

# Mathematica 11.3 Integration Test Results

Test results for the 190 problems in "7.5.1 u (a+b arcsech(c x))^n.m"

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int x^6 (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 142 leaves, 8 steps):

$$\begin{aligned} & -\frac{5 b x \sqrt{1-c x}}{112 c^6 \sqrt{\frac{1}{1+c x}}} - \frac{5 b x^3 \sqrt{1-c x}}{168 c^4 \sqrt{\frac{1}{1+c x}}} - \frac{b x^5 \sqrt{1-c x}}{42 c^2 \sqrt{\frac{1}{1+c x}}} + \\ & \frac{1}{7} x^7 (a + b \operatorname{ArcSech}[c x]) + \frac{5 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcSin}[c x]}{112 c^7} \end{aligned}$$

Result (type 3, 143 leaves):

$$\begin{aligned} & \frac{a x^7}{7} + b \sqrt{\frac{1-c x}{1+c x}} \left( -\frac{5 x}{112 c^6} - \frac{5 x^2}{112 c^5} - \frac{5 x^3}{168 c^4} - \frac{5 x^4}{168 c^3} - \frac{x^5}{42 c^2} - \frac{x^6}{42 c} \right) + \\ & \frac{1}{7} b x^7 \operatorname{ArcSech}[c x] + \frac{5 i b \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right]}{112 c^7} \end{aligned}$$

Problem 21: Result unnecessarily involves imaginary or complex numbers.

$$\int x^4 (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 110 leaves, 6 steps):

$$\begin{aligned} & -\frac{3 b x \sqrt{1-c x}}{40 c^4 \sqrt{\frac{1}{1+c x}}} - \frac{b x^3 \sqrt{1-c x}}{20 c^2 \sqrt{\frac{1}{1+c x}}} + \frac{1}{5} x^5 (a + b \operatorname{ArcSech}[c x]) + \frac{3 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcSin}[c x]}{40 c^5} \end{aligned}$$

Result (type 3, 123 leaves):

$$\frac{a x^5}{5} + b \sqrt{\frac{1-c x}{1+c x}} \left( -\frac{3 x}{40 c^4} - \frac{3 x^2}{40 c^3} - \frac{x^3}{20 c^2} - \frac{x^4}{20 c} \right) + \frac{1}{5} b x^5 \operatorname{ArcSech}[c x] + \frac{3 i b \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right]}{40 c^5}$$

**Problem 23: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 78 leaves, 4 steps):

$$-\frac{b x \sqrt{1-c x}}{6 c^2 \sqrt{\frac{1}{1+c x}}} + \frac{1}{3} x^3 (a + b \operatorname{ArcSech}[c x]) + \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcSin}[c x]}{6 c^3}$$

Result (type 3, 103 leaves):

$$\frac{a x^3}{3} + b \sqrt{\frac{1-c x}{1+c x}} \left( -\frac{x}{6 c^2} - \frac{x^2}{6 c} \right) + \frac{1}{3} b x^3 \operatorname{ArcSech}[c x] + \frac{i b \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right]}{6 c^3}$$

**Problem 45: Result more than twice size of optimal antiderivative.**

$$\int (a + b \operatorname{ArcSech}[c x])^3 dx$$

Optimal (type 4, 140 leaves, 9 steps):

$$x (a + b \operatorname{ArcSech}[c x])^3 - \frac{6 b (a + b \operatorname{ArcSech}[c x])^2 \operatorname{ArcTan}\left[e^{\operatorname{ArcSech}[c x]}\right]}{c} + \frac{6 i b^2 (a + b \operatorname{ArcSech}[c x]) \operatorname{PolyLog}\left[2, -i e^{\operatorname{ArcSech}[c x]}\right]}{c} - \frac{6 i b^2 (a + b \operatorname{ArcSech}[c x]) \operatorname{PolyLog}\left[2, i e^{\operatorname{ArcSech}[c x]}\right]}{c} - \frac{6 i b^3 \operatorname{PolyLog}\left[3, -i e^{\operatorname{ArcSech}[c x]}\right]}{c} + \frac{6 i b^3 \operatorname{PolyLog}\left[3, i e^{\operatorname{ArcSech}[c x]}\right]}{c}$$

Result (type 4, 282 leaves):

$$\begin{aligned}
 & a^3 x + 3 a^2 b x \operatorname{ArcSech}[c x] - \frac{3 a^2 b \operatorname{ArcTan}\left[\frac{c x \sqrt{\frac{1-c x}{1+c x}}}{-1+c x}\right]}{c} + \frac{1}{c} \\
 & 3 i a b^2 \left( \operatorname{ArcSech}[c x] \left( -i c x \operatorname{ArcSech}[c x] + 2 \operatorname{Log}\left[1 - i e^{-\operatorname{ArcSech}[c x]}\right] - 2 \operatorname{Log}\left[1 + i e^{-\operatorname{ArcSech}[c x]}\right]\right) + \right. \\
 & \quad \left. 2 \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcSech}[c x]}\right] - 2 \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcSech}[c x]}\right]\right) + \frac{1}{c} \\
 & b^3 \left( c x \operatorname{ArcSech}[c x]^3 - 3 i \left( -\operatorname{ArcSech}[c x]^2 \left( \operatorname{Log}\left[1 - i e^{-\operatorname{ArcSech}[c x]}\right] - \operatorname{Log}\left[1 + i e^{-\operatorname{ArcSech}[c x]}\right]\right) - \right. \right. \\
 & \quad \left. \left. 2 \operatorname{ArcSech}[c x] \left( \operatorname{PolyLog}\left[2, -i e^{-\operatorname{ArcSech}[c x]}\right] - \operatorname{PolyLog}\left[2, i e^{-\operatorname{ArcSech}[c x]}\right]\right) - \right. \right. \\
 & \quad \left. \left. 2 \left( \operatorname{PolyLog}\left[3, -i e^{-\operatorname{ArcSech}[c x]}\right] - \operatorname{PolyLog}\left[3, i e^{-\operatorname{ArcSech}[c x]}\right]\right) \right) \right)
 \end{aligned}$$

**Problem 71: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (d x)^m (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 5, 87 leaves, 3 steps):

$$\frac{(d x)^{1+m} (a + b \operatorname{ArcSech}[c x])}{d (1+m)} + \frac{b (d x)^{1+m} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right]}{d (1+m)^2}$$

Result (type 6, 183 leaves):

$$\begin{aligned}
 & \frac{1}{1+m} (d x)^m \left( a x - \left( 12 b \sqrt{\frac{1-c x}{1+c x}} (1+c x) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) / \\
 & \left( c (-1+c x) \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \right. \right. \\
 & \quad \left. \left. (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right]\right) \right) \right) \right) + b x \operatorname{ArcSech}[c x]
 \end{aligned}$$

