

# Mathematica 11.3 Integration Test Results

Test results for the 361 problems in "7.3.7 Inverse hyperbolic tangent functions.m"

Problem 16: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{9/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 196 leaves, 6 steps):

$$-\frac{60 d^2 \sqrt{x} \sqrt{d+e x^2}}{847 e^{5/2}} + \frac{36 d x^{5/2} \sqrt{d+e x^2}}{847 e^{3/2}} - \frac{4 x^{9/2} \sqrt{d+e x^2}}{121 \sqrt{e}} + \frac{2}{11} x^{11/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] + \left( \frac{30 d^{11/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{847 e^{11/4} \sqrt{d+e x^2}} \right) /$$

Result (type 4, 161 leaves):

$$\frac{2}{847} \sqrt{x} \left( -\frac{2 \sqrt{d+e x^2} (15 d^2 - 9 d e x^2 + 7 e^2 x^4)}{e^{5/2}} + 77 x^5 \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] \right) + \frac{60 d^{5/2} \sqrt{\frac{i \sqrt{d}}{\sqrt{e}}} \sqrt{1 + \frac{d}{e x^2}} x \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{847 e^2 \sqrt{d+e x^2}}$$

Problem 17: Result unnecessarily involves imaginary or complex numbers.

$$\int x^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 168 leaves, 5 steps):

$$\frac{20 d \sqrt{x} \sqrt{d+e x^2}}{147 e^{3/2}} - \frac{4 x^{5/2} \sqrt{d+e x^2}}{49 \sqrt{e}} + \frac{2}{7} x^{7/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] - \left(10 d^{7/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]\right) / (147 e^{7/4} \sqrt{d+e x^2})$$

Result (type 4, 147 leaves):

$$\frac{2}{147} \sqrt{x} \left( \frac{2 (5 d - 3 e x^2) \sqrt{d+e x^2}}{e^{3/2}} + 21 x^3 \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] \right) + \frac{20 \sqrt{d} \left(\frac{i \sqrt{d}}{\sqrt{e}}\right)^{5/2} \sqrt{1 + \frac{d}{e x^2}} x \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{147 \sqrt{d+e x^2}}$$

Problem 18: Result unnecessarily involves imaginary or complex numbers.

$$\int \sqrt{x} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 142 leaves, 4 steps):

$$-\frac{4 \sqrt{x} \sqrt{d+e x^2}}{9 \sqrt{e}} + \frac{2}{3} x^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] + \frac{2 d^{3/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{9 e^{3/4} \sqrt{d+e x^2}}$$

Result (type 4, 135 leaves):

$$\frac{2}{9} \sqrt{x} \left( -\frac{2 \sqrt{d+e x^2}}{\sqrt{e}} + 3 x \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] \right) + \frac{4 \sqrt{d} \sqrt{\frac{i \sqrt{d}}{\sqrt{e}}} \sqrt{1 + \frac{d}{e x^2}} x \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{9 \sqrt{d+e x^2}}$$

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{x^{3/2}} dx$$

Optimal (type 4, 113 leaves, 3 steps):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{\sqrt{x}} + \frac{2 e^{1/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{d^{1/4} \sqrt{d+e x^2}}$$

Result (type 4, 111 leaves):

$$-\frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{\sqrt{x}} + \frac{4 i \sqrt{e} \sqrt{1 + \frac{d}{e x^2}} x \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}} \sqrt{d+e x^2}}$$

**Problem 20: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{x^{7/2}} dx$$

Optimal (type 4, 145 leaves, 4 steps):

$$-\frac{4 \sqrt{e} \sqrt{d+e x^2}}{15 d x^{3/2}} - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{5 x^{5/2}} - \frac{2 e^{5/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{15 d^{5/4} \sqrt{d+e x^2}}$$

Result (type 4, 142 leaves):

$$-\frac{2 \left( 2 \sqrt{e} x \sqrt{d+e x^2} + 3 d \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] \right)}{15 d x^{5/2}} - \frac{4 \sqrt{\frac{i \sqrt{d}}{\sqrt{e}}} e^2 \sqrt{1 + \frac{d}{e x^2}} x \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{\frac{i \sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{15 d^{3/2} \sqrt{d+e x^2}}$$

**Problem 21: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{x^{11/2}} dx$$

Optimal (type 4, 173 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{4\sqrt{e}\sqrt{d+ex^2}}{63dx^{7/2}} + \frac{20e^{3/2}\sqrt{d+ex^2}}{189d^2x^{3/2}} - \frac{2\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{9x^{9/2}} + \\
 & \frac{10e^{9/4}\left(\sqrt{d} + \sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d} + \sqrt{e}x\right)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{189d^{9/4}\sqrt{d+ex^2}}
 \end{aligned}$$

Result (type 4, 154 leaves):

$$\begin{aligned}
 & \frac{4\sqrt{e}x\sqrt{d+ex^2}\left(-3d+5ex^2\right)-42d^2\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{189d^2x^{9/2}} + \\
 & \frac{20\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}\ e^3\sqrt{1+\frac{d}{ex^2}}x\text{EllipticF}\left[i\text{ArcSinh}\left[\frac{\sqrt{\frac{i\sqrt{d}}{\sqrt{e}}}}{\sqrt{x}}\right], -1\right]}{189d^{5/2}\sqrt{d+ex^2}}
 \end{aligned}$$

**Problem 22: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{x^{15/2}} dx$$

Optimal (type 4, 201 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{4\sqrt{e}\sqrt{d+ex^2}}{143dx^{11/2}} + \frac{36e^{3/2}\sqrt{d+ex^2}}{1001d^2x^{7/2}} - \frac{60e^{5/2}\sqrt{d+ex^2}}{1001d^3x^{3/2}} - \frac{2\text{ArcTanh}\left[\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right]}{13x^{13/2}} - \\
 & \left( \frac{30e^{13/4}\left(\sqrt{d} + \sqrt{e}x\right)\sqrt{\frac{d+ex^2}{\left(\sqrt{d} + \sqrt{e}x\right)^2}}\text{EllipticF}\left[2\text{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{1001d^{13/4}\sqrt{d+ex^2}} \right) /
 \end{aligned}$$

Result (type 4, 163 leaves):

$$\frac{1}{1001 x^{13/2}} \left( 2 \frac{2 \sqrt{e} x \sqrt{d+e x^2} (7 d^2 - 9 d e x^2 + 15 e^2 x^4)}{d^3} - 77 \operatorname{ArcTanh} \left[ \frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right] - \frac{30 \sqrt{\frac{i \sqrt{d}}{\sqrt{e}}} e^4 \sqrt{1 + \frac{d}{e x^2}} x^{15/2} \operatorname{EllipticF} \left[ i \operatorname{ArcSinh} \left[ \sqrt{\frac{i \sqrt{d}}{\sqrt{e} x}} \right], -1 \right]}{d^{7/2} \sqrt{d+e x^2}} \right)$$

**Problem 23: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^{7/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right] dx$$

Optimal (type 4, 297 leaves, 7 steps):

$$\frac{28 d x^{3/2} \sqrt{d+e x^2}}{405 e^{3/2}} - \frac{4 x^{7/2} \sqrt{d+e x^2}}{81 \sqrt{e}} - \frac{28 d^2 \sqrt{x} \sqrt{d+e x^2}}{135 e^2 (\sqrt{d} + \sqrt{e} x)} + \frac{2}{9} x^{9/2} \operatorname{ArcTanh} \left[ \frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right] + \frac{28 d^{9/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticE} \left[ 2 \operatorname{ArcTan} \left[ \frac{e^{1/4} \sqrt{x}}{d^{1/4}} \right], \frac{1}{2} \right]}{135 e^{9/4} \sqrt{d+e x^2}} - \frac{14 d^{9/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF} \left[ 2 \operatorname{ArcTan} \left[ \frac{e^{1/4} \sqrt{x}}{d^{1/4}} \right], \frac{1}{2} \right]}{135 e^{9/4} \sqrt{d+e x^2}}$$

