. \right. \\
 & \quad \left. \left. \left(\sqrt{-a^2} (2 - 3 a x - 3 a^2 x^2) - 3 i a (-1 + a x) (1 + a x)^2 \text{EllipticF} [i \text{ArcSinh} [\sqrt{-a^2} x], 1] \right) \right) \right) / \\
 & \quad \left(8 (-a^2)^{3/2} (-1 + a x) (c + a c x)^2 \sqrt{c - a^2 c x^2} \right)
 \end{aligned}$$

Problem 1214: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-\text{ArcTanh}[a x]}}{(c - a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 276 leaves, 5 steps):

$$\begin{aligned}
 & \frac{\sqrt{1 - a^2 x^2}}{32 a c^3 (1 - a x)^2 \sqrt{c - a^2 c x^2}} + \frac{\sqrt{1 - a^2 x^2}}{8 a c^3 (1 - a x) \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{24 a c^3 (1 + a x)^3 \sqrt{c - a^2 c x^2}} - \\
 & \frac{3 \sqrt{1 - a^2 x^2}}{32 a c^3 (1 + a x)^2 \sqrt{c - a^2 c x^2}} - \frac{3 \sqrt{1 - a^2 x^2}}{16 a c^3 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{5 \sqrt{1 - a^2 x^2} \text{ArcTanh}[a x]}{16 a c^3 \sqrt{c - a^2 c x^2}}
 \end{aligned}$$

Result (type 4, 136 leaves):

$$\begin{aligned}
 & - \left(\left(a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (-8 + 25 a x + 25 a^2 x^2 - 15 a^3 x^3 - 15 a^4 x^4) - \right. \right. \right. \\
 & \quad \left. \left. \left. 15 i a (-1 + a x)^2 (1 + a x)^3 \text{EllipticF} [i \text{ArcSinh} [\sqrt{-a^2} x], 1] \right) \right) \right) / \\
 & \quad \left(48 (-a^2)^{3/2} (-1 + a x)^2 (c + a c x)^3 \sqrt{c - a^2 c x^2} \right)
 \end{aligned}$$

Problem 1215: Unable to integrate problem.

$$\int e^{-\text{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Optimal (type 5, 137 leaves, 5 steps):

$$\begin{aligned}
 & \frac{1}{1+m} x^{1+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1} \left[\frac{1+m}{2}, \frac{1}{2} - p, \frac{3+m}{2}, a^2 x^2 \right] - \\
 & \frac{1}{2+m} a x^{2+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1} \left[\frac{2+m}{2}, \frac{1}{2} - p, \frac{4+m}{2}, a^2 x^2 \right]
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int e^{-\text{ArcTanh}[a x]} x^m (c - a^2 c x^2)^p dx$$

Problem 1216: Result more than twice size of optimal antiderivative.

$$\int e^{-\text{ArcTanh}[a x]} x^3 (1 - a^2 x^2)^p dx$$

Optimal (type 5, 85 leaves, 6 steps):

$$-\frac{(1-a^2 x^2)^{\frac{1}{2}+p}}{a^4(1+2p)} + \frac{(1-a^2 x^2)^{\frac{3}{2}+p}}{a^4(3+2p)} - \frac{1}{5} a x^5 \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1}{2}-p, \frac{7}{2}, a^2 x^2\right]$$

Result (type 5, 183 leaves):

$$\frac{1}{3 a^4} \left(3 a x \text{Hypergeometric2F1}\left[\frac{1}{2}, -\frac{1}{2}-p, \frac{3}{2}, a^2 x^2\right] + \frac{1}{3+2p} \left(-3 + 3(1-a^2 x^2)^{\frac{1}{2}+p} - 3 a^2 x^2 (1-a^2 x^2)^{\frac{1}{2}+p} + a^3(3+2p)x^3 \text{Hypergeometric2F1}\left[\frac{3}{2}, -\frac{1}{2}-p, \frac{5}{2}, a^2 x^2\right] + 3(1-ax)(1+ax)^{-\frac{1}{2}-p}(2-2a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[\frac{1}{2}-p, \frac{3}{2}+p, \frac{5}{2}+p, \frac{1}{2}-\frac{ax}{2}\right] \right) \right)$$

Problem 1220: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-\text{ArcTanh}[ax]} (1-a^2 x^2)^p}{x} dx$$

Optimal (type 5, 73 leaves, 5 steps):

$$-a x \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-p, \frac{3}{2}, a^2 x^2\right] - \frac{(1-a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[1, \frac{1}{2}+p, \frac{3}{2}+p, 1-a^2 x^2\right]}{1+2p}$$

Result (type 5, 148 leaves):

$$(1-a^2 x^2)^{\frac{1}{2}+p} \left(\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2}-p, -\frac{1}{2}-p, \frac{1}{2}-p, \frac{1}{a^2 x^2}\right]}{\left(1-\frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p} + 2p \left(1-\frac{1}{a^2 x^2}\right)^{\frac{1}{2}+p}} + \frac{1}{3+2p} \right) + 2^{\frac{1}{2}+p} (1-ax)(1+ax)^{-\frac{1}{2}-p} \text{Hypergeometric2F1}\left[\frac{1}{2}-p, \frac{3}{2}+p, \frac{5}{2}+p, \frac{1}{2}-\frac{ax}{2}\right]$$

Problem 1221: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{-\text{ArcTanh}[ax]} (1-a^2 x^2)^p}{x^2} dx$$