**Problem 74: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d + e x)^3 (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 264 leaves, 9 steps):

$$\begin{aligned} & - \frac{b e (9 c^2 d^2 + e^2) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{6 c^4} - \frac{b d e^2 x \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{2 c^2} - \\ & \frac{b e^3 x^2 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{12 c^2} + \frac{(d+e x)^4 (a+b \operatorname{ArcSech}[c x])}{4 e} + \\ & \frac{b d (2 c^2 d^2 + e^2) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcSin}[c x]}{2 c^3} - \frac{b d^4 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}[\sqrt{1-c^2 x^2}]}{4 e} \end{aligned}$$

Result (type 3, 190 leaves):

$$\begin{aligned} & \frac{1}{4} \left( 4 a d^3 x + 6 a d^2 e x^2 + 4 a d e^2 x^3 + a e^3 x^4 - \frac{b e \sqrt{\frac{1-c x}{1+c x}} (1+c x) (2 e^2 + c^2 (18 d^2 + 6 d e x + e^2 x^2))}{3 c^4} + \right. \\ & \left. b x (4 d^3 + 6 d^2 e x + 4 d e^2 x^2 + e^3 x^3) \operatorname{ArcSech}[c x] + \right. \\ & \left. \frac{2 i b d (2 c^2 d^2 + e^2) \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right]}{c^3} \right) \end{aligned}$$

**Problem 75: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)^2 (a+b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 201 leaves, 8 steps):

$$\begin{aligned} & - \frac{b d e \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{c^2} - \\ & \frac{b e^2 x \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{6 c^2} + \frac{(d+e x)^3 (a+b \operatorname{ArcSech}[c x])}{3 e} + \\ & \frac{b (6 c^2 d^2 + e^2) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcSin}[c x]}{6 c^3} - \frac{b d^3 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}[\sqrt{1-c^2 x^2}]}{3 e} \end{aligned}$$

Result (type 3, 147 leaves):

$$\frac{1}{6 c^3} \left( -b c e \sqrt{\frac{1-c x}{1+c x}} (1+c x) (6 d+e x) + 2 a c^3 x (3 d^2+3 d e x+e^2 x^2) + \right. \\ \left. 2 b c^3 x (3 d^2+3 d e x+e^2 x^2) \operatorname{ArcSech}[c x] + i b (6 c^2 d^2+e^2) \operatorname{Log}\left[-2 i c x+2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right] \right)$$

**Problem 78: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{d+e x} dx$$

Optimal (type 4, 229 leaves, 4 steps):

$$-\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+e^{-2 \operatorname{ArcSech}[c x]}\right]}{e} + \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{\left(e-\sqrt{-c^2 d^2+e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right]}{e} + \\ \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{\left(e+\sqrt{-c^2 d^2+e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right]}{e} + \frac{b \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcSech}[c x]}\right]}{2 e} - \\ \frac{b \operatorname{PolyLog}\left[2,-\frac{\left(e-\sqrt{-c^2 d^2+e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right]}{e} - \frac{b \operatorname{PolyLog}\left[2,-\frac{\left(e+\sqrt{-c^2 d^2+e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right]}{e}$$

Result (type 4, 393 leaves):

$$\frac{a \operatorname{Log}[d + e x]}{e} + \frac{1}{2e} b \left( \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right] - \right.$$

$$2 \left( -4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{e}{c d}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(-c d + e) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{-c^2 d^2 + e^2}}\right] + \operatorname{ArcSech}[c x] \right.$$

$$\operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] - \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{\left(e - \sqrt{-c^2 d^2 + e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right] +$$

$$2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{e}{c d}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{\left(e - \sqrt{-c^2 d^2 + e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right] -$$

$$\operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{\left(e + \sqrt{-c^2 d^2 + e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right] -$$

$$2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{e}{c d}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{\left(e + \sqrt{-c^2 d^2 + e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right] + \operatorname{PolyLog}\left[2, \right.$$

$$\left. \frac{\left(-e + \sqrt{-c^2 d^2 + e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right] + \operatorname{PolyLog}\left[2, -\frac{\left(e + \sqrt{-c^2 d^2 + e^2}\right) e^{-\operatorname{ArcSech}[c x]}}{c d}\right] \right)$$

**Problem 80: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcSech}[c x]}{(d + e x)^3} dx$$

Optimal (type 3, 306 leaves, 11 steps):

$$\frac{b e \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{2 d (c^2 d^2 - e^2) (d+e x)} - \frac{a + b \operatorname{ArcSech}[c x]}{2 e (d+e x)^2} +$$

$$\frac{b c^2 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTan}\left[\frac{e+c^2 d x}{\sqrt{c^2 d^2 - e^2} \sqrt{1-c^2 x^2}}\right]}{2 (c^2 d^2 - e^2)^{3/2}} +$$

$$\frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTan}\left[\frac{e+c^2 d x}{\sqrt{c^2 d^2 - e^2} \sqrt{1-c^2 x^2}}\right]}{2 d^2 \sqrt{c^2 d^2 - e^2}} + \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\sqrt{1-c^2 x^2}\right]}{2 d^2 e}$$

Result (type 3, 342 leaves):

$$\frac{1}{2} \left( -\frac{a}{e (d+e x)^2} + \frac{b \sqrt{\frac{1-c x}{1+c x}} (e+c e x)}{d (c d - e) (c d + e) (d+e x)} - \right.$$

$$\frac{b \operatorname{ArcSech}[c x]}{e (d+e x)^2} - \frac{b \operatorname{Log}[x]}{d^2 e} + \frac{b \operatorname{Log}\left[1 + \sqrt{\frac{1-c x}{1+c x}} + c x \sqrt{\frac{1-c x}{1+c x}}\right]}{d^2 e} - \left( i b (2 c^2 d^2 - e^2) \right.$$

$$\left. \operatorname{Log}\left[4 d^2 e \sqrt{c^2 d^2 - e^2} \left( i e + i c^2 d x + \sqrt{c^2 d^2 - e^2} \sqrt{\frac{1-c x}{1+c x}} + c \sqrt{c^2 d^2 - e^2} x \sqrt{\frac{1-c x}{1+c x}} \right) \right] \right) /$$

$$\left. \left( b (2 c^2 d^2 - e^2) (d+e x) \right) \right) / \left( d^2 (c d - e) (c d + e) \sqrt{c^2 d^2 - e^2} \right)$$

**Problem 81: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d+e x)^{3/2} (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 4, 343 leaves, 21 steps):

$$\begin{aligned}
 & - \frac{4 b e \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x} \sqrt{1-c^2 x^2}}{15 c^2} + \frac{2 (d+e x)^{5/2} (a+b \operatorname{ArcSech}[c x])}{5 e} - \\
 & \frac{28 b d \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{15 c \sqrt{\frac{c(d+e x)}{c d+e}}} - \frac{1}{15 c^3 \sqrt{d+e x}} \\
 & 4 b (2 c^2 d^2 + e^2) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right] - \\
 & \frac{1}{5 e \sqrt{d+e x}} 4 b d^3 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]
 \end{aligned}$$