Result (type 4, 224 leaves):

$$\left( 2 \sqrt{x} \left( \sqrt{e} x \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \left( 14 d^2 + 4 d e x^2 - 10 e^2 x^4 + 45 e^{3/2} x^3 \sqrt{d+e x^2} \operatorname{ArcTanh} \left[ \frac{\sqrt{e} x}{\sqrt{d+e x^2}} \right] \right) - 42 d^{5/2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticE} \left[ i \operatorname{ArcSinh} \left[ \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \right], -1 \right] + 42 d^{5/2} \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF} \left[ i \operatorname{ArcSinh} \left[ \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \right], -1 \right] \right) \right) / \left( 405 e^2 \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \sqrt{d+e x^2} \right)$$

**Problem 24: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] dx$$

Optimal (type 4, 269 leaves, 6 steps):

$$\begin{aligned} & -\frac{4 x^{3/2} \sqrt{d+e x^2}}{25 \sqrt{e}} + \frac{12 d \sqrt{x} \sqrt{d+e x^2}}{25 e (\sqrt{d} + \sqrt{e} x)} + \frac{2}{5} x^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right] - \\ & \frac{12 d^{5/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{25 e^{5/4} \sqrt{d+e x^2}} + \\ & \frac{6 d^{5/4} (\sqrt{d} + \sqrt{e} x) \sqrt{\frac{d+e x^2}{(\sqrt{d} + \sqrt{e} x)^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{25 e^{5/4} \sqrt{d+e x^2}} \end{aligned}$$

Result (type 4, 211 leaves):

$$\begin{aligned} & -\left(2 \sqrt{x} \left(\sqrt{e} x \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}\left(2 d+2 e x^2-5 \sqrt{e} x \sqrt{d+e x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]\right)\right.\right. \\ & \quad \left.6 d^{3/2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}\right],-1\right]+ \right. \\ & \quad \left.6 d^{3/2} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}}\right],-1\right]\right) / \left(25 e \sqrt{\frac{i \sqrt{e} x}{\sqrt{d}}} \sqrt{d+e x^2}\right) \end{aligned}$$

**Problem 25: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{e} x}{\sqrt{d+e x^2}}\right]}{\sqrt{x}} dx$$

Optimal (type 4, 232 leaves, 5 steps):

$$\begin{aligned}
 & -\frac{4\sqrt{x}\sqrt{d+ex^2}}{\sqrt{d}+\sqrt{ex}} + 2\sqrt{x}\operatorname{ArcTanh}\left[\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right] + \\
 & \frac{4d^{1/4}(\sqrt{d}+\sqrt{ex})\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{ex})^2}}\operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{e^{1/4}\sqrt{d+ex^2}} - \\
 & \frac{2d^{1/4}(\sqrt{d}+\sqrt{ex})\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{ex})^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{e^{1/4}\sqrt{d+ex^2}}
 \end{aligned}$$

Result (type 4, 182 leaves):

$$\begin{aligned}
 & \left( 2\sqrt{x}\left(\sqrt{\frac{i\sqrt{ex}}{\sqrt{d}}}\sqrt{d+ex^2}\operatorname{ArcTanh}\left[\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right] - \right. \right. \\
 & \quad \left. \left. 2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticE}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{ex}}{\sqrt{d}}}\right], -1\right] + \right. \right. \\
 & \quad \left. \left. 2\sqrt{d}\sqrt{1+\frac{ex^2}{d}}\operatorname{EllipticF}\left[i\operatorname{ArcSinh}\left[\sqrt{\frac{i\sqrt{ex}}{\sqrt{d}}}\right], -1\right]\right)\right) / \left(\sqrt{\frac{i\sqrt{ex}}{\sqrt{d}}}\sqrt{d+ex^2}\right)
 \end{aligned}$$

**Problem 26: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right]}{x^{5/2}} dx$$

Optimal (type 4, 272 leaves, 6 steps):

$$\begin{aligned}
 & -\frac{4\sqrt{e}\sqrt{d+ex^2}}{3d\sqrt{x}} + \frac{4e\sqrt{x}\sqrt{d+ex^2}}{3d(\sqrt{d}+\sqrt{ex})} - \frac{2\operatorname{ArcTanh}\left[\frac{\sqrt{ex}}{\sqrt{d+ex^2}}\right]}{3x^{3/2}} - \\
 & \frac{4e^{3/4}(\sqrt{d}+\sqrt{ex})\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{ex})^2}}\operatorname{EllipticE}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{3d^{3/4}\sqrt{d+ex^2}} + \\
 & \frac{2e^{3/4}(\sqrt{d}+\sqrt{ex})\sqrt{\frac{d+ex^2}{(\sqrt{d}+\sqrt{ex})^2}}\operatorname{EllipticF}\left[2\operatorname{ArcTan}\left[\frac{e^{1/4}\sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{3d^{3/4}\sqrt{d+ex^2}}
 \end{aligned}$$

Result (type 4, 214 leaves):

$$\left( -2 \sqrt{\frac{i \sqrt{e x}}{\sqrt{d}}} \left( 2 \sqrt{e x} (d + e x^2) + d \sqrt{d + e x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{e x}}{\sqrt{d + e x^2}}\right] \right) + \right. \\ \left. 4 \sqrt{d} e x^2 \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e x}}{\sqrt{d}}}\right], -1\right] - \right. \\ \left. 4 \sqrt{d} e x^2 \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e x}}{\sqrt{d}}}\right], -1\right] \right) / \left( 3 d x^{3/2} \sqrt{\frac{i \sqrt{e x}}{\sqrt{d}}} \sqrt{d + e x^2} \right)$$

**Problem 27: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{e x}}{\sqrt{d + e x^2}}\right]}{x^{9/2}} dx$$

Optimal (type 4, 302 leaves, 7 steps):

$$- \frac{4 \sqrt{e} \sqrt{d + e x^2}}{35 d x^{5/2}} + \frac{12 e^{3/2} \sqrt{d + e x^2}}{35 d^2 \sqrt{x}} - \frac{12 e^2 \sqrt{x} \sqrt{d + e x^2}}{35 d^2 (\sqrt{d} + \sqrt{e x})} - \frac{2 \operatorname{ArcTanh}\left[\frac{\sqrt{e x}}{\sqrt{d + e x^2}}\right]}{7 x^{7/2}} + \\ \frac{12 e^{7/4} (\sqrt{d} + \sqrt{e x}) \sqrt{\frac{d + e x^2}{(\sqrt{d} + \sqrt{e x})^2}} \operatorname{EllipticE}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{35 d^{7/4} \sqrt{d + e x^2}} - \\ \frac{6 e^{7/4} (\sqrt{d} + \sqrt{e x}) \sqrt{\frac{d + e x^2}{(\sqrt{d} + \sqrt{e x})^2}} \operatorname{EllipticF}\left[2 \operatorname{ArcTan}\left[\frac{e^{1/4} \sqrt{x}}{d^{1/4}}\right], \frac{1}{2}\right]}{35 d^{7/4} \sqrt{d + e x^2}}$$

Result (type 4, 234 leaves):

$$\left( 2 \left( \sqrt{\frac{i \sqrt{e x}}{\sqrt{d}}} \left( 2 \sqrt{e x} (-d^2 + 2 d e x^2 + 3 e^2 x^4) - 5 d^2 \sqrt{d + e x^2} \operatorname{ArcTanh}\left[\frac{\sqrt{e x}}{\sqrt{d + e x^2}}\right] \right) - \right. \right. \\ \left. \left. 6 \sqrt{d} e^2 x^4 \sqrt{1 + \frac{e x^2}{d}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e x}}{\sqrt{d}}}\right], -1\right] + 6 \sqrt{d} e^2 x^4 \sqrt{1 + \frac{e x^2}{d}} \right. \right. \\ \left. \left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{\frac{i \sqrt{e x}}{\sqrt{d}}}\right], -1\right] \right) \right) / \left( 35 d^2 x^{7/2} \sqrt{\frac{i \sqrt{e x}}{\sqrt{d}}} \sqrt{d + e x^2} \right)$$