Optimal (type 5, 74 leaves, 5 steps):

$$-\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2}-p, \frac{1}{2}, a^2 x^2\right]}{x} + \frac{a(1-a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[1, \frac{1}{2}+p, \frac{3}{2}+p, 1-a^2 x^2\right]}{1+2p}$$

Result (type 5, 171 leaves):

$$\begin{aligned}
 & -\frac{\text{Hypergeometric2F1}\left[-\frac{1}{2}, -\frac{1}{2}-p, \frac{1}{2}, a^2 x^2\right]}{x} - \frac{1}{1+2p} \\
 & a \left(1 - \frac{1}{a^2 x^2}\right)^{-\frac{1}{2}-p} (1 - a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[-\frac{1}{2}-p, -\frac{1}{2}-p, \frac{1}{2}-p, \frac{1}{a^2 x^2}\right] + \frac{1}{3+2p} \\
 & a (-1 + a x) (1 + a x)^{-\frac{1}{2}-p} (2 - 2 a^2 x^2)^{\frac{1}{2}+p} \text{Hypergeometric2F1}\left[\frac{1}{2}-p, \frac{3}{2}+p, \frac{5}{2}+p, \frac{1}{2} - \frac{a x}{2}\right]
 \end{aligned}$$

Problem 1222: Result more than twice size of optimal antiderivative.

$$\int e^{-\text{ArcTanh}[a x]} x^3 (c - a^2 c x^2)^p dx$$

Optimal (type 5, 134 leaves, 7 steps):

$$\begin{aligned}
 & -\frac{\sqrt{1-a^2 x^2} (c - a^2 c x^2)^p}{a^4 (1+2p)} + \frac{(1-a^2 x^2)^{3/2} (c - a^2 c x^2)^p}{a^4 (3+2p)} - \\
 & \frac{1}{5} a x^5 (1-a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{5}{2}, \frac{1}{2}-p, \frac{7}{2}, a^2 x^2\right]
 \end{aligned}$$

Result (type 5, 290 leaves):

$$\begin{aligned}
 & \frac{1}{a^4 (1+2p) (3+2p) (5+2p) (7+2p)} \\
 & 4^{1+p} \left(\frac{e^{\text{ArcTanh}[a x]}}{1 + e^{2 \text{ArcTanh}[a x]}} \right)^{1+2p} (1 + e^{2 \text{ArcTanh}[a x]})^{1+2p} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \\
 & \left(- (105 + 142 p + 60 p^2 + 8 p^3) \text{Hypergeometric2F1}\left[\frac{1}{2}+p, 5+2p, \frac{3}{2}+p, -e^{2 \text{ArcTanh}[a x]}\right] + \right. \\
 & \quad e^{2 \text{ArcTanh}[a x]} (1+2p) \\
 & \quad \left(3 (35 + 24 p + 4 p^2) \text{Hypergeometric2F1}\left[\frac{3}{2}+p, 5+2p, \frac{5}{2}+p, -e^{2 \text{ArcTanh}[a x]}\right] + \right. \\
 & \quad \quad e^{2 \text{ArcTanh}[a x]} (3+2p) \left(-3 (7+2p) \text{Hypergeometric2F1}\left[\frac{5}{2}+p, 5+2p, \frac{7}{2}+p, -e^{2 \text{ArcTanh}[a x]}\right] + \right. \\
 & \quad \quad \quad \left. \left. \left. e^{2 \text{ArcTanh}[a x]} (5+2p) \text{Hypergeometric2F1}\left[\frac{7}{2}+p, 5+2p, \frac{9}{2}+p, -e^{2 \text{ArcTanh}[a x]}\right]\right)\right)\right) \left. \right)
 \end{aligned}$$

Problem 1226: Unable to integrate problem.

$$\int \frac{e^{-\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Optimal (type 5, 111 leaves, 6 steps):

$$\begin{aligned}
 & -a x (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1}{2}-p, \frac{3}{2}, a^2 x^2\right] - \\
 & \frac{1}{1+2p} \sqrt{1-a^2 x^2} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[1, \frac{1}{2}+p, \frac{3}{2}+p, 1-a^2 x^2\right]
 \end{aligned}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{-\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x} dx$$

Problem 1227: Unable to integrate problem.

$$\int \frac{e^{-\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Optimal (type 5, 112 leaves, 6 steps):

$$-\frac{(1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1}{2} - p, \frac{1}{2}, a^2 x^2\right]}{x} + \frac{1}{1 + 2p} a \sqrt{1 - a^2 x^2} (c - a^2 c x^2)^p \text{Hypergeometric2F1}\left[1, \frac{1}{2} + p, \frac{3}{2} + p, 1 - a^2 x^2\right]$$

Result (type 8, 27 leaves):

$$\int \frac{e^{-\text{ArcTanh}[a x]} (c - a^2 c x^2)^p}{x^2} dx$$

Problem 1232: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-2 \text{ArcTanh}[a x]}}{c - a^2 c x^2} dx$$

Optimal (type 1, 15 leaves, 2 steps):

$$-\frac{1}{a c (1 + a x)}$$

Result (type 3, 18 leaves):

$$-\frac{e^{-2 \text{ArcTanh}[a x]}}{2 a c}$$

Problem 1252: Result unnecessarily involves higher level functions.

$$\int e^{-2 \text{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Optimal (type 5, 172 leaves, 7 steps):

$$-\frac{x^{1+m} \sqrt{c - a^2 c x^2}}{2 + m} + \frac{c (3 + 2m) x^{1+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{(1 + m) (2 + m) \sqrt{c - a^2 c x^2}} - \frac{2 a c x^{2+m} \sqrt{1 - a^2 x^2} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+m}{2}, \frac{4+m}{2}, a^2 x^2\right]}{(2 + m) \sqrt{c - a^2 c x^2}}$$