Result(type 4, 575 leaves):



$$\begin{aligned}
 & -\frac{4 b e \sqrt{\frac{1-c x}{1+c x}} (1+c x) \sqrt{d+e x}}{15 c^2} + \frac{2 a (d+e x)^{5/2}}{5 e} + \\
 & \frac{2 b (d+e x)^{5/2} \operatorname{ArcSech}[c x]}{5 e} + \frac{1}{15 c^2 \sqrt{-\frac{c d+e}{c}} \sqrt{d+e x} (e-c x)} \\
 & 4 b \sqrt{\frac{1-c x}{1+c x}} \left( 7 c^2 d^3 \sqrt{-\frac{c d+e}{c}} - 7 d e^2 \sqrt{-\frac{c d+e}{c}} - 14 c^2 d^2 \sqrt{-\frac{c d+e}{c}} (d+e x) + \right. \\
 & 7 c^2 d \sqrt{-\frac{c d+e}{c}} (d+e x)^2 - 7 i c d (c d+e) \sqrt{\frac{e(-1+c x)}{c(d+e x)}} \\
 & \left. (d+e x)^{3/2} \sqrt{\frac{e+c x}{c d+c x}} \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}} \right], \frac{c d-e}{c d+e} \right] + \right. \\
 & i (6 c^2 d^2 + 7 c d e + e^2) \sqrt{\frac{e(-1+c x)}{c(d+e x)}} (d+e x)^{3/2} \sqrt{\frac{e+c x}{c d+c x}} \\
 & \left. \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}} \right], \frac{c d-e}{c d+e} \right] + 3 i c^2 d^2 \sqrt{\frac{e(-1+c x)}{c(d+e x)}} \right. \\
 & \left. (d+e x)^{3/2} \sqrt{\frac{e+c x}{c d+c x}} \operatorname{EllipticPi}\left[ \frac{c d}{c d+e}, i \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}} \right], \frac{c d-e}{c d+e} \right] \right)
 \end{aligned}$$

**Problem 82: Result unnecessarily involves imaginary or complex numbers.**

$$\int \sqrt{d+e x} (a+b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 4, 279 leaves, 14 steps):

$$\frac{2 (d+e x)^{3/2} (a+b \operatorname{ArcSech}[c x])}{3 e}$$

$$\frac{4 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 c \sqrt{\frac{c(d+e x)}{c d+e}}}$$

$$\frac{4 b d \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 c \sqrt{d+e x}} - \frac{1}{3 e \sqrt{d+e x}}$$

$$4 b d^2 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]$$

Result (type 4, 279 leaves):

$$\frac{2}{3} \left( \frac{a (d+e x)^{3/2}}{e} + \frac{b (d+e x)^{3/2} \operatorname{ArcSech}[c x]}{e} - \frac{1}{c^2 \sqrt{\frac{e-c e x}{c d+e}}} 2 i b \sqrt{-\frac{c}{c d+e}} \sqrt{\frac{1-c x}{1+c x}} \right. \\ \left. \sqrt{\frac{e(1+c x)}{-c d+e}} \left( (c d-e) \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] + \right. \right. \\ \left. \left. (-2 c d+e) \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] + \right. \right. \\ \left. \left. c d \operatorname{EllipticPi}\left[1 + \frac{e}{c d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{c}{c d+e}} \sqrt{d+e x}\right], \frac{c d+e}{c d-e}\right] \right) \right)$$

**Problem 83: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{\sqrt{d+e x}} dx$$

Optimal (type 4, 187 leaves, 8 steps):

$$\frac{2 \sqrt{d+e x} (a+b \operatorname{ArcSech}[c x])}{e} - \frac{4 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{c \sqrt{d+e x}} - \frac{1}{e \sqrt{d+e x}}$$

$$4 b d \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]$$

Result (type 4, 286 leaves):

$$\left( 2 \sqrt{d+e x} \left( (-1+c x) \sqrt{-\frac{c(d+e x)}{c d+e}} (a+b \operatorname{ArcSech}[c x]) + 2 i b \sqrt{\frac{1-c x}{1+c x}} \sqrt{\frac{e(1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \right. \right.$$

$$\left. \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\sqrt{-\frac{c(d+e x)}{c d+e}}\right], \frac{c d+e}{c d-e}\right] - 2 i b \sqrt{\frac{1-c x}{1+c x}} \sqrt{\frac{e(1+c x)}{-c d+e}} \sqrt{\frac{e-c e x}{c d+e}} \right.$$

$$\left. \left. \operatorname{EllipticPi}\left[1+\frac{e}{c d}, i \operatorname{ArcSinh}\left[\sqrt{-\frac{c(d+e x)}{c d+e}}\right], \frac{c d+e}{c d-e}\right] \right) \right) / \left( e (-1+c x) \sqrt{-\frac{c(d+e x)}{c d+e}} \right)$$

**Problem 84: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{(d+e x)^{3/2}} dx$$

Optimal (type 4, 105 leaves, 5 steps):

$$-\frac{2(a+b \operatorname{ArcSech}[c x])}{e \sqrt{d+e x}} + \frac{4 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{e \sqrt{d+e x}}$$

Result (type 4, 223 leaves):

$$\begin{aligned}
 & -\frac{2 a}{e \sqrt{d+e x}}-\frac{2 b \operatorname{ArcSech}[c x]}{e \sqrt{d+e x}}- \\
 & \left(4 i b \sqrt{\frac{1-c x}{1+c x}} \sqrt{\frac{e+c e x}{c d+c e x}} \left(\operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}}\right], \frac{c d-e}{c d+e}\right]-\right. \right. \\
 & \left. \left.\operatorname{EllipticPi}\left[\frac{c d}{c d+e}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}}\right], \frac{c d-e}{c d+e}\right]\right)\right) / \left(c d \sqrt{-\frac{c d+e}{c}} \sqrt{\frac{e(-1+c x)}{c(d+e x)}}\right)
 \end{aligned}$$

**Problem 85: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{(d+e x)^{5 / 2}} d x$$

Optimal (type 4, 278 leaves, 11 steps):

$$\begin{aligned}
 & \frac{4 b e \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{3 d\left(c^2 d^2-e^2\right) \sqrt{d+e x}}-\frac{2(a+b \operatorname{ArcSech}[c x])}{3 e(d+e x)^{3 / 2}}- \\
 & \frac{4 b c \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 d\left(c^2 d^2-e^2\right) \sqrt{\frac{c(d+e x)}{c d+e}}}+ \\
 & \frac{4 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{3 d e \sqrt{d+e x}}
 \end{aligned}$$