### Problem 31: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

Optimal (type 4, 409 leaves, 9 steps):

$$\begin{aligned} & \frac{2 \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3 \operatorname{ArcTanh}\left[1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{c} + \\ & - \frac{3 b \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} - \\ & \frac{3 b \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2 \operatorname{PolyLog}\left[2, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} - \\ & \frac{3 b^2 \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} + \\ & \frac{3 b^2 \left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right) \operatorname{PolyLog}\left[3, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{2 c} + \\ & \frac{3 b^3 \operatorname{PolyLog}\left[4, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{4 c} - \frac{3 b^3 \operatorname{PolyLog}\left[4, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}}\right]}{4 c} \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^3}{1 - c^2 x^2} dx$$

### Problem 32: Unable to integrate problem.

$$\int \frac{\left(a + b \operatorname{ArcTanh}\left[\frac{\sqrt{1-cx}}{\sqrt{1+cx}}\right]\right)^2}{1 - c^2 x^2} dx$$

Optimal (type 4, 268 leaves, 7 steps):

$$\begin{aligned}
 & \frac{2 \left( a + b \operatorname{ArcTanh} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2 \operatorname{ArcTanh} \left[ 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{c} + \\
 & \frac{b \left( a + b \operatorname{ArcTanh} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right) \operatorname{PolyLog} \left[ 2, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{c} - \\
 & \frac{b \left( a + b \operatorname{ArcTanh} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right) \operatorname{PolyLog} \left[ 2, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{c} - \\
 & \frac{b^2 \operatorname{PolyLog} \left[ 3, 1 - \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{2c} + \frac{b^2 \operatorname{PolyLog} \left[ 3, -1 + \frac{2}{1 - \frac{\sqrt{1-cx}}{\sqrt{1+cx}}} \right]}{2c}
 \end{aligned}$$

Result (type 8, 42 leaves):

$$\int \frac{\left( a + b \operatorname{ArcTanh} \left[ \frac{\sqrt{1-cx}}{\sqrt{1+cx}} \right] \right)^2}{1 - c^2 x^2} dx$$

**Problem 47: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{ArcTanh} [\operatorname{Tanh} [a + b x]]^2 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcTanh} [\operatorname{Tanh} [a + b x]]^3}{3b} - \frac{\operatorname{ArcTanh} [\operatorname{Tanh} [a + b x]]^4}{12b^2}$$

Result (type 3, 74 leaves):

$$\frac{1}{12b^2} (a + b x) \left( - (3a - b x) (a + b x)^2 + 4 (2a^2 + a b x - b^2 x^2) \operatorname{ArcTanh} [\operatorname{Tanh} [a + b x]] - 6 (a - b x) \operatorname{ArcTanh} [\operatorname{Tanh} [a + b x]]^2 \right)$$

**Problem 57: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{ArcTanh} [\operatorname{Tanh} [a + b x]]^3 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcTanh} [\operatorname{Tanh} [a + b x]]^4}{4b} - \frac{\operatorname{ArcTanh} [\operatorname{Tanh} [a + b x]]^5}{20b^2}$$

Result (type 3, 99 leaves):

$$\frac{1}{20 b^2} (a + b x) \left( (4 a - b x) (a + b x)^3 - 5 (3 a - b x) (a + b x)^2 \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]] + \right. \\ \left. 10 (2 a^2 + a b x - b^2 x^2) \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^2 - 10 (a - b x) \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^3 \right)$$

**Problem 71: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^4 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^5}{5 b} - \frac{\operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^6}{30 b^2}$$

Result (type 3, 125 leaves):

$$-\frac{1}{30 b^2} (a + b x) \left( (5 a - b x) (a + b x)^4 - 6 (4 a - b x) (a + b x)^3 \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]] + \right. \\ \left. 15 (3 a - b x) (a + b x)^2 \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^2 - \right. \\ \left. 20 (2 a^2 + a b x - b^2 x^2) \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^3 + 15 (a - b x) \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^4 \right)$$

**Problem 78: Result more than twice size of optimal antiderivative.**

$$\int \frac{\operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^4}{x^6} dx$$

Optimal (type 3, 31 leaves, 1 step):

$$\frac{\operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^5}{5 x^5 (b x - \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]])}$$

Result (type 3, 66 leaves):

$$-\frac{1}{5 x^5} (b^4 x^4 + b^3 x^3 \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]] + \\ b^2 x^2 \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^2 + b x \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^3 + \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^4)$$

**Problem 84: Result more than twice size of optimal antiderivative.**

$$\int x \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^6 dx$$

Optimal (type 3, 34 leaves, 3 steps):

$$\frac{x \operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^7}{7 b} - \frac{\operatorname{ArcTanh}[\operatorname{Tanh}[a + b x]]^8}{56 b^2}$$

Result (type 3, 177 leaves):

$$\begin{aligned}
 & -\frac{1}{56 b^2} (a+b x) \left( (7 a-b x) (a+b x)^6 - 8 (6 a-b x) (a+b x)^5 \operatorname{ArcTanh}[\operatorname{Tanh}[a+b x]] + \right. \\
 & \quad 28 (5 a-b x) (a+b x)^4 \operatorname{ArcTanh}[\operatorname{Tanh}[a+b x]]^2 - 56 (4 a-b x) (a+b x)^3 \\
 & \quad \operatorname{ArcTanh}[\operatorname{Tanh}[a+b x]]^3 + 70 (3 a-b x) (a+b x)^2 \operatorname{ArcTanh}[\operatorname{Tanh}[a+b x]]^4 - \\
 & \quad \left. 56 (2 a^2+a b x-b^2 x^2) \operatorname{ArcTanh}[\operatorname{Tanh}[a+b x]]^5 + 28 (a-b x) \operatorname{ArcTanh}[\operatorname{Tanh}[a+b x]]^6 \right)
 \end{aligned}$$

**Problem 286: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}[c+d \operatorname{Tanh}[a+b x]] dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$\begin{aligned}
 & x \operatorname{ArcTanh}[c+d \operatorname{Tanh}[a+b x]] + \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1-c-d) e^{2a+2bx}}{1-c+d}\right] - \\
 & \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1+c+d) e^{2a+2bx}}{1+c-d}\right] + \frac{\operatorname{PolyLog}\left[2, -\frac{(1-c-d) e^{2a+2bx}}{1-c+d}\right]}{4 b} - \frac{\operatorname{PolyLog}\left[2, -\frac{(1+c+d) e^{2a+2bx}}{1+c-d}\right]}{4 b}
 \end{aligned}$$

Result (type 4, 366 leaves):

$$\begin{aligned}
 & x \operatorname{ArcTanh}[c+d \operatorname{Tanh}[a+b x]] + \\
 & \frac{1}{2 b} \left( (a+b x) \operatorname{Log}\left[1 - \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{1-c+d}}\right] + (a+b x) \operatorname{Log}\left[1 + \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{1-c+d}}\right] - \right. \\
 & \quad (a+b x) \operatorname{Log}\left[1 - \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{-1-c+d}}\right] - (a+b x) \operatorname{Log}\left[1 + \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{-1-c+d}}\right] + \\
 & \quad a \operatorname{Log}\left[1+c-d+e^{2(a+bx)}+c e^{2(a+bx)}+d e^{2(a+bx)}\right] - \\
 & \quad a \operatorname{Log}\left[1+d+e^{2(a+bx)}-d e^{2(a+bx)}-c\left(1+e^{2(a+bx)}\right)\right] + \\
 & \quad \operatorname{PolyLog}\left[2, -\frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{1-c+d}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{-1+c+d} e^{a+bx}}{\sqrt{1-c+d}}\right] - \\
 & \quad \left. \operatorname{PolyLog}\left[2, -\frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{-1-c+d}}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{1+c+d} e^{a+bx}}{\sqrt{-1-c+d}}\right] \right)
 \end{aligned}$$