Result (type 6, 192 leaves):

$$\frac{1}{1+m} x^{1+m} \left(-\frac{\sqrt{c-a^2 c x^2} \operatorname{Hypergeometric2F1}\left[-\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, a^2 x^2\right]}{\sqrt{1-a^2 x^2}} - \right. \\ \left. \left(4(2+m) \sqrt{c-a c x} \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -a x, a x\right] \right) / \right. \\ \left. \left(\sqrt{1+a x} \left(-2(2+m) \operatorname{AppellF1}\left[1+m, \frac{1}{2}, -\frac{1}{2}, 2+m, -a x, a x\right] + a x \left(\operatorname{AppellF1}\left[2+m, \frac{3}{2}, -\frac{1}{2}, 3+m, -a x, a x\right] + \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, 1+\frac{m}{2}\right\}, \left\{2+\frac{m}{2}\right\}, a^2 x^2\right]\right) \right) \right) \right)$$

Problem 1253: Result more than twice size of optimal antiderivative.

$$\int e^{-2 \operatorname{ArcTanh}[a x]} (c - a^2 c x^2)^p dx$$

Optimal (type 5, 54 leaves, 3 steps):

$$\frac{1}{a p} 2^{1+p} (1 - a x)^{-p} (c - a^2 c x^2)^p \operatorname{Hypergeometric2F1}\left[-1-p, p, 1+p, \frac{1}{2}(1+a x)\right]$$

Result (type 5, 125 leaves):

$$\frac{1}{a(1+p)} 2^p (1+a x)^{-p} (1-a^2 x^2)^{-p} (c - a^2 c x^2)^p \\ \left(-a(1+p) x \left(\frac{1}{2} + \frac{a x}{2} \right)^p \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, a^2 x^2\right] + \right. \\ \left. (-1+a x) (1-a^2 x^2)^p \operatorname{Hypergeometric2F1}\left[1-p, 1+p, 2+p, \frac{1}{2} - \frac{a x}{2}\right] \right)$$

Problem 1277: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[a x]}}{(c - a^2 c x^2)^{5/2}} dx$$

Optimal (type 3, 182 leaves, 5 steps):

$$-\frac{\sqrt{1-a^2 x^2}}{6 a c^2 (1+a x)^3 \sqrt{c-a^2 c x^2}} - \frac{\sqrt{1-a^2 x^2}}{8 a c^2 (1+a x)^2 \sqrt{c-a^2 c x^2}} - \\ \frac{\sqrt{1-a^2 x^2}}{8 a c^2 (1+a x) \sqrt{c-a^2 c x^2}} + \frac{\sqrt{1-a^2 x^2} \operatorname{ArcTanh}[a x]}{8 a c^2 \sqrt{c-a^2 c x^2}}$$

Result (type 4, 108 leaves):

$$\left(a \sqrt{1-a^2 x^2} \left(\sqrt{-a^2} (10+9 a x+3 a^2 x^2) + 3 i a (1+a x)^3 \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-a^2} x\right], 1\right] \right) \right) / \\ \left(24 (-a^2)^{3/2} c^2 (1+a x)^3 \sqrt{c-a^2 c x^2} \right)$$

Problem 1278: Result unnecessarily involves higher level functions.

$$\int \frac{e^{-3 \operatorname{ArcTanh}[a x]}}{(c - a^2 c x^2)^{7/2}} dx$$

Optimal (type 3, 275 leaves, 5 steps):

$$\frac{\sqrt{1 - a^2 x^2}}{32 a c^3 (1 - a x) \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{16 a c^3 (1 + a x)^4 \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{12 a c^3 (1 + a x)^3 \sqrt{c - a^2 c x^2}} - \frac{3 \sqrt{1 - a^2 x^2}}{32 a c^3 (1 + a x)^2 \sqrt{c - a^2 c x^2}} - \frac{\sqrt{1 - a^2 x^2}}{8 a c^3 (1 + a x) \sqrt{c - a^2 c x^2}} + \frac{5 \sqrt{1 - a^2 x^2} \operatorname{ArcTanh}[a x]}{32 a c^3 \sqrt{c - a^2 c x^2}}$$

Result (type 4, 136 leaves):

$$- \left(\left(a \sqrt{1 - a^2 x^2} \left(\sqrt{-a^2} (32 + 15 a x - 35 a^2 x^2 - 45 a^3 x^3 - 15 a^4 x^4) - 15 i a (-1 + a x) (1 + a x)^4 \operatorname{EllipticF}[i \operatorname{ArcSinh}[\sqrt{-a^2} x], 1] \right) \right) / \left(96 (-a^2)^{3/2} c^3 (-1 + a x) (1 + a x)^4 \sqrt{c - a^2 c x^2} \right) \right)$$

Problem 1279: Unable to integrate problem.

$$\int e^{-3 \operatorname{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Optimal (type 5, 136 leaves, 5 steps):

$$- \frac{3 x^{1+m} \sqrt{c - a^2 c x^2}}{(1 + m) \sqrt{1 - a^2 x^2}} + \frac{a x^{2+m} \sqrt{c - a^2 c x^2}}{(2 + m) \sqrt{1 - a^2 x^2}} + \frac{4 x^{1+m} \sqrt{c - a^2 c x^2} \operatorname{Hypergeometric2F1}[1, 1 + m, 2 + m, -a x]}{(1 + m) \sqrt{1 - a^2 x^2}}$$