Result (type 4, 698 leaves):

$$\begin{aligned}
 & \frac{1}{3 (d+ex)^{3/2}} \\
 & \left( \frac{2a}{e} + \frac{4b \sqrt{\frac{1-cx}{1+cx}} (d+ex) (e+cx)}{d(c d-e)(c d+e)} - \frac{2b \operatorname{ArcSech}[cx]}{e} - \frac{1}{d^2 e \sqrt{-\frac{cd+e}{c}} (-c^2 d^2 + e^2) (-1+cx)} \right. \\
 & 4b \sqrt{\frac{1-cx}{1+cx}} (d+ex) \left( -c^2 d^3 \sqrt{-\frac{cd+e}{c}} + d e^2 \sqrt{-\frac{cd+e}{c}} + \right. \\
 & 2c^2 d^2 \sqrt{-\frac{cd+e}{c}} (d+ex) - c^2 d \sqrt{-\frac{cd+e}{c}} (d+ex)^2 + i c d (c d+e) \sqrt{\frac{e(-1+cx)}{c(d+ex)}} \\
 & (d+ex)^{3/2} \sqrt{\frac{e+cx}{cd+cx}} \operatorname{EllipticE}\left[ i \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}} \right], \frac{cd-e}{cd+e} \right] - \\
 & i (2c^2 d^2 + c d e - e^2) \sqrt{\frac{e(-1+cx)}{c(d+ex)}} (d+ex)^{3/2} \sqrt{\frac{e+cx}{cd+cx}} \\
 & \operatorname{EllipticF}\left[ i \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}} \right], \frac{cd-e}{cd+e} \right] + i c^2 d^2 \sqrt{\frac{e(-1+cx)}{c(d+ex)}} (d+ex)^{3/2} \\
 & \sqrt{\frac{e+cx}{cd+cx}} \operatorname{EllipticPi}\left[ \frac{cd}{cd+e}, i \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}} \right], \frac{cd-e}{cd+e} \right] - i e^2 \sqrt{\frac{e(-1+cx)}{c(d+ex)}} \\
 & \left. \left. (d+ex)^{3/2} \sqrt{\frac{e+cx}{cd+cx}} \operatorname{EllipticPi}\left[ \frac{cd}{cd+e}, i \operatorname{ArcSinh}\left[ \frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}} \right], \frac{cd-e}{cd+e} \right] \right) \right)
 \end{aligned}$$

**Problem 86: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcSech}[cx]}{(d+ex)^{7/2}} dx$$

Optimal (type 4, 609 leaves, 18 steps):

$$\frac{4 b e \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{15 d \left(c^2 d^2 - e^2\right) (d+e x)^{3/2}} + \frac{16 b c^2 e \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{15 \left(c^2 d^2 - e^2\right)^2 \sqrt{d+e x}} +$$

$$\frac{4 b e \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{5 d^2 \left(c^2 d^2 - e^2\right) \sqrt{d+e x}} - \frac{2 (a+b \operatorname{ArcSech}[c x])}{5 e (d+e x)^{5/2}} -$$

$$\frac{16 b c^3 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{15 \left(c^2 d^2 - e^2\right)^2 \sqrt{\frac{c(d+e x)}{c d+e}}} -$$

$$\frac{4 b c \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x} \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{5 d^2 \left(c^2 d^2 - e^2\right) \sqrt{\frac{c(d+e x)}{c d+e}}} +$$

$$\frac{4 b c \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{15 d \left(c^2 d^2 - e^2\right) \sqrt{d+e x}} +$$

$$\frac{4 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{\frac{c(d+e x)}{c d+e}} \operatorname{EllipticPi}\left[2, \operatorname{ArcSin}\left[\frac{\sqrt{1-c x}}{\sqrt{2}}\right], \frac{2 e}{c d+e}\right]}{5 d^2 e \sqrt{d+e x}}$$

Result (type 4, 1193 leaves):

$$-\frac{2 a}{5 e (d+e x)^{5/2}} + \sqrt{\frac{1-c x}{1+c x}} \sqrt{d+e x}$$

$$\left( \frac{4 b c (7 c^2 d^2 - 3 e^2)}{15 d^2 (c^2 d^2 - e^2)^2} - \frac{4 b}{15 d (c d+e) (d+e x)^2} - \frac{4 b (6 c^2 d^2 - c d e - 3 e^2)}{15 d^2 (c d-e) (c d+e)^2 (d+e x)} \right) -$$

$$\frac{2 b \operatorname{ArcSech}[c x]}{5 e (d+e x)^{5/2}} - \frac{1}{15 d^3 \sqrt{-\frac{c d+e}{c}} (c^2 d^2 - e^2)^2 \left(\frac{e}{d+e x} + c \left(-1 + \frac{d}{d+e x}\right)\right)}$$

$$4 b \sqrt{d+e x} \sqrt{\frac{c - \frac{c d}{d+e x} - \frac{e}{d+e x}}{c - \frac{c d}{d+e x} + \frac{e}{d+e x}}} \left( -7 c^4 d^3 \sqrt{-\frac{c d+e}{c}} + 3 c^2 d e^2 \sqrt{-\frac{c d+e}{c}} - \frac{7 c^4 d^5 \sqrt{-\frac{c d+e}{c}}}{(d+e x)^2} + \right.$$

$$\begin{aligned}
 & \frac{10 c^2 d^3 e^2 \sqrt{-\frac{c d+e}{c}}}{(d+e x)^2} - \frac{3 d e^4 \sqrt{-\frac{c d+e}{c}}}{(d+e x)^2} + \frac{14 c^4 d^4 \sqrt{-\frac{c d+e}{c}}}{d+e x} - \frac{6 c^2 d^2 e^2 \sqrt{-\frac{c d+e}{c}}}{d+e x} + \\
 & \frac{1}{\sqrt{d+e x}} i c d (7 c^3 d^3 + 7 c^2 d^2 e - 3 c d e^2 - 3 e^3) \sqrt{1 - \frac{d}{d+e x} - \frac{e}{c (d+e x)}} \\
 & \sqrt{1 - \frac{d}{d+e x} + \frac{e}{c (d+e x)}} \operatorname{EllipticE}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}}\right], \frac{c d-e}{c d+e}\right] - \\
 & \frac{1}{\sqrt{d+e x}} i (9 c^4 d^4 + 7 c^3 d^3 e - 8 c^2 d^2 e^2 - 3 c d e^3 + 3 e^4) \sqrt{1 - \frac{d}{d+e x} - \frac{e}{c (d+e x)}} \\
 & \sqrt{1 - \frac{d}{d+e x} + \frac{e}{c (d+e x)}} \operatorname{EllipticF}\left[i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}}\right], \frac{c d-e}{c d+e}\right] + \\
 & \frac{1}{\sqrt{d+e x}} 3 i c^4 d^4 \sqrt{1 - \frac{d}{d+e x} - \frac{e}{c (d+e x)}} \sqrt{1 - \frac{d}{d+e x} + \frac{e}{c (d+e x)}} \\
 & \operatorname{EllipticPi}\left[\frac{c d}{c d+e}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}}\right], \frac{c d-e}{c d+e}\right] - \frac{1}{\sqrt{d+e x}} \\
 & 6 i c^2 d^2 e^2 \sqrt{1 - \frac{d}{d+e x} - \frac{e}{c (d+e x)}} \sqrt{1 - \frac{d}{d+e x} + \frac{e}{c (d+e x)}} \operatorname{EllipticPi}\left[\frac{c d}{c d+e}, \right. \\
 & \left. i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}}\right], \frac{c d-e}{c d+e}\right] + \frac{1}{\sqrt{d+e x}} 3 i e^4 \sqrt{1 - \frac{d}{d+e x} - \frac{e}{c (d+e x)}} \\
 & \left. \sqrt{1 - \frac{d}{d+e x} + \frac{e}{c (d+e x)}} \operatorname{EllipticPi}\left[\frac{c d}{c d+e}, i \operatorname{ArcSinh}\left[\frac{\sqrt{-\frac{c d+e}{c}}}{\sqrt{d+e x}}\right], \frac{c d-e}{c d+e}\right]\right)
 \end{aligned}$$