**Problem 291: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}[1+d+d \operatorname{Tanh}[a+b x]] dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\frac{b x^2}{2} + x \operatorname{ArcTanh}[1+d+d \operatorname{Tanh}[a+b x]] - \frac{1}{2} x \operatorname{Log}\left[1 + (1+d) e^{2a+2bx}\right] - \frac{\operatorname{PolyLog}\left[2, -(1+d) e^{2a+2bx}\right]}{4 b}$$

Result (type 4, 168 leaves):

$$\begin{aligned}
 & x \operatorname{ArcTanh}[1 + d + d \operatorname{Tanh}[a + b x]] - \frac{1}{2b} \\
 & \left( b x \left( -b x - \operatorname{Log}\left[ e^{-a-bx} + (1+d) e^{a+bx} \right] + \operatorname{Log}\left[ 1 - e^{bx} \sqrt{-(1+d) e^{2a}} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[ 1 + e^{bx} \sqrt{-(1+d) e^{2a}} \right] + \operatorname{Log}\left[ (2+d) \operatorname{Cosh}[a + b x] + d \operatorname{Sinh}[a + b x] \right] \right) + \right. \\
 & \quad \left. \operatorname{PolyLog}\left[ 2, -e^{bx} \sqrt{-(1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[ 2, e^{bx} \sqrt{-(1+d) e^{2a}} \right] \right)
 \end{aligned}$$

**Problem 296: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}[1 - d - d \operatorname{Tanh}[a + b x]] dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$\frac{b x^2}{2} + x \operatorname{ArcTanh}[1 - d - d \operatorname{Tanh}[a + b x]] - \frac{1}{2} x \operatorname{Log}\left[ 1 + (1-d) e^{2a+2bx} \right] - \frac{\operatorname{PolyLog}\left[ 2, -(1-d) e^{2a+2bx} \right]}{4b}$$

Result (type 4, 171 leaves):

$$\begin{aligned}
 & x \operatorname{ArcTanh}[1 - d - d \operatorname{Tanh}[a + b x]] - \frac{1}{2b} \\
 & \left( b x \left( -b x - \operatorname{Log}\left[ e^{-a-bx} (-1 + (-1+d) e^{2(a+bx)}) \right] + \operatorname{Log}\left[ 1 - e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[ 1 + e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \operatorname{Log}\left[ (-2+d) \operatorname{Cosh}[a + b x] + d \operatorname{Sinh}[a + b x] \right] \right) + \right. \\
 & \quad \left. \operatorname{PolyLog}\left[ 2, -e^{bx} \sqrt{(-1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[ 2, e^{bx} \sqrt{(-1+d) e^{2a}} \right] \right)
 \end{aligned}$$

**Problem 300: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}[c + d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 150 leaves, 7 steps):

$$\begin{aligned}
 & x \operatorname{ArcTanh}[c + d \operatorname{Coth}[a + b x]] + \frac{1}{2} x \operatorname{Log}\left[ 1 - \frac{(1-c-d) e^{2a+2bx}}{1-c+d} \right] - \\
 & \frac{1}{2} x \operatorname{Log}\left[ 1 - \frac{(1+c+d) e^{2a+2bx}}{1+c-d} \right] + \frac{\operatorname{PolyLog}\left[ 2, \frac{(1-c-d) e^{2a+2bx}}{1-c+d} \right]}{4b} - \frac{\operatorname{PolyLog}\left[ 2, \frac{(1+c+d) e^{2a+2bx}}{1+c-d} \right]}{4b}
 \end{aligned}$$

Result (type 4, 369 leaves):

$$\begin{aligned}
 & x \operatorname{ArcTanh}[c + d \operatorname{Coth}[a + b x]] - \\
 & \frac{1}{2b} \left( - (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{-1 + c - d}}\right] - (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{-1 + c - d}}\right] + \right. \\
 & \quad (a + b x) \operatorname{Log}\left[1 - \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{1 + c - d}}\right] + (a + b x) \operatorname{Log}\left[1 + \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{1 + c - d}}\right] + \\
 & \quad a \operatorname{Log}\left[1 + d - e^{2(a+bx)} + d e^{2(a+bx)} + c(-1 + e^{2(a+bx)})\right] - \\
 & \quad a \operatorname{Log}\left[1 + c - e^{2(a+bx)} - c e^{2(a+bx)} - d(1 + e^{2(a+bx)})\right] - \\
 & \quad \operatorname{PolyLog}\left[2, -\frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{-1 + c - d}}\right] - \operatorname{PolyLog}\left[2, \frac{\sqrt{-1 + c + d} e^{a+bx}}{\sqrt{-1 + c - d}}\right] + \\
 & \quad \left. \operatorname{PolyLog}\left[2, -\frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{1 + c - d}}\right] + \operatorname{PolyLog}\left[2, \frac{\sqrt{1 + c + d} e^{a+bx}}{\sqrt{1 + c - d}}\right] \right)
 \end{aligned}$$

**Problem 305: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}[1 + d + d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 69 leaves, 5 steps):

$$\frac{bx^2}{2} + x \operatorname{ArcTanh}[1 + d + d \operatorname{Coth}[a + b x]] - \frac{1}{2} x \operatorname{Log}[1 - (1 + d) e^{2a+2bx}] - \frac{\operatorname{PolyLog}[2, (1 + d) e^{2a+2bx}]}{4b}$$

Result (type 4, 168 leaves):

$$\begin{aligned}
 & x \operatorname{ArcTanh}[1 + d + d \operatorname{Coth}[a + b x]] - \frac{1}{2b} \\
 & \left( b x \left( -b x - \operatorname{Log}[e^{-a-bx}(-1 + (1 + d) e^{2(a+bx)})] \right) + \operatorname{Log}\left[1 - e^{bx} \sqrt{(1 + d) e^{2a}}\right] + \right. \\
 & \quad \left. \operatorname{Log}\left[1 + e^{bx} \sqrt{(1 + d) e^{2a}}\right] + \operatorname{Log}[d \operatorname{Cosh}[a + b x] + (2 + d) \operatorname{Sinh}[a + b x]] \right) + \\
 & \quad \operatorname{PolyLog}\left[2, -e^{bx} \sqrt{(1 + d) e^{2a}}\right] + \operatorname{PolyLog}\left[2, e^{bx} \sqrt{(1 + d) e^{2a}}\right]
 \end{aligned}$$

**Problem 310: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}[1 - d - d \operatorname{Coth}[a + b x]] dx$$

Optimal (type 4, 76 leaves, 5 steps):

$$\frac{bx^2}{2} + x \operatorname{ArcTanh}[1 - d - d \operatorname{Coth}[a + b x]] - \frac{1}{2} x \operatorname{Log}[1 - (1 - d) e^{2a+2bx}] - \frac{\operatorname{PolyLog}[2, (1 - d) e^{2a+2bx}]}{4b}$$

Result (type 4, 175 leaves):

$$\begin{aligned}
 & x \operatorname{ArcTanh}[1 - d - d \operatorname{Coth}[a + b x]] - \frac{1}{2b} \\
 & \left( b x \left( -b x - \operatorname{Log}\left[ e^{-a-bx} \left( 1 + (-1+d) e^{2(a+bx)} \right) \right] + \operatorname{Log}\left[ 1 - e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \right. \right. \\
 & \quad \left. \left. \operatorname{Log}\left[ 1 + e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \operatorname{Log}\left[ d \operatorname{Cosh}[a + b x] + (-2+d) \operatorname{Sinh}[a + b x] \right] \right) + \right. \\
 & \quad \left. \operatorname{PolyLog}\left[ 2, -e^{bx} \sqrt{-(-1+d) e^{2a}} \right] + \operatorname{PolyLog}\left[ 2, e^{bx} \sqrt{-(-1+d) e^{2a}} \right] \right)
 \end{aligned}$$