Result (type 8, 29 leaves):

$$\int e^{-3 \operatorname{ArcTanh}[a x]} x^m \sqrt{c - a^2 c x^2} dx$$

Problem 1281: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (1 - a^2 x^2)^{5/2} dx$$

Optimal (type 3, 359 leaves, 18 steps):

$$\begin{aligned}
 & \frac{231 (1 - a x)^{1/4} (1 + a x)^{3/4}}{512 a} + \frac{231 (1 - a x)^{5/4} (1 + a x)^{3/4}}{1280 a} + \\
 & \frac{77 (1 - a x)^{9/4} (1 + a x)^{3/4}}{960 a} - \frac{77 (1 - a x)^{13/4} (1 + a x)^{3/4}}{480 a} - \frac{11 (1 - a x)^{13/4} (1 + a x)^{7/4}}{60 a} - \\
 & \frac{(1 - a x)^{13/4} (1 + a x)^{11/4}}{6 a} + \frac{231 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{512 \sqrt{2} a} - \frac{231 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{512 \sqrt{2} a} + \\
 & \frac{231 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{1024 \sqrt{2} a} - \frac{231 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{1024 \sqrt{2} a}
 \end{aligned}$$

Result (type 7, 422 leaves):

$$\begin{aligned}
 & \frac{1}{1920 a (1 + e^{2 \operatorname{ArcTanh}[a x]})^6} \left(960 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} (1 + e^{2 \operatorname{ArcTanh}[a x]})^4 (-1 + 3 e^{2 \operatorname{ArcTanh}[a x]}) - \right. \\
 & 360 (1 + e^{2 \operatorname{ArcTanh}[a x]})^6 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1} \&\right] + \\
 & 80 (1 + e^{2 \operatorname{ArcTanh}[a x]})^2 \left(\frac{39 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1} \&\right]}{(-1 + a^2 x^2)^2} - \right. \\
 & \left. \left(\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] \right) \right. \\
 & \left. \left(13 \operatorname{Cosh}[3 \operatorname{ArcTanh}[a x]] + \frac{7 - 166 a x + 26 \sqrt{1 - a^2 x^2} \operatorname{Sinh}[3 \operatorname{ArcTanh}[a x]]}{\sqrt{1 - a^2 x^2}} \right) \right) \\
 & \left(\operatorname{Cosh}[4 \operatorname{ArcTanh}[a x]] + \operatorname{Sinh}[4 \operatorname{ArcTanh}[a x]] \right) - \\
 & \left(- \frac{3300 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1} \&\right]}{(-1 + a^2 x^2)^3} - \right. \\
 & \left(\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] \right) \\
 & \left(\frac{286}{\sqrt{1 - a^2 x^2}} + \frac{12556 a x}{\sqrt{1 - a^2 x^2}} - 129 \operatorname{Cosh}[3 \operatorname{ArcTanh}[a x]] + 275 \operatorname{Cosh}[5 \operatorname{ArcTanh}[a x]] - \right. \\
 & \left. 7374 \operatorname{Sinh}[3 \operatorname{ArcTanh}[a x]] + 550 \operatorname{Sinh}[5 \operatorname{ArcTanh}[a x]] \right) \left. \right) \\
 & \left. \left(\operatorname{Cosh}[6 \operatorname{ArcTanh}[a x]] + \operatorname{Sinh}[6 \operatorname{ArcTanh}[a x]] \right) \right)
 \end{aligned}$$

Problem 1282: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} (1 - a^2 x^2)^{3/2} dx$$

Optimal (type 3, 307 leaves, 16 steps):

$$\frac{35 (1 - a x)^{1/4} (1 + a x)^{3/4}}{64 a} + \frac{7 (1 - a x)^{5/4} (1 + a x)^{3/4}}{32 a} - \frac{7 (1 - a x)^{9/4} (1 + a x)^{3/4}}{24 a} -$$

$$\frac{(1 - a x)^{9/4} (1 + a x)^{7/4}}{4 a} + \frac{35 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a} - \frac{35 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a} +$$

$$\frac{35 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a} - \frac{35 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a}$$

Result (type 7, 249 leaves):

$$\frac{1}{48 a (1 + e^{2 \operatorname{ArcTanh}[a x]})^4} \left(24 e^{\frac{3}{2} \operatorname{ArcTanh}[a x]} (1 + e^{2 \operatorname{ArcTanh}[a x]})^2 (-1 + 3 e^{2 \operatorname{ArcTanh}[a x]}) - \right.$$

$$9 (1 + e^{2 \operatorname{ArcTanh}[a x]})^4 \operatorname{RootSum}\left[1 + \#1^4, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \left. + \right.$$

$$\left(\frac{39 \operatorname{RootSum}\left[1 + \#1^4 \&, \frac{\operatorname{ArcTanh}[a x] - 2 \operatorname{Log}\left[e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \&}{(-1 + a^2 x^2)^2} - \right.$$

$$\left(\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcTanh}[a x]\right] \right) \left(\right.$$

$$\left. \left. \left(13 \operatorname{Cosh}\left[3 \operatorname{ArcTanh}[a x]\right] + \frac{7 - 166 a x + 26 \sqrt{1 - a^2 x^2} \operatorname{Sinh}\left[3 \operatorname{ArcTanh}[a x]\right]}{\sqrt{1 - a^2 x^2}} \right) \right) \right.$$

$$\left. \left. \left(\operatorname{Cosh}\left[4 \operatorname{ArcTanh}[a x]\right] + \operatorname{Sinh}\left[4 \operatorname{ArcTanh}[a x]\right] \right) \right) \right)$$