**Problem 88: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^4 (d+e x^2) (a+b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 229 leaves, 6 steps):

$$\frac{b (42 c^2 d + 25 e) x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{560 c^6} - \frac{b (42 c^2 d + 25 e) x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{840 c^4}$$

$$\frac{b e x^5 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{42 c^2} + \frac{1}{5} d x^5 (a + b \operatorname{ArcSech}[c x]) +$$

$$\frac{1}{7} e x^7 (a + b \operatorname{ArcSech}[c x]) + \frac{b (42 c^2 d + 25 e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcSin}[c x]}{560 c^7}$$

Result (type 3, 162 leaves):

$$\frac{1}{1680 c^7}$$

$$\left( 48 a c^7 x^5 (7 d + 5 e x^2) - b c x \sqrt{\frac{1-cx}{1+cx}} (1+cx) (75 e + 2 c^2 (63 d + 25 e x^2) + c^4 (84 d x^2 + 40 e x^4)) + \right.$$

$$\left. 48 b c^7 x^5 (7 d + 5 e x^2) \operatorname{ArcSech}[c x] + 3 i b (42 c^2 d + 25 e) \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-cx}{1+cx}} (1+cx)\right] \right)$$

**Problem 89: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 (d + e x^2) (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 174 leaves, 5 steps):

$$\frac{b (20 c^2 d + 9 e) x \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{120 c^4}$$

$$\frac{b e x^3 \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2}}{20 c^2} + \frac{1}{3} d x^3 (a + b \operatorname{ArcSech}[c x]) +$$

$$\frac{1}{5} e x^5 (a + b \operatorname{ArcSech}[c x]) + \frac{b (20 c^2 d + 9 e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcSin}[c x]}{120 c^5}$$

Result (type 3, 144 leaves):



$$\frac{1}{120 c^5} \left( 8 a c^5 x^3 (5 d + 3 e x^2) - b c x \sqrt{\frac{1-c x}{1+c x}} (1+c x) (9 e + c^2 (20 d + 6 e x^2)) + \right. \\ \left. 8 b c^5 x^3 (5 d + 3 e x^2) \operatorname{ArcSech}[c x] + i b (20 c^2 d + 9 e) \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right] \right)$$

**Problem 90: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d + e x^2) (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 112 leaves, 4 steps):

$$-\frac{b e x \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{6 c^2} + d x (a + b \operatorname{ArcSech}[c x]) + \\ \frac{1}{3} e x^3 (a + b \operatorname{ArcSech}[c x]) + \frac{b (6 c^2 d + e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcSin}[c x]}{6 c^3}$$

Result (type 3, 181 leaves):

$$a d x + \frac{1}{3} a e x^3 + b e \sqrt{\frac{1-c x}{1+c x}} \left( -\frac{x}{6 c^2} - \frac{x^2}{6 c} \right) + b d x \operatorname{ArcSech}[c x] + \frac{1}{3} b e x^3 \operatorname{ArcSech}[c x] + \\ \frac{2 b d \sqrt{\frac{1-c x}{1+c x}} \sqrt{1-c^2 x^2} \operatorname{ArcSin}\left[\frac{\sqrt{1+c x}}{\sqrt{2}}\right]}{c - c^2 x} + \frac{i b e \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right]}{6 c^3}$$

**Problem 100: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^2 (d + e x^2)^2 (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 275 leaves, 6 steps):

$$\frac{b(280c^4d^2 + 252c^2de + 75e^2)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{1680c^6} - \frac{be(84c^2d + 25e)x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{840c^4} - \frac{be^2x^5\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{42c^2} + \frac{1}{3}d^2x^3(a+b\operatorname{ArcSech}[cx]) + \frac{2}{5}dex^5(a+b\operatorname{ArcSech}[cx]) + \frac{1}{7}e^2x^7(a+b\operatorname{ArcSech}[cx]) + \frac{b(280c^4d^2 + 252c^2de + 75e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{ArcSin}[cx]}{1680c^7}$$

Result (type 3, 207 leaves):

$$\frac{1}{1680c^7} \left( 16ac^7x^3(35d^2 + 42dex^2 + 15e^2x^4) - bcx\sqrt{\frac{1-cx}{1+cx}}(1+cx)(75e^2 + 2c^2e(126d + 25ex^2) + 8c^4(35d^2 + 21dex^2 + 5e^2x^4)) + 16bc^7x^3(35d^2 + 42dex^2 + 15e^2x^4)\operatorname{ArcSech}[cx] + i b(280c^4d^2 + 252c^2de + 75e^2)\operatorname{Log}\left[-2ix + 2\sqrt{\frac{1-cx}{1+cx}}(1+cx)\right] \right)$$

**Problem 101: Result unnecessarily involves imaginary or complex numbers.**

$$\int (d + ex^2)^2 (a + b \operatorname{ArcSech}[cx]) dx$$

Optimal (type 3, 204 leaves, 5 steps):

$$\frac{be(40c^2d + 9e)x\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{120c^4} - \frac{be^2x^3\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\sqrt{1-c^2x^2}}{20c^2} + d^2x(a+b\operatorname{ArcSech}[cx]) + \frac{2}{3}dex^3(a+b\operatorname{ArcSech}[cx]) + \frac{1}{5}e^2x^5(a+b\operatorname{ArcSech}[cx]) + \frac{b(120c^4d^2 + 40c^2de + 9e^2)\sqrt{\frac{1}{1+cx}}\sqrt{1+cx}\operatorname{ArcSin}[cx]}{120c^5}$$