**Problem 312: Result more than twice size of optimal antiderivative.**

$$\int (e + f x)^3 \operatorname{ArcTanh}[\operatorname{Tan}[a + b x]] dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\begin{aligned}
 & \frac{i (e + f x)^4 \operatorname{ArcTan}\left[ e^{2i(a+bx)} \right]}{4f} + \frac{(e + f x)^4 \operatorname{ArcTanh}[\operatorname{Tan}[a + b x]]}{4f} - \\
 & \frac{i (e + f x)^3 \operatorname{PolyLog}\left[ 2, -i e^{2i(a+bx)} \right]}{4b} + \frac{i (e + f x)^3 \operatorname{PolyLog}\left[ 2, i e^{2i(a+bx)} \right]}{4b} + \\
 & \frac{3f (e + f x)^2 \operatorname{PolyLog}\left[ 3, -i e^{2i(a+bx)} \right]}{8b^2} - \frac{3f (e + f x)^2 \operatorname{PolyLog}\left[ 3, i e^{2i(a+bx)} \right]}{8b^2} + \\
 & \frac{3i f^2 (e + f x) \operatorname{PolyLog}\left[ 4, -i e^{2i(a+bx)} \right]}{8b^3} - \frac{3i f^2 (e + f x) \operatorname{PolyLog}\left[ 4, i e^{2i(a+bx)} \right]}{8b^3} - \\
 & \frac{3f^3 \operatorname{PolyLog}\left[ 5, -i e^{2i(a+bx)} \right]}{16b^4} + \frac{3f^3 \operatorname{PolyLog}\left[ 5, i e^{2i(a+bx)} \right]}{16b^4}
 \end{aligned}$$

Result (type 4, 654 leaves):

$$\begin{aligned}
 & \frac{1}{4} x \left( 4e^3 + 6e^2 f x + 4e f^2 x^2 + f^3 x^3 \right) \operatorname{ArcTanh}[\operatorname{Tan}[a + b x]] + \\
 & \frac{1}{16b^4} \left( -8b^4 e^3 x \operatorname{Log}\left[ 1 - i e^{2i(a+bx)} \right] - 12b^4 e^2 f x^2 \operatorname{Log}\left[ 1 - i e^{2i(a+bx)} \right] - \right. \\
 & \quad 8b^4 e f^2 x^3 \operatorname{Log}\left[ 1 - i e^{2i(a+bx)} \right] - 2b^4 f^3 x^4 \operatorname{Log}\left[ 1 - i e^{2i(a+bx)} \right] + 8b^4 e^3 x \operatorname{Log}\left[ 1 + i e^{2i(a+bx)} \right] + \\
 & \quad 12b^4 e^2 f x^2 \operatorname{Log}\left[ 1 + i e^{2i(a+bx)} \right] + 8b^4 e f^2 x^3 \operatorname{Log}\left[ 1 + i e^{2i(a+bx)} \right] + \\
 & \quad 2b^4 f^3 x^4 \operatorname{Log}\left[ 1 + i e^{2i(a+bx)} \right] - 4i b^3 (e + f x)^3 \operatorname{PolyLog}\left[ 2, -i e^{2i(a+bx)} \right] + \\
 & \quad 4i b^3 (e + f x)^3 \operatorname{PolyLog}\left[ 2, i e^{2i(a+bx)} \right] + 6b^2 e^2 f \operatorname{PolyLog}\left[ 3, -i e^{2i(a+bx)} \right] + \\
 & \quad 12b^2 e f^2 x \operatorname{PolyLog}\left[ 3, -i e^{2i(a+bx)} \right] + 6b^2 f^3 x^2 \operatorname{PolyLog}\left[ 3, -i e^{2i(a+bx)} \right] - \\
 & \quad 6b^2 e^2 f \operatorname{PolyLog}\left[ 3, i e^{2i(a+bx)} \right] - 12b^2 e f^2 x \operatorname{PolyLog}\left[ 3, i e^{2i(a+bx)} \right] - \\
 & \quad 6b^2 f^3 x^2 \operatorname{PolyLog}\left[ 3, i e^{2i(a+bx)} \right] + 6i b e f^2 \operatorname{PolyLog}\left[ 4, -i e^{2i(a+bx)} \right] + \\
 & \quad 6i b f^3 x \operatorname{PolyLog}\left[ 4, -i e^{2i(a+bx)} \right] - 6i b e f^2 \operatorname{PolyLog}\left[ 4, i e^{2i(a+bx)} \right] - \\
 & \quad \left. 6i b f^3 x \operatorname{PolyLog}\left[ 4, i e^{2i(a+bx)} \right] - 3f^3 \operatorname{PolyLog}\left[ 5, -i e^{2i(a+bx)} \right] + 3f^3 \operatorname{PolyLog}\left[ 5, i e^{2i(a+bx)} \right] \right)
 \end{aligned}$$

**Problem 319: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}[c + d \operatorname{Tan}[a + b x]] dx$$

Optimal (type 4, 194 leaves, 7 steps):

$$\begin{aligned}
& x \operatorname{ArcTanh}[c + d \operatorname{Tan}[a + b x]] + \\
& \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1 - c + i d) e^{2ia + 2ibx}}{1 - c - i d}\right] - \frac{1}{2} x \operatorname{Log}\left[1 + \frac{(1 + c - i d) e^{2ia + 2ibx}}{1 + c + i d}\right] - \\
& \frac{i \operatorname{PolyLog}\left[2, -\frac{(1 - c + i d) e^{2ia + 2ibx}}{1 - c - i d}\right]}{4 b} + \frac{i \operatorname{PolyLog}\left[2, -\frac{(1 + c - i d) e^{2ia + 2ibx}}{1 + c + i d}\right]}{4 b}
\end{aligned}$$

Result (type 4, 4654 leaves):

$$\begin{aligned}
& x \operatorname{ArcTanh}[c + d \operatorname{Tan}[a + b x]] + \\
& \left( d \left[ -a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 ((-1 + c) \operatorname{Cos}[a + b x] + d \operatorname{Sin}[a + b x])\right]\right] + \right. \\
& a \operatorname{Log}\left[\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]^2 (\operatorname{Cos}[a + b x] + c \operatorname{Cos}[a + b x] + d \operatorname{Sin}[a + b x])\right] + \\
& (a + b x) \operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& i \operatorname{Log}\left[\frac{(-1 + c) \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-1 + c + i d - i \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& i \operatorname{Log}\left[-\frac{(-1 + c) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{i - i c - d + \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{-d + \sqrt{1 - 2c + c^2 + d^2}}{-1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& (a + b x) \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& i \operatorname{Log}\left[\frac{(-1 + c) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{i - i c + d + \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& i \operatorname{Log}\left[\frac{(-1 + c) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-i + i c + d + \sqrt{1 - 2c + c^2 + d^2}}\right] \operatorname{Log}\left[\frac{d + \sqrt{1 - 2c + c^2 + d^2}}{1 - c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& (a + b x) \operatorname{Log}\left[-\frac{d + \sqrt{1 + 2c + c^2 + d^2}}{1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& i \operatorname{Log}\left[\frac{(1 + c) \left(-i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{-i - i c + d + \sqrt{1 + 2c + c^2 + d^2}}\right] \operatorname{Log}\left[-\frac{d + \sqrt{1 + 2c + c^2 + d^2}}{1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] + \\
& i \operatorname{Log}\left[\frac{(1 + c) \left(i + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{i + i c + d + \sqrt{1 + 2c + c^2 + d^2}}\right] \operatorname{Log}\left[-\frac{d + \sqrt{1 + 2c + c^2 + d^2}}{1 + c} + \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right] - \\
& (a + b x) \operatorname{Log}\left[\frac{-d + \sqrt{1 + 2c + c^2 + d^2} + (1 + c) \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 + c}\right] + \\
& i \operatorname{Log}\left[\frac{(1 + c) \left(1 - i \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]\right)}{1 + c - i d + i \sqrt{1 + 2c + c^2 + d^2}}\right] \\
& \operatorname{Log}\left[\frac{-d + \sqrt{1 + 2c + c^2 + d^2} + (1 + c) \operatorname{Tan}\left[\frac{1}{2}(a + b x)\right]}{1 + c}\right] - i \operatorname{Log}\left[
\end{aligned}$$