Problem 1283: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \operatorname{ArcTanh}[a x]} \sqrt{1 - a^2 x^2} \, dx$$

Optimal (type 3, 255 leaves, 14 steps):

$$\frac{3 (1 - a x)^{1/4} (1 + a x)^{3/4}}{4 a} - \frac{(1 - a x)^{5/4} (1 + a x)^{3/4}}{2 a} + \frac{3 \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a} -$$

$$\frac{3 \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{4 \sqrt{2} a} + \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a} - \frac{3 \operatorname{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{8 \sqrt{2} a}$$

Result (type 7, 83 leaves):

$$\frac{1}{16 a} \left(\frac{8 e^{\frac{3}{2} \text{ArcTanh}[a x]} (-1 + 3 e^{2 \text{ArcTanh}[a x]})}{(1 + e^{2 \text{ArcTanh}[a x]})^2} - \right. \\ \left. 3 \text{RootSum}\left[1 + \#1^4 \ \&, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1} \ \&\right] \right)$$

Problem 1284: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2} \text{ArcTanh}[a x]}}{\sqrt{1 - a^2 x^2}} dx$$

Optimal (type 3, 193 leaves, 12 steps):

$$\frac{\sqrt{2} \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{a} - \frac{\sqrt{2} \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{a} + \\ \frac{\text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a} - \frac{\text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{\sqrt{2} a}$$

Result (type 7, 46 leaves):

$$\frac{\text{RootSum}\left[1 + \#1^4 \ \&, \frac{-\text{ArcTanh}[a x] + 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1} \ \&\right]}{2 a}$$

Problem 1289: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \text{ArcTanh}[a x]} (c - a^2 c x^2)^{5/2} dx$$

Optimal (type 3, 679 leaves, 19 steps):

$$\frac{231 c^2 (1 - a x)^{1/4} (1 + a x)^{3/4} \sqrt{c - a^2 c x^2}}{512 a \sqrt{1 - a^2 x^2}} + \frac{231 c^2 (1 - a x)^{5/4} (1 + a x)^{3/4} \sqrt{c - a^2 c x^2}}{1280 a \sqrt{1 - a^2 x^2}} + \\ \frac{77 c^2 (1 - a x)^{9/4} (1 + a x)^{3/4} \sqrt{c - a^2 c x^2}}{960 a \sqrt{1 - a^2 x^2}} - \frac{77 c^2 (1 - a x)^{13/4} (1 + a x)^{3/4} \sqrt{c - a^2 c x^2}}{480 a \sqrt{1 - a^2 x^2}} - \\ \frac{11 c^2 (1 - a x)^{13/4} (1 + a x)^{7/4} \sqrt{c - a^2 c x^2}}{60 a \sqrt{1 - a^2 x^2}} - \frac{c^2 (1 - a x)^{13/4} (1 + a x)^{11/4} \sqrt{c - a^2 c x^2}}{6 a \sqrt{1 - a^2 x^2}} + \\ \frac{231 c^2 \sqrt{c - a^2 c x^2} \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{512 \sqrt{2} a \sqrt{1 - a^2 x^2}} - \frac{231 c^2 \sqrt{c - a^2 c x^2} \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{512 \sqrt{2} a \sqrt{1 - a^2 x^2}} + \\ \frac{231 c^2 \sqrt{c - a^2 c x^2} \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{1024 \sqrt{2} a \sqrt{1 - a^2 x^2}} - \frac{231 c^2 \sqrt{c - a^2 c x^2} \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{1024 \sqrt{2} a \sqrt{1 - a^2 x^2}}$$

Result (type 7, 167 leaves):

$$\begin{aligned}
 & - \left(\left(c^3 \sqrt{1 - a^2 x^2} \left(-8 e^{\frac{3}{2} \text{ArcTanh}[a x]} \left(-1155 - 6435 e^{2 \text{ArcTanh}[a x]} - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. 14670 e^{4 \text{ArcTanh}[a x]} + 48202 e^{6 \text{ArcTanh}[a x]} + 20097 e^{8 \text{ArcTanh}[a x]} + 3465 e^{10 \text{ArcTanh}[a x]} \right) + \right. \right. \\
 & \quad \left. \left. 3465 \left(1 + e^{2 \text{ArcTanh}[a x]} \right)^6 \text{RootSum}\left[1 + \#1^4, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right] \right) / \\
 & \left. \left(30720 a \left(1 + e^{2 \text{ArcTanh}[a x]} \right)^6 \sqrt{c - a^2 c x^2} \right) \right)
 \end{aligned}$$

Problem 1290: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \text{ArcTanh}[a x]} (c - a^2 c x^2)^{3/2} dx$$