Result (type 3, 174 leaves):

$$\frac{1}{120 c^5} \left( 8 a c^5 x (15 d^2 + 10 d e x^2 + 3 e^2 x^4) - b c e x \sqrt{\frac{1-c x}{1+c x}} (1+c x) (9 e + c^2 (40 d + 6 e x^2)) + \right. \\ \left. 8 b c^5 x (15 d^2 + 10 d e x^2 + 3 e^2 x^4) \operatorname{ArcSech}[c x] + \right. \\ \left. i b (120 c^4 d^2 + 40 c^2 d e + 9 e^2) \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right] \right)$$

**Problem 102: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{ArcSech}[c x])}{x^2} dx$$

Optimal (type 3, 177 leaves, 5 steps):

$$\frac{b d^2 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{x} - \frac{b e^2 x \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{6 c^2} - \\ \frac{d^2 (a + b \operatorname{ArcSech}[c x])}{x} + 2 d e x (a + b \operatorname{ArcSech}[c x]) + \\ \frac{1}{3} e^2 x^3 (a + b \operatorname{ArcSech}[c x]) + \frac{b e (12 c^2 d + e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcSin}[c x]}{6 c^3}$$

Result (type 3, 158 leaves):

$$\frac{1}{6 c^3 x} \left( -b c \sqrt{\frac{1-c x}{1+c x}} (1+c x) (-6 c^2 d^2 + e^2 x^2) + \right. \\ \left. 2 a c^3 (-3 d^2 + 6 d e x^2 + e^2 x^4) + 2 b c^3 (-3 d^2 + 6 d e x^2 + e^2 x^4) \operatorname{ArcSech}[c x] + \right. \\ \left. i b e (12 c^2 d + e) x \operatorname{Log}\left[-2 i c x + 2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right] \right)$$

**Problem 103: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{(d + e x^2)^2 (a + b \operatorname{ArcSech}[c x])}{x^4} dx$$

Optimal (type 3, 176 leaves, 5 steps):

$$\frac{b d^2 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{9 x^3} + \frac{2 b d (c^2 d+9 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{9 x} -$$

$$\frac{d^2 (a+b \operatorname{ArcSech}[c x])}{3 x^3} - \frac{2 d e (a+b \operatorname{ArcSech}[c x])}{x} +$$

$$e^2 x (a+b \operatorname{ArcSech}[c x]) + \frac{b e^2 \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcSin}[c x]}{c}$$

Result (type 3, 149 leaves):

$$\frac{1}{9 c x^3} \left( b c d \sqrt{\frac{1-c x}{1+c x}} (1+c x) (d+2 c^2 d x^2+18 e x^2) - 3 a c (d^2+6 d e x^2-3 e^2 x^4) - \right.$$

$$\left. 3 b c (d^2+6 d e x^2-3 e^2 x^4) \operatorname{ArcSech}[c x] + 9 i b e^2 x^3 \operatorname{Log}\left[-2 i c x+2 \sqrt{\frac{1-c x}{1+c x}} (1+c x)\right] \right)$$

**Problem 110: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 (a+b \operatorname{ArcSech}[c x])}{d+e x^2} dx$$

Optimal (type 4, 519 leaves, 24 steps):

$$\begin{aligned}
 & \frac{x (a + b \operatorname{ArcSech}[c x])}{e} - \frac{b \operatorname{ArcTan}\left[\sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}\right]}{c e} + \\
 & \frac{\sqrt{-d} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^{3/2}} - \\
 & \frac{\sqrt{-d} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^{3/2}} + \\
 & \frac{\sqrt{-d} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^{3/2}} - \\
 & \frac{\sqrt{-d} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^{3/2}} - \\
 & \frac{b \sqrt{-d} \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^{3/2}} + \frac{b \sqrt{-d} \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^{3/2}} - \\
 & \frac{b \sqrt{-d} \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^{3/2}} + \frac{b \sqrt{-d} \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^{3/2}}
 \end{aligned}$$

Result (type 4, 921 leaves):

$$\begin{aligned}
 & \frac{1}{4 c e^{3/2}} \left( 4 a c \sqrt{e} x - 4 a c \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + \right. \\
 & \left. b \left( 4 \sqrt{e} \left( c x \operatorname{ArcSech}[c x] - 2 \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]\right]\right) - 2 i c \sqrt{d} \right. \right. \\
 & \left. \left( -4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \operatorname{ArcSech}[c x] \right. \right. \\
 & \left. \left. \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] - \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i (\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2, \right. \\
 & \left. \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\left. \right]+ \\
 & 2 i c \sqrt{d}\left(-4 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(-i c \sqrt{d}+\sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]+ \right. \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+e^{-2 \operatorname{ArcSech}[c x]}\right]- \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2, \right. \\
 & \left. \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\left. \right)\left. \right)\left. \right)
 \end{aligned}$$

**Problem 111: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x(a+b \operatorname{ArcSech}[c x])}{d+e x^2} d x$$

Optimal (type 4, 441 leaves, 26 steps):

$$\begin{aligned}
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c\sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c\sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c\sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c\sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} - \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcSech}[c x]}\right]}{e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c\sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \\
 & \frac{b \operatorname{PolyLog}\left[2, \frac{c\sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c\sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} + \\
 & \frac{b \operatorname{PolyLog}\left[2, \frac{c\sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e} - \frac{b \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcSech}[c x]}\right]}{2 e}
 \end{aligned}$$

Result (type 4, 860 leaves):

$$\begin{aligned}
 & \frac{1}{2 e} \left( 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) + \\
 & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] - 2 b \operatorname{ArcSech}[c x] \\
 & \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] + b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c\sqrt{d}}\right] - \\
 & 2 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c\sqrt{d}}\right] + \\
 & b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c\sqrt{d}}\right] - \\
 & 2 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c\sqrt{d}}\right] + \\
 & b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c\sqrt{d}}\right] +
 \end{aligned}$$

$$\begin{aligned}
 & 2 i b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 2 i b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+a \operatorname{Log}[d+e x^2]+ \\
 & b \operatorname{PolyLog}\left[2,-e^{-2 \operatorname{ArcSech}[c x]}\right]-b \operatorname{PolyLog}\left[2,\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & b \operatorname{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & b \operatorname{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & \left. b \operatorname{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\right)
 \end{aligned}$$

**Problem 112: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{d+e x^2} d x$$

Optimal (type 4, 469 leaves, 19 steps):

$$\begin{aligned}
 & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 \sqrt{-d} \sqrt{e}}- \\
 & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 \sqrt{-d} \sqrt{e}}+\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 \sqrt{-d} \sqrt{e}}- \\
 & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 \sqrt{-d} \sqrt{e}}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 \sqrt{-d} \sqrt{e}}+ \\
 & \frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 \sqrt{-d} \sqrt{e}}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 \sqrt{-d} \sqrt{e}}+\frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 \sqrt{-d} \sqrt{e}}
 \end{aligned}$$