$$\begin{aligned}
 & \left. \frac{(1+c) \left(1 + i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{1+c+i d-i \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right] + \\
 & i \operatorname{PolyLog}\left[2, \frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i-i c+d+\sqrt{1-2c+c^2+d^2}}\right] - \\
 & i \operatorname{PolyLog}\left[2, \frac{d+\sqrt{1-2c+c^2+d^2}-(-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i+i c+d+\sqrt{1-2c+c^2+d^2}}\right] - \\
 & i \operatorname{PolyLog}\left[2, \frac{-d+\sqrt{1-2c+c^2+d^2}+(-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i-i c-d+\sqrt{1-2c+c^2+d^2}}\right] + \\
 & i \operatorname{PolyLog}\left[2, \frac{-d+\sqrt{1-2c+c^2+d^2}+(-1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i+i c-d+\sqrt{1-2c+c^2+d^2}}\right] - \\
 & i \operatorname{PolyLog}\left[2, \frac{d+\sqrt{1+2c+c^2+d^2}-(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c+d+\sqrt{1+2c+c^2+d^2}}\right] + \\
 & i \operatorname{PolyLog}\left[2, \frac{d+\sqrt{1+2c+c^2+d^2}-(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c+d+\sqrt{1+2c+c^2+d^2}}\right] + \\
 & i \operatorname{PolyLog}\left[2, \frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c-d+\sqrt{1+2c+c^2+d^2}}\right] - \\
 & i \operatorname{PolyLog}\left[2, \frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c-d+\sqrt{1+2c+c^2+d^2}}\right] \Big) \\
 & \left. \left(-\left(\frac{2a}{b}\right) / \left(b\left(-1+c^2+d^2-\operatorname{Cos}\left[2(a+bx)\right]+c^2 \operatorname{Cos}\left[2(a+bx)\right]-d^2 \operatorname{Cos}\left[2(a+bx)\right]\right)+\right.\right. \right. \\
 & \left. \left. 2cd \operatorname{Sin}\left[2(a+bx)\right]\right)\right) + \left(2(a+bx)\right) / \left(b\left(-1+c^2+d^2-\operatorname{Cos}\left[2(a+bx)\right]\right)+\right. \right. \\
 & \left. \left. c^2 \operatorname{Cos}\left[2(a+bx)\right]-d^2 \operatorname{Cos}\left[2(a+bx)\right]+2cd \operatorname{Sin}\left[2(a+bx)\right]\right)\right) \Big) \Big) / \\
 & \left( \operatorname{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] + \operatorname{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] - \right. \\
 & \operatorname{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] - \\
 & \operatorname{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right] + \\
 & \left. \frac{\operatorname{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(1-i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\text{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(1+i \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{\text{Log}\left[\frac{-d+\sqrt{1+2c+c^2+d^2}+(1+c)\text{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c}\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(1+i \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i \text{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \text{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-i + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \text{Log}\left[\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \text{Log}\left[\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i \text{Log}\left[-\frac{d+\sqrt{1+2c+c^2+d^2}}{1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{(a+bx) \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i \text{Log}\left[\frac{(-1+c)\left(1+i \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1+c+i d-i \sqrt{1-2c+c^2+d^2}}\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \text{Log}\left[-\frac{(-1+c)\left(i+\text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{i-i c-d+\sqrt{1-2c+c^2+d^2}}\right] \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{-d+\sqrt{1-2c+c^2+d^2}}{-1+c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{(a+bx) \text{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(\frac{d+\sqrt{1-2c+c^2+d^2}}{1-c} + \text{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i \operatorname{Log} \left[ \frac{(-1+c) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{i - i c + d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( \frac{d + \sqrt{1-2c+c^2+d^2}}{1-c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \operatorname{Log} \left[ \frac{(-1+c) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{-i + i c + d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( \frac{d + \sqrt{1-2c+c^2+d^2}}{1-c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{(a+bx) \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \operatorname{Log} \left[ \frac{(1+c) \left( -i + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{-i - i c + d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \operatorname{Log} \left[ \frac{(1+c) \left( i + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{i + i c + d + \sqrt{1+2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -\frac{d + \sqrt{1+2c+c^2+d^2}}{1+c} + \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \left( i (-1+c) \operatorname{Log} \left[ 1 - \frac{d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right]}{i - i c + d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
 & \left( 2 \left( d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right) \right) - \\
 & \left( i (-1+c) \operatorname{Log} \left[ 1 - \frac{d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right]}{-i + i c + d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
 & \left( 2 \left( d + \sqrt{1-2c+c^2+d^2} - (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right) \right) + \\
 & \left( i (-1+c) \operatorname{Log} \left[ 1 - \frac{-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right]}{i - i c - d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
 & \left( 2 \left( -d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right) \right) - \\
 & \left( i (-1+c) \operatorname{Log} \left[ 1 - \frac{-d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right]}{-i + i c - d + \sqrt{1-2c+c^2+d^2}} \right] \operatorname{Sec} \left[ \frac{1}{2} (a+bx) \right]^2 \right) / \\
 & \left( 2 \left( -d + \sqrt{1-2c+c^2+d^2} + (-1+c) \operatorname{Tan} \left[ \frac{1}{2} (a+bx) \right] \right) \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i(1+c) \operatorname{Log}\left[1 - \frac{d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c+d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i(1+c) \operatorname{Log}\left[1 - \frac{d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c+d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(d + \sqrt{1+2c+c^2+d^2} - (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{(1+c)(a+bx) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i(1+c) \operatorname{Log}\left[\frac{(1+c)\left(1-i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{1+c-i d+i \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i(1+c) \operatorname{Log}\left[\frac{(1+c)\left(1+i \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{1+c+i d-i \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i(1+c) \operatorname{Log}\left[1 - \frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-i-i c-d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i(1+c) \operatorname{Log}\left[1 - \frac{-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{i+i c-d+\sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-d + \sqrt{1+2c+c^2+d^2} + (1+c) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \left. \begin{aligned}
 & \left(a \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right)^2 \left(-\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]\right)^2 (d \operatorname{Cos}[a+bx] - (-1+c) \operatorname{Sin}[a+bx]) - \\
 & \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 \left((-1+c) \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]\right) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \Big) \Big) / \\
 & \left((-1+c) \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]\right) + \\
 & \left(a \operatorname{Cos}\left[\frac{1}{2}(a+bx)\right]\right)^2 \left(\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]\right)^2 (d \operatorname{Cos}[a+bx] - \operatorname{Sin}[a+bx] - c \operatorname{Sin}[a+bx]) + \\
 & \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2 (\operatorname{Cos}[a+bx] + c \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]) \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right] \Big) \Big) / \\
 & \left. \left(\operatorname{Cos}[a+bx] + c \operatorname{Cos}[a+bx] + d \operatorname{Sin}[a+bx]\right) \right)
 \end{aligned}
 \right\}
 \end{aligned}$$

Problem 329: Result more than twice size of optimal antiderivative.

$$\int (e + f x)^3 \operatorname{ArcTanh}[\operatorname{Cot}[a + b x]] dx$$

Optimal (type 4, 302 leaves, 12 steps):

$$\begin{aligned} & \frac{i (e + f x)^4 \operatorname{ArcTan}[e^{2i(a+bx)}]}{4f} + \frac{(e + f x)^4 \operatorname{ArcTanh}[\operatorname{Cot}[a + b x]]}{4f} - \\ & \frac{i (e + f x)^3 \operatorname{PolyLog}[2, -i e^{2i(a+bx)}]}{4b} + \frac{i (e + f x)^3 \operatorname{PolyLog}[2, i e^{2i(a+bx)}]}{4b} + \\ & \frac{3f (e + f x)^2 \operatorname{PolyLog}[3, -i e^{2i(a+bx)}]}{8b^2} - \frac{3f (e + f x)^2 \operatorname{PolyLog}[3, i e^{2i(a+bx)}]}{8b^2} + \\ & \frac{3i f^2 (e + f x) \operatorname{PolyLog}[4, -i e^{2i(a+bx)}]}{8b^3} - \frac{3i f^2 (e + f x) \operatorname{PolyLog}[4, i e^{2i(a+bx)}]}{8b^3} - \\ & \frac{3f^3 \operatorname{PolyLog}[5, -i e^{2i(a+bx)}]}{16b^4} + \frac{3f^3 \operatorname{PolyLog}[5, i e^{2i(a+bx)}]}{16b^4} \end{aligned}$$