Optimal (type 3, 547 leaves, 17 steps):

$$\begin{aligned}
 & \frac{35 c (1 - a x)^{1/4} (1 + a x)^{3/4} \sqrt{c - a^2 c x^2}}{64 a \sqrt{1 - a^2 x^2}} + \frac{7 c (1 - a x)^{5/4} (1 + a x)^{3/4} \sqrt{c - a^2 c x^2}}{32 a \sqrt{1 - a^2 x^2}} - \\
 & \frac{7 c (1 - a x)^{9/4} (1 + a x)^{3/4} \sqrt{c - a^2 c x^2}}{24 a \sqrt{1 - a^2 x^2}} - \frac{c (1 - a x)^{9/4} (1 + a x)^{7/4} \sqrt{c - a^2 c x^2}}{4 a \sqrt{1 - a^2 x^2}} + \\
 & \frac{35 c \sqrt{c - a^2 c x^2} \text{ArcTan}\left[1 - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a \sqrt{1 - a^2 x^2}} - \frac{35 c \sqrt{c - a^2 c x^2} \text{ArcTan}\left[1 + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{64 \sqrt{2} a \sqrt{1 - a^2 x^2}} + \\
 & \frac{35 c \sqrt{c - a^2 c x^2} \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} - \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a \sqrt{1 - a^2 x^2}} - \frac{35 c \sqrt{c - a^2 c x^2} \text{Log}\left[1 + \frac{\sqrt{1 - a x}}{\sqrt{1 + a x}} + \frac{\sqrt{2} (1 - a x)^{1/4}}{(1 + a x)^{1/4}}\right]}{128 \sqrt{2} a \sqrt{1 - a^2 x^2}}
 \end{aligned}$$

Result (type 7, 147 leaves):

$$\begin{aligned}
 & - \left(\left(c^2 \sqrt{1 - a^2 x^2} \left(-8 e^{\frac{3}{2} \text{ArcTanh}[a x]} \left(-35 - 125 e^{2 \text{ArcTanh}[a x]} + 399 e^{4 \text{ArcTanh}[a x]} + 105 e^{6 \text{ArcTanh}[a x]} \right) + \right. \right. \right. \\
 & \quad \left. \left. 105 \left(1 + e^{2 \text{ArcTanh}[a x]} \right)^4 \text{RootSum}\left[1 + \#1^4, \frac{\text{ArcTanh}[a x] - 2 \text{Log}\left[e^{\frac{1}{2} \text{ArcTanh}[a x]} - \#1\right]}{\#1}\right] \& \right] \right) / \\
 & \left. \left(768 a \left(1 + e^{2 \text{ArcTanh}[a x]} \right)^4 \sqrt{c - a^2 c x^2} \right) \right)
 \end{aligned}$$

Problem 1291: Result is not expressed in closed-form.

$$\int e^{\frac{1}{2} \text{ArcTanh}[a x]} \sqrt{c - a^2 c x^2} dx$$

Optimal (type 3, 429 leaves, 15 steps):

$$\frac{3(1-ax)^{1/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{4a\sqrt{1-a^2x^2}} - \frac{(1-ax)^{5/4}(1+ax)^{3/4}\sqrt{c-a^2cx^2}}{2a\sqrt{1-a^2x^2}} +$$

$$\frac{3\sqrt{c-a^2cx^2}\operatorname{ArcTan}\left[1-\frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a\sqrt{1-a^2x^2}} - \frac{3\sqrt{c-a^2cx^2}\operatorname{ArcTan}\left[1+\frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{4\sqrt{2}a\sqrt{1-a^2x^2}} +$$

$$\frac{3\sqrt{c-a^2cx^2}\operatorname{Log}\left[1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a\sqrt{1-a^2x^2}} - \frac{3\sqrt{c-a^2cx^2}\operatorname{Log}\left[1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{8\sqrt{2}a\sqrt{1-a^2x^2}}$$

Result (type 7, 126 leaves):

$$\left(c\sqrt{1-a^2x^2} \left(8e^{\frac{3}{2}\operatorname{ArcTanh}[ax]} (-1+3e^{2\operatorname{ArcTanh}[ax]}) - \right. \right.$$

$$\left. \left. 3(1+e^{2\operatorname{ArcTanh}[ax]})^2 \operatorname{RootSum}\left[1+\#1^4, \frac{\operatorname{ArcTanh}[ax]-2\operatorname{Log}\left[e^{\frac{1}{2}\operatorname{ArcTanh}[ax]}-\#1\right]}{\#1}\right] \& \right] \right) /$$

$$\left(16a(1+e^{2\operatorname{ArcTanh}[ax]})^2\sqrt{c(1-a^2x^2)} \right)$$

Problem 1292: Result is not expressed in closed-form.

$$\int \frac{e^{\frac{1}{2}\operatorname{ArcTanh}[ax]}}{\sqrt{c-a^2cx^2}} dx$$

Optimal (type 3, 309 leaves, 13 steps):

$$\frac{\sqrt{2}\sqrt{1-a^2x^2}\operatorname{ArcTan}\left[1-\frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{a\sqrt{c-a^2cx^2}} - \frac{\sqrt{2}\sqrt{1-a^2x^2}\operatorname{ArcTan}\left[1+\frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{a\sqrt{c-a^2cx^2}} +$$

$$\frac{\sqrt{1-a^2x^2}\operatorname{Log}\left[1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}-\frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a\sqrt{c-a^2cx^2}} - \frac{\sqrt{1-a^2x^2}\operatorname{Log}\left[1+\frac{\sqrt{1-ax}}{\sqrt{1+ax}}+\frac{\sqrt{2}(1-ax)^{1/4}}{(1+ax)^{1/4}}\right]}{\sqrt{2}a\sqrt{c-a^2cx^2}}$$

Result (type 7, 79 leaves):

$$\frac{\sqrt{c(1-a^2x^2)}\operatorname{RootSum}\left[1+\#1^4, \frac{-\operatorname{ArcTanh}[ax]+2\operatorname{Log}\left[e^{\frac{1}{2}\operatorname{ArcTanh}[ax]}-\#1\right]}{\#1}\right] \&}{2ac\sqrt{1-a^2x^2}}$$

Problem 1307: Unable to integrate problem.