Result (type 4, 849 leaves):



$$\frac{1}{2\sqrt{d}\sqrt{e}}$$

$$\left( 2 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] - 4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] \right) +$$

$$4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] -$$

$$i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] -$$

$$2 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$2 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] -$$

$$2 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] -$$

$$i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$2 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] -$$

$$i b \operatorname{PolyLog}\left[2, \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$i b \operatorname{PolyLog}\left[2, \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$\left. \begin{aligned} & i b \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\ & i b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \end{aligned} \right)$$

**Problem 113: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{x\left(d+e x^2\right)} d x$$

Optimal (type 4, 417 leaves, 19 steps):

$$\begin{aligned} & \frac{(a+b \operatorname{ArcSech}[c x])^2}{2 b d}-\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d}- \\ & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d}-\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d}- \\ & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d}- \\ & \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d}-\frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d} \end{aligned}$$

Result (type 4, 841 leaves):

$$\begin{aligned} & -\frac{1}{2 d} \\ & \left( b \operatorname{ArcSech}[c x]^2+4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(-i c \sqrt{d}+\sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]\right)+ \\ & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(i c \sqrt{d}+\sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]+ \\ & b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \end{aligned}$$

$$\begin{aligned}
 & 2 i b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 i b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 2 i b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 2 i b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 a \operatorname{Log}[x]+a \operatorname{Log}[d+e x^2]-b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & b \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & b \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & \left. b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\right)
 \end{aligned}$$

**Problem 114: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{x^2(d+e x^2)} dx$$

Optimal (type 4, 523 leaves, 24 steps):

$$\frac{b c \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{d} - \frac{a}{d x} - \frac{b \operatorname{ArcSech}[c x]}{d x} + \frac{\sqrt{e} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e - \sqrt{c^2 d + e}}}\right]}{2 (-d)^{3/2}} -$$

$$\frac{\sqrt{e} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e - \sqrt{c^2 d + e}}}\right]}{2 (-d)^{3/2}} +$$

$$\frac{\sqrt{e} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e + \sqrt{c^2 d + e}}}\right]}{2 (-d)^{3/2}} -$$

$$\frac{\sqrt{e} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e + \sqrt{c^2 d + e}}}\right]}{2 (-d)^{3/2}} -$$

$$\frac{b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e - \sqrt{c^2 d + e}}}\right]}{2 (-d)^{3/2}} + \frac{b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e - \sqrt{c^2 d + e}}}\right]}{2 (-d)^{3/2}} -$$

$$\frac{b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e + \sqrt{c^2 d + e}}}\right]}{2 (-d)^{3/2}} + \frac{b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e + \sqrt{c^2 d + e}}}\right]}{2 (-d)^{3/2}} -$$

Result (type 4, 933 leaves):

$$\frac{1}{4 d^{3/2} x}$$

$$\left( -4 a \sqrt{d} - 4 a \sqrt{e} x \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] + b \left( 4 \sqrt{d} \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x) - 4 \sqrt{d} \operatorname{ArcSech}[c x] - 2 i \sqrt{e} x \right. \right.$$

$$\left. \left( -4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \operatorname{ArcSech}[c x] \right. \right.$$

$$\left. \left. \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] - \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i (\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \right. \right.$$

$$\left. \left. 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i (\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \right. \right.$$

$$\left. \left. \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i (\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \right) -$$

$$\begin{aligned}
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \operatorname{PolyLog}\left[2,\right. \\
 & \left.\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \operatorname{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\right]+ \\
 & 2 i \sqrt{e} x\left(-4 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(-i c \sqrt{d}+\sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]+ \right. \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+e^{-2 \operatorname{ArcSech}[c x]}\right]- \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \operatorname{PolyLog}\left[2,\right. \\
 & \left.\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \operatorname{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\right)
 \end{aligned}$$

Problem 115: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a+b \operatorname{ArcSech}[c x])}{(d+e x^2)^2} dx$$

Optimal (type 4, 611 leaves, 32 steps):

$$\begin{aligned}
 & - \frac{b \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} x}{2 c e^2} + \frac{d (a + b \operatorname{ArcSech}[c x])}{2 e^2 \left(e + \frac{d}{x^2}\right)} + \frac{x^2 (a + b \operatorname{ArcSech}[c x])}{2 e^2} - \\
 & \frac{b d \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}}}\right]}{2 e^{5/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} - \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \\
 & \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \\
 & \frac{d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} + \frac{2 d (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcSech}[c x]}\right]}{e^3} - \\
 & \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{e^3} - \\
 & \frac{b d \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} - \frac{b d \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{e^3} + \frac{b d \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcSech}[c x]}\right]}{e^3}
 \end{aligned}$$

Result (type 4, 1397 leaves):