Result (type 4, 654 leaves):

$$\begin{aligned} & \frac{1}{4} x (4e^3 + 6e^2 f x + 4e f^2 x^2 + f^3 x^3) \operatorname{ArcTanh}[\operatorname{Cot}[a + b x]] + \\ & \frac{1}{16b^4} \left( -8b^4 e^3 x \operatorname{Log}[1 - i e^{2i(a+bx)}] - 12b^4 e^2 f x^2 \operatorname{Log}[1 - i e^{2i(a+bx)}] - \right. \\ & \quad 8b^4 e f^2 x^3 \operatorname{Log}[1 - i e^{2i(a+bx)}] - 2b^4 f^3 x^4 \operatorname{Log}[1 - i e^{2i(a+bx)}] + 8b^4 e^3 x \operatorname{Log}[1 + i e^{2i(a+bx)}] + \\ & \quad 12b^4 e^2 f x^2 \operatorname{Log}[1 + i e^{2i(a+bx)}] + 8b^4 e f^2 x^3 \operatorname{Log}[1 + i e^{2i(a+bx)}] + \\ & \quad 2b^4 f^3 x^4 \operatorname{Log}[1 + i e^{2i(a+bx)}] - 4i b^3 (e + f x)^3 \operatorname{PolyLog}[2, -i e^{2i(a+bx)}] + \\ & \quad 4i b^3 (e + f x)^3 \operatorname{PolyLog}[2, i e^{2i(a+bx)}] + 6b^2 e^2 f \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] + \\ & \quad 12b^2 e f^2 x \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] + 6b^2 f^3 x^2 \operatorname{PolyLog}[3, -i e^{2i(a+bx)}] - \\ & \quad 6b^2 e^2 f \operatorname{PolyLog}[3, i e^{2i(a+bx)}] - 12b^2 e f^2 x \operatorname{PolyLog}[3, i e^{2i(a+bx)}] - \\ & \quad 6b^2 f^3 x^2 \operatorname{PolyLog}[3, i e^{2i(a+bx)}] + 6i b e f^2 \operatorname{PolyLog}[4, -i e^{2i(a+bx)}] + \\ & \quad 6i b f^3 x \operatorname{PolyLog}[4, -i e^{2i(a+bx)}] - 6i b e f^2 \operatorname{PolyLog}[4, i e^{2i(a+bx)}] - \\ & \quad \left. 6i b f^3 x \operatorname{PolyLog}[4, i e^{2i(a+bx)}] - 3f^3 \operatorname{PolyLog}[5, -i e^{2i(a+bx)}] + 3f^3 \operatorname{PolyLog}[5, i e^{2i(a+bx)}] \right) \end{aligned}$$

**Problem 336: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}[c + d \operatorname{Cot}[a + b x]] dx$$

Optimal (type 4, 194 leaves, 7 steps):

$$\begin{aligned} & x \operatorname{ArcTanh}[c + d \operatorname{Cot}[a + b x]] + \\ & \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1 - c - i d) e^{2ia+2ibx}}{1 - c + i d}\right] - \frac{1}{2} x \operatorname{Log}\left[1 - \frac{(1 + c + i d) e^{2ia+2ibx}}{1 + c - i d}\right] - \\ & \frac{i \operatorname{PolyLog}\left[2, \frac{(1 - c - i d) e^{2ia+2ibx}}{1 - c + i d}\right]}{4b} + \frac{i \operatorname{PolyLog}\left[2, \frac{(1 + c + i d) e^{2ia+2ibx}}{1 + c - i d}\right]}{4b} \end{aligned}$$

Result (type 4, 4463 leaves):

$$\begin{aligned} & x \operatorname{ArcTanh}[c + d \operatorname{Cot}[a + b x]] - \\ & \left( d \left( a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a + b x)\right]\right]^2 (d \operatorname{Cos}[a + b x] + (-1 + c) \operatorname{Sin}[a + b x]) \right) - \right. \end{aligned}$$

$$\begin{aligned}
 & a \operatorname{Log}\left[-\operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2(d\cos[a+bx]+\sin[a+bx]+c\sin[a+bx])\right]- \\
 & (a+bx)\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]- \\
 & i\operatorname{Log}\left[\frac{d(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{-1+c-i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]+ \\
 & i\operatorname{Log}\left[\frac{d(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{-1+c+i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]+ \\
 & (a+bx)\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]+ \\
 & i\operatorname{Log}\left[\frac{d(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1+c-i d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]- \\
 & i\operatorname{Log}\left[\frac{d(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1+c+i d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d}+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right]- \\
 & (a+bx)\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]- \\
 & i\operatorname{Log}\left[-\frac{d(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1-c+i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]+ \\
 & i\operatorname{Log}\left[-\frac{d(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{1-c-i d+\sqrt{1-2c+c^2+d^2}}\right]\operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]+ \\
 & (a+bx)\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]+ \\
 & i\operatorname{Log}\left[-\frac{d(-i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{-1-c+i d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]- \\
 & i\operatorname{Log}\left[-\frac{d(i+\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])}{-1-c-i d+\sqrt{1+2c+c^2+d^2}}\right]\operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right]- \\
 & i\operatorname{PolyLog}\left[2,\frac{-1+c+\sqrt{1-2c+c^2+d^2}-d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-1+c-i d+\sqrt{1-2c+c^2+d^2}}\right]+ \\
 & i\operatorname{PolyLog}\left[2,\frac{-1+c+\sqrt{1-2c+c^2+d^2}-d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-1+c+i d+\sqrt{1-2c+c^2+d^2}}\right]- \\
 & i\operatorname{PolyLog}\left[2,\frac{1+c-\sqrt{1+2c+c^2+d^2}-d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c+i d-\sqrt{1+2c+c^2+d^2}}\right]+ \\
 & i\operatorname{PolyLog}\left[2,\frac{1+c+\sqrt{1+2c+c^2+d^2}-d\operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c-i d+\sqrt{1+2c+c^2+d^2}}\right]-
 \end{aligned}$$

$$\begin{aligned}
 & \left( \begin{aligned}
 & \text{i PolyLog}\left[2, \frac{1+c+\sqrt{1+2c+c^2+d^2}-d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1+c+i d+\sqrt{1+2c+c^2+d^2}}\right] + \\
 & \text{i PolyLog}\left[2, \frac{1-c+\sqrt{1-2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1-c-i d+\sqrt{1-2c+c^2+d^2}}\right] - \\
 & \text{i PolyLog}\left[2, \frac{1-c+\sqrt{1-2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{1-c+i d+\sqrt{1-2c+c^2+d^2}}\right] + \\
 & \text{i PolyLog}\left[2, \frac{-1-c+\sqrt{1+2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-1-c+i d+\sqrt{1+2c+c^2+d^2}}\right] \right) \\
 & \left( (2a) / (b(1-c^2-d^2-\cos[2(a+bx)] + c^2 \cos[2(a+bx)] - d^2 \cos[2(a+bx)] - \right. \\
 & \quad \left. 2cd \sin[2(a+bx)])) - (2(a+bx)) / (b(1-c^2-d^2-\cos[2(a+bx)] + \right. \\
 & \quad \left. c^2 \cos[2(a+bx)] - d^2 \cos[2(a+bx)] - 2cd \sin[2(a+bx)])) \right) \Big/ \\
 & \left( \begin{aligned}
 & -\operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] + \\
 & \operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] - \\
 & \operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] + \\
 & \operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] - \\
 & \frac{\text{i Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])} + \\
 & \frac{\text{i Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])} - \\
 & \frac{\text{i Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])} + \\
 & \frac{\text{i Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right])} +
 \end{aligned} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i \operatorname{Log}\left[-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \operatorname{Log}\left[-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i \operatorname{Log}\left[\frac{1-c+\sqrt{1-2c+c^2+d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \operatorname{Log}\left[\frac{-1-c+\sqrt{1+2c+c^2+d^2} + d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{d}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{(a+bx) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \operatorname{Log}\left[\frac{d\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1+c-i d + \sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{i \operatorname{Log}\left[\frac{d\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{-1+c+i d + \sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{-1+c+\sqrt{1-2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \\
 & \frac{(a+bx) \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} + \frac{i \operatorname{Log}\left[\frac{d\left(-i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{1+c-i d + \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i \operatorname{Log}\left[\frac{d\left(i + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)}{1+c+i d + \sqrt{1+2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-\frac{1+c+\sqrt{1+2c+c^2+d^2}}{d} + \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} - \\
 & \frac{i d \operatorname{Log}\left[1 - \frac{-1+c+\sqrt{1-2c+c^2+d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]}{-1+c-i d + \sqrt{1-2c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+bx)\right]^2}{2\left(-1+c+\sqrt{1-2c+c^2+d^2} - d \operatorname{Tan}\left[\frac{1}{2}(a+bx)\right]\right)} +
 \end{aligned}$$