$$\int \frac{e^{\frac{1}{2}\operatorname{ArcTanh}[ax]}}{x(c-a^2cx^2)^{9/8}} dx$$

Optimal (type 6, 73 leaves, 3 steps):

$$-\frac{1}{c (c - a^2 c x^2)^{1/8}} 2 \times 2^{5/8} (1 + a x)^{1/8} (1 - a^2 x^2)^{1/8} \text{AppellF1}\left[\frac{1}{8}, \frac{11}{8}, 1, \frac{9}{8}, \frac{1}{2} (1 + a x), 1 + a x\right]$$

Result (type 8, 31 leaves):

$$\int \frac{e^{\frac{1}{2} \text{ArcTanh}[a x]}}{x (c - a^2 c x^2)^{9/8}} dx$$

Problem 1309: Result more than twice size of optimal antiderivative.

$$\int e^{n \text{ArcTanh}[a x]} (c - a^2 c x^2)^2 dx$$

Optimal (type 5, 70 leaves, 2 steps):

$$-\frac{1}{a (6 - n)} 2^{3 + \frac{n}{2}} c^2 (1 - a x)^{3 - \frac{n}{2}} \text{Hypergeometric2F1}\left[-2 - \frac{n}{2}, 3 - \frac{n}{2}, 4 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]$$

Result (type 5, 184 leaves):

$$\frac{1}{120 a} c^2 e^{n \text{ArcTanh}[a x]} \left(22 n - n^3 + 120 a x - 22 a n^2 x + a n^4 x - 28 a^2 n x^2 + a^2 n^3 x^2 - 80 a^3 x^3 + 2 a^3 n^2 x^3 + 6 a^4 n x^4 + 24 a^5 x^5 - e^{2 \text{ArcTanh}[a x]} n (32 - 16 n - 2 n^2 + n^3) \text{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, -e^{2 \text{ArcTanh}[a x]}\right] + (64 - 20 n^2 + n^4) \text{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, -e^{2 \text{ArcTanh}[a x]}\right] \right)$$

Problem 1310: Result more than twice size of optimal antiderivative.

$$\int e^{n \text{ArcTanh}[a x]} (c - a^2 c x^2)^3 dx$$

Optimal (type 5, 70 leaves, 2 steps):

$$-\frac{1}{a (8 - n)} 2^{4 + \frac{n}{2}} c^3 (1 - a x)^{4 - \frac{n}{2}} \text{Hypergeometric2F1}\left[-3 - \frac{n}{2}, 4 - \frac{n}{2}, 5 - \frac{n}{2}, \frac{1}{2} (1 - a x)\right]$$

Result (type 5, 272 leaves):

$$\frac{1}{5040 a} c^3 e^{n \text{ArcTanh}[a x]} \left(-912 n + 58 n^3 - n^5 - 5040 a x + 912 a n^2 x - 58 a n^4 x + a n^6 x + 1368 a^2 n x^2 - 64 a^2 n^3 x^2 + a^2 n^5 x^2 + 5040 a^3 x^3 - 152 a^3 n^2 x^3 + 2 a^3 n^4 x^3 - 576 a^4 n x^4 + 6 a^4 n^3 x^4 - 3024 a^5 x^5 + 24 a^5 n^2 x^5 + 120 a^6 n x^6 + 720 a^7 x^7 - e^{2 \text{ArcTanh}[a x]} n (-1152 + 576 n + 104 n^2 - 52 n^3 - 2 n^4 + n^5) \text{Hypergeometric2F1}\left[1, 1 + \frac{n}{2}, 2 + \frac{n}{2}, -e^{2 \text{ArcTanh}[a x]}\right] + (-2304 + 784 n^2 - 56 n^4 + n^6) \text{Hypergeometric2F1}\left[1, \frac{n}{2}, 1 + \frac{n}{2}, -e^{2 \text{ArcTanh}[a x]}\right] \right)$$

Problem 1356: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^2 dx$$

Optimal (type 6, 42 leaves, 2 steps):

$$\frac{c^2 x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{1}{2}(-4+n), -2-\frac{n}{2}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 27 leaves):

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2)^2 dx$$

Problem 1357: Unable to integrate problem.

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2) dx$$

Optimal (type 6, 40 leaves, 2 steps):

$$\frac{c x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{1}{2}(-2+n), -1-\frac{n}{2}, 2+m, a x, -a x\right]}{1+m}$$

Result (type 8, 25 leaves):

$$\int e^{n \operatorname{ArcTanh}[a x]} x^m (c - a^2 c x^2) dx$$

Problem 1358: Result more than twice size of optimal antiderivative.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]} x^m}{c - a^2 c x^2} dx$$

Optimal (type 6, 42 leaves, 2 steps):

$$\frac{x^{1+m} \operatorname{AppellF1}\left[1+m, \frac{2+n}{2}, 1-\frac{n}{2}, 2+m, a x, -a x\right]}{c(1+m)}$$

Result (type 6, 106 leaves):

$$\frac{1}{a c n} e^{n \operatorname{ArcTanh}[a x]} \left(-1 + e^{-2 \operatorname{ArcTanh}[a x]}\right)^m \left(1 + e^{-2 \operatorname{ArcTanh}[a x]}\right)^m \left(-e^{-4 \operatorname{ArcTanh}[a x]} \left(-1 + e^{2 \operatorname{ArcTanh}[a x]}\right)^2\right)^{-m} x^m \operatorname{AppellF1}\left[-\frac{n}{2}, m, -m, 1-\frac{n}{2}, -e^{-2 \operatorname{ArcTanh}[a x]}, e^{-2 \operatorname{ArcTanh}[a x]}\right]$$

Problem 1359: Unable to integrate problem.

$$\int \frac{e^{n \operatorname{ArcTanh}[a x]} x^m}{(c - a^2 c x^2)^2} dx$$

Optimal (type 6, 42 leaves, 2 steps):

$$\frac{x^{1+m} \text{AppellF1}\left[1+m, \frac{4+n}{2}, 2-\frac{n}{2}, 2+m, ax, -ax\right]}{c^2 (1+m)}$$

Result (type 8, 27 leaves):

$$\int \frac{e^{n \text{ArcTanh}[ax]} x^m}{(c - a^2 c x^2)^2} dx$$

Problem 1360: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[ax]} x^m (c - a^2 c x^2)^p dx$$

Optimal (type 6, 70 leaves, 3 steps):

$$\frac{1}{1+m} x^{1+m} (1 - a^2 x^2)^{-p} (c - a^2 c x^2)^p \text{AppellF1}\left[1+m, \frac{1}{2}(n-2p), -\frac{n}{2}-p, 2+m, ax, -ax\right]$$

Result (type 8, 27 leaves):

$$\int e^{n \text{ArcTanh}[ax]} x^m (c - a^2 c x^2)^p dx$$

Problem 1361: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[ax]} x (c - a^2 c x^2)^p dx$$

Optimal (type 5, 177 leaves, 4 steps):

$$\frac{(1-ax)^{1-\frac{n}{2}+p} (1+ax)^{1+\frac{n}{2}+p} (1-a^2 x^2)^{-p} (c-a^2 c x^2)^p}{2 a^2 (1+p)} - \left(2^{\frac{n}{2}+p} n (1-ax)^{1-\frac{n}{2}+p} (1-a^2 x^2)^{-p} (c-a^2 c x^2)^p \text{Hypergeometric2F1}\left[-\frac{n}{2}-p, 1-\frac{n}{2}+p, 2-\frac{n}{2}+p, \frac{1}{2}(1-ax)\right] \right) / (a^2 (1+p) (2-n+2p))$$

Result (type 8, 25 leaves):

$$\int e^{n \text{ArcTanh}[ax]} x (c - a^2 c x^2)^p dx$$

Problem 1362: Unable to integrate problem.

$$\int e^{n \text{ArcTanh}[ax]} (c - a^2 c x^2)^p dx$$

Optimal (type 5, 103 leaves, 3 steps):

$$-\frac{1}{a(2-n+2p)} 2^{1+\frac{n}{2}+p} (1-ax)^{1-\frac{n}{2}+p} (1-a^2x^2)^{-p} \\ (c-a^2cx^2)^p \text{Hypergeometric2F1}\left[-\frac{n}{2}-p, 1-\frac{n}{2}+p, 2-\frac{n}{2}+p, \frac{1}{2}(1-ax)\right]$$

Result (type 8, 24 leaves):

$$\int e^{n \text{ArcTanh}[ax]} (c-a^2cx^2)^p dx$$

Problem 1363: Unable to integrate problem.

$$\int e^{2(1+p) \text{ArcTanh}[ax]} (1-a^2x^2)^{-p} dx$$

Optimal (type 3, 41 leaves, 3 steps):

$$\frac{(1-ax)^{1-2p}}{a(1-2p)} + \frac{(1-ax)^{-2p}}{ap}$$

Result (type 8, 28 leaves):

$$\int e^{2(1+p) \text{ArcTanh}[ax]} (1-a^2x^2)^{-p} dx$$

Problem 1364: Unable to integrate problem.

$$\int e^{2(1+p) \text{ArcTanh}[ax]} (c-a^2cx^2)^{-p} dx$$

Optimal (type 3, 95 leaves, 4 steps):

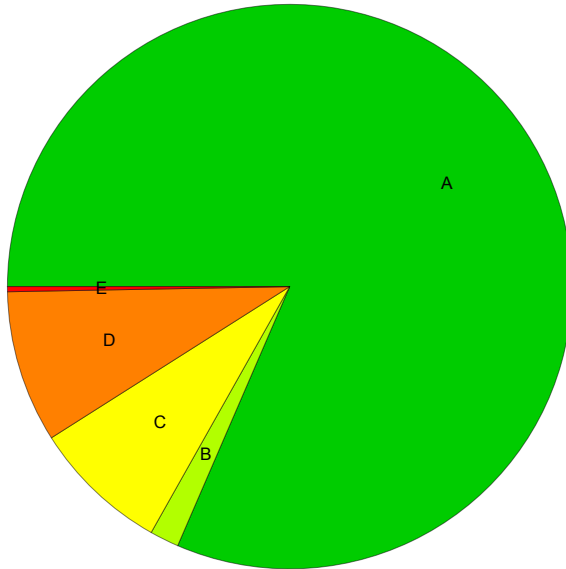
$$\frac{(1-ax)^{1-2p} (1-a^2x^2)^p (c-a^2cx^2)^{-p}}{a(1-2p)} + \frac{(1-ax)^{-2p} (1-a^2x^2)^p (c-a^2cx^2)^{-p}}{ap}$$

Result (type 8, 29 leaves):

$$\int e^{2(1+p) \text{ArcTanh}[ax]} (c-a^2cx^2)^{-p} dx$$

Summary of Integration Test Results

1378 integration problems



- A - 1123 optimal antiderivatives
- B - 23 more than twice size of optimal antiderivatives
- C - 108 unnecessarily complex antiderivatives
- D - 120 unable to integrate problems
- E - 4 integration timeouts