$$\begin{aligned}
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{-1+c+\sqrt{1-2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{-1+c+i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -1+c+\sqrt{1-2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1+c-\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1+c+i \, d-\sqrt{1+2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1+c-\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1+c+\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1+c-i \, d+\sqrt{1+2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1+c+\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1+c+\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1+c+i \, d+\sqrt{1+2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1+c+\sqrt{1+2c+c^2+d^2} - d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{d (a+bx) \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \, d \, \text{Log} \left[ -\frac{d \left( -i+\text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{1-c+i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \, d \, \text{Log} \left[ -\frac{d \left( i+\text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{1-c-i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} - \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1-c-i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \, d \, \text{Log} \left[ 1 - \frac{1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right]}{1-c+i \, d+\sqrt{1-2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( 1-c+\sqrt{1-2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{d (a+bx) \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -1-c+\sqrt{1+2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} + \\
 & \frac{i \, d \, \text{Log} \left[ -\frac{d \left( -i+\text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)}{-1-c+i \, d+\sqrt{1+2c+c^2+d^2}} \right] \, \text{Sec} \left[ \frac{1}{2} (a+bx) \right]^2}{2 \left( -1-c+\sqrt{1+2c+c^2+d^2} + d \, \text{Tan} \left[ \frac{1}{2} (a+bx) \right] \right)} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{i d \operatorname{Log}\left[-\frac{d\left(i+\operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)}{-1-c-i d+\sqrt{1+2 c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1-c+\sqrt{1+2 c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)} - \\
 & \frac{i d \operatorname{Log}\left[1-\frac{-1-c+\sqrt{1+2 c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]}{-1-c+i d+\sqrt{1+2 c+c^2+d^2}}\right] \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2}{2\left(-1-c+\sqrt{1+2 c+c^2+d^2}+d \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right)} - \\
 & \left(a \operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right)^2\left(-\operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]\right)^2\left((-1+c) \operatorname{Cos}[a+b x]-d \operatorname{Sin}[a+b x]\right)- \\
 & \quad \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2\left(d \operatorname{Cos}[a+b x]+(-1+c) \operatorname{Sin}[a+b x]\right) \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right) / \\
 & \quad (d \operatorname{Cos}[a+b x]+(-1+c) \operatorname{Sin}[a+b x])+ \\
 & \left(a \operatorname{Cos}\left[\frac{1}{2}(a+b x)\right]\right)^2\left(-\operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]\right)^2\left(\operatorname{Cos}[a+b x]+c \operatorname{Cos}[a+b x]-d \operatorname{Sin}[a+b x]\right)- \\
 & \quad \operatorname{Sec}\left[\frac{1}{2}(a+b x)\right]^2\left(d \operatorname{Cos}[a+b x]+\operatorname{Sin}[a+b x]+c \operatorname{Sin}[a+b x]\right) \operatorname{Tan}\left[\frac{1}{2}(a+b x)\right]\right) / \\
 & \quad \left.(d \operatorname{Cos}[a+b x]+\operatorname{Sin}[a+b x]+c \operatorname{Sin}[a+b x])\right)
 \end{aligned}$$

**Problem 346: Result more than twice size of optimal antiderivative.**

$$\int \operatorname{ArcTanh}\left[e^x\right] dx$$

Optimal (type 4, 21 leaves, 2 steps):

$$-\frac{1}{2} \operatorname{PolyLog}\left[2,-e^x\right]+\frac{1}{2} \operatorname{PolyLog}\left[2,e^x\right]$$

Result (type 4, 46 leaves):

$$x \operatorname{ArcTanh}\left[e^x\right]+\frac{1}{2}\left(-x\left(-\operatorname{Log}\left[1-e^x\right]+\operatorname{Log}\left[1+e^x\right]\right)-\operatorname{PolyLog}\left[2,-e^x\right]+\operatorname{PolyLog}\left[2,e^x\right]\right)$$

**Problem 361: Result unnecessarily involves higher level functions.**

$$\int \frac{(a+b \operatorname{ArcTanh}\left[c x^n\right])\left(d+e \operatorname{Log}\left[f x^m\right]\right)}{x} dx$$

Optimal (type 4, 136 leaves, 11 steps):

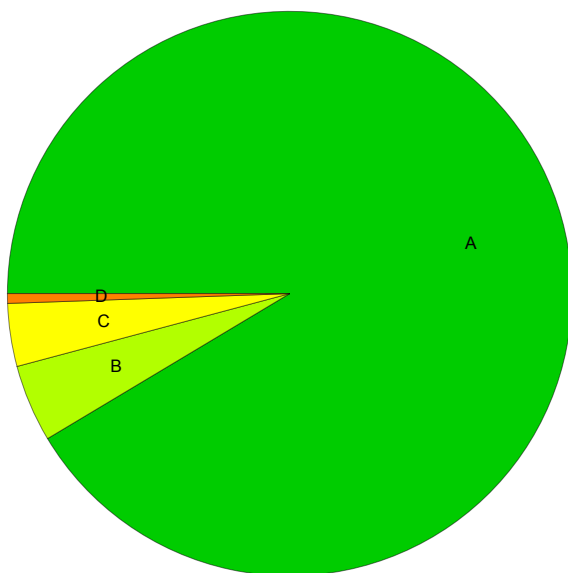
$$\begin{aligned}
 & a d \operatorname{Log}[x] + \frac{a e \operatorname{Log}[f x^m]^2}{2 m} - \frac{b d \operatorname{PolyLog}[2, -c x^n]}{2 n} - \\
 & \frac{b e \operatorname{Log}[f x^m] \operatorname{PolyLog}[2, -c x^n]}{2 n} + \frac{b d \operatorname{PolyLog}[2, c x^n]}{2 n} + \\
 & \frac{b e \operatorname{Log}[f x^m] \operatorname{PolyLog}[2, c x^n]}{2 n} + \frac{b e m \operatorname{PolyLog}[3, -c x^n]}{2 n^2} - \frac{b e m \operatorname{PolyLog}[3, c x^n]}{2 n^2}
 \end{aligned}$$

Result (type 5, 114 leaves):

$$\begin{aligned}
 & - \frac{b c e m x^n \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2 n}\right]}{n^2} + \frac{1}{n} \\
 & b c x^n \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, 1\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, c^2 x^{2 n}\right] (d + e \operatorname{Log}[f x^m]) + \\
 & \frac{1}{2} a \operatorname{Log}[x] (2 d - e m \operatorname{Log}[x] + 2 e \operatorname{Log}[f x^m])
 \end{aligned}$$

## Summary of Integration Test Results

361 integration problems



A - 330 optimal antiderivatives

B - 16 more than twice size of optimal antiderivatives

C - 13 unnecessarily complex antiderivatives

D - 2 unable to integrate problems

E - 0 integration timeouts