$$\begin{aligned}
 & \frac{a x^2}{2 e^2} - \frac{a d^2}{2 e^3 (d + e x^2)} - \frac{a d \operatorname{Log}[d + e x^2]}{e^3} + \\
 & b \left( \frac{-\frac{\sqrt{\frac{1-cx}{1+cx}} (1+cx)}{c^2} + x^2 \operatorname{ArcSech}[c x]}{2 e^2} + \frac{1}{4 e^{5/2}} i d^{3/2} \left( -\frac{\operatorname{ArcSech}[c x]}{i \sqrt{d} \sqrt{e + e x}} + \frac{1}{\sqrt{d}} \right. \right. \\
 & \left. \left. + \frac{\operatorname{Log}[x]}{\sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}} + \frac{\operatorname{Log}\left[\frac{2 i \sqrt{e} \left(\sqrt{d} \sqrt{\frac{1-cx}{1+cx}} (1+cx) + \frac{\sqrt{d} \sqrt{e + i c^2 d x}}{\sqrt{c^2 d + e}}\right)}{i \sqrt{d} + \sqrt{e} x}\right]}{\sqrt{c^2 d + e}} \right) \right) - \\
 & \frac{1}{4 e^{5/2}} i d^{3/2} \left( -\frac{\operatorname{ArcSech}[c x]}{-i \sqrt{d} \sqrt{e + e x}} - \frac{1}{\sqrt{d}} i \left( \frac{\operatorname{Log}[x]}{\sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}} + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left. \left. \frac{\text{Log}\left[\frac{2\sqrt{e}\left(i\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx)+\frac{i\sqrt{d}\sqrt{e+c^2dx}}{\sqrt{c^2d+e}}\right)}{-i\sqrt{d}+\sqrt{e}x}\right]}{\sqrt{c^2d+e}}\right]}{\right)} - \frac{1}{2e^3}d \left( \text{PolyLog}\left[2, -e^{-2\text{ArcSech}[cx]}\right] - \right. \\
 & 2 \left( -4i \text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTanh}\left[\frac{(ic\sqrt{d}+\sqrt{e})\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[cx]\right]}{\sqrt{c^2d+e}}\right] + \text{ArcSech}[ \right. \\
 & \left. cx] \text{Log}\left[1+e^{-2\text{ArcSech}[cx]}\right] - \text{ArcSech}[cx] \text{Log}\left[1+\frac{i(\sqrt{e}-\sqrt{c^2d+e})e^{-\text{ArcSech}[cx]}}{c\sqrt{d}}\right] + \right. \\
 & 2i \text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{i(\sqrt{e}-\sqrt{c^2d+e})e^{-\text{ArcSech}[cx]}}{c\sqrt{d}}\right] - \\
 & \text{ArcSech}[cx] \text{Log}\left[1+\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\text{ArcSech}[cx]}}{c\sqrt{d}}\right] - \\
 & 2i \text{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\text{ArcSech}[cx]}}{c\sqrt{d}}\right] + \\
 & \text{PolyLog}\left[2, \frac{i(-\sqrt{e}+\sqrt{c^2d+e})e^{-\text{ArcSech}[cx]}}{c\sqrt{d}}\right] + \\
 & \left. \left. \left. \text{PolyLog}\left[2, -\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\text{ArcSech}[cx]}}{c\sqrt{d}}\right] \right] \right) \right) + \\
 & \frac{1}{2e^3}d \left( -\text{PolyLog}\left[2, -e^{-2\text{ArcSech}[cx]}\right] + 2 \left( -4i \text{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTanh}\left[ \right. \right. \\
 & \left. \left. \frac{(-ic\sqrt{d}+\sqrt{e})\text{Tanh}\left[\frac{1}{2}\text{ArcSech}[cx]\right]}{\sqrt{c^2d+e}}\right] + \text{ArcSech}[cx] \text{Log}\left[1+e^{-2\text{ArcSech}[cx]}\right] - \right. \\
 & \left. \left. \text{ArcSech}[cx] \text{Log}\left[1+\frac{i(-\sqrt{e}+\sqrt{c^2d+e})e^{-\text{ArcSech}[cx]}}{c\sqrt{d}}\right] + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & 2 \operatorname{Im} \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 + \frac{i \left( -\sqrt{e} + \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}} \right] - \\
 & \operatorname{ArcSech}[c x] \operatorname{Log} \left[ 1 - \frac{i \left( \sqrt{e} + \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}} \right] - \\
 & 2 \operatorname{Im} \operatorname{ArcSin} \left[ \frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log} \left[ 1 - \frac{i \left( \sqrt{e} + \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}} \right] + \operatorname{PolyLog} \left[ 2, \right. \\
 & \left. \frac{i \left( \sqrt{e} - \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}} \right] + \operatorname{PolyLog} \left[ 2, \frac{i \left( \sqrt{e} + \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}} \right] \left. \right] \left. \right) \left. \right)
 \end{aligned}$$

**Problem 116: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{x^3 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 4, 562 leaves, 30 steps):



$$\begin{aligned}
 & -\frac{a + b \operatorname{ArcSech}[c x]}{2 e \left( e + \frac{d}{x^2} \right)} + \frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[ \frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x} \right]}{2 e^{3/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} + \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[ 1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^2} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[ 1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^2} + \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[ 1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^2} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[ 1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^2} - \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[ 1 + e^{2 \operatorname{ArcSech}[c x]} \right]}{e^2} + \frac{b \operatorname{PolyLog}\left[ 2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^2} + \\
 & \frac{b \operatorname{PolyLog}\left[ 2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}} \right]}{2 e^2} + \frac{b \operatorname{PolyLog}\left[ 2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^2} + \\
 & \frac{b \operatorname{PolyLog}\left[ 2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}} \right]}{2 e^2} - \frac{b \operatorname{PolyLog}\left[ 2, -e^{2 \operatorname{ArcSech}[c x]} \right]}{2 e^2}
 \end{aligned}$$

Result (type 4, 1208 leaves):

$$\begin{aligned}
 & \frac{1}{4 e^2} \left( \frac{2 a d}{d + e x^2} + \frac{b \sqrt{d} \operatorname{ArcSech}[c x]}{\sqrt{d} - i \sqrt{e} x} + \frac{b \sqrt{d} \operatorname{ArcSech}[c x]}{\sqrt{d} + i \sqrt{e} x} \right) + \\
 & 8 i b \operatorname{ArcSin}\left[ \frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{ArcTanh}\left[ \frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[ \frac{1}{2} \operatorname{ArcSech}[c x] \right]}{\sqrt{c^2 d + e}} \right] + \\
 & 8 i b \operatorname{ArcSin}\left[ \frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{ArcTanh}\left[ \frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[ \frac{1}{2} \operatorname{ArcSech}[c x] \right]}{\sqrt{c^2 d + e}} \right] - \\
 & 4 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[ 1 + e^{-2 \operatorname{ArcSech}[c x]} \right] + \\
 & 2 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[ 1 + \frac{i \left( \sqrt{e} - \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}} \right] - \\
 & 4 i b \operatorname{ArcSin}\left[ \frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}} \right] \operatorname{Log}\left[ 1 + \frac{i \left( \sqrt{e} - \sqrt{c^2 d + e} \right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}} \right] +
 \end{aligned}$$

$$\begin{aligned}
 & 2 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i \left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i \left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 2 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 - \frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 2 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 2 b \operatorname{Log}[x] + 2 a \operatorname{Log}[d + e x^2] - 2 b \operatorname{Log}\left[1 + \sqrt{\frac{1 - c x}{1 + c x}} + c x \sqrt{\frac{1 - c x}{1 + c x}}\right] + \\
 & \frac{b \sqrt{e} \operatorname{Log}\left[\frac{2 i \sqrt{e} \left(\sqrt{d} \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x) + \frac{\sqrt{d} \sqrt{e + i c^2 d x}}{\sqrt{c^2 d + e}}\right)}{i \sqrt{d} + \sqrt{e} x}\right]}{\sqrt{c^2 d + e}} + \frac{b \sqrt{e} \operatorname{Log}\left[\frac{2 \sqrt{e} \left(i \sqrt{d} \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x) + \frac{i \sqrt{d} \sqrt{e + c^2 d x}}{\sqrt{c^2 d + e}}\right)}{-i \sqrt{d} + \sqrt{e} x}\right]}{\sqrt{c^2 d + e}} + \\
 & 2 b \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right] - 2 b \operatorname{PolyLog}\left[2, \frac{i \left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 2 b \operatorname{PolyLog}\left[2, \frac{i \left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 2 b \operatorname{PolyLog}\left[2, -\frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & \left. 2 b \operatorname{PolyLog}\left[2, \frac{i \left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \right)
 \end{aligned}$$

Problem 117: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^2} dx$$

Optimal (type 3, 147 leaves, 8 steps):

$$-\frac{a + b \operatorname{ArcSech}[c x]}{2 e (d + e x^2)} + \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}[\sqrt{1-c^2 x^2}]}{2 d e}$$

$$\frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{\sqrt{c^2 d+e}}\right]}{2 d \sqrt{e} \sqrt{c^2 d+e}}$$

Result (type 3, 345 leaves):

$$-\frac{1}{4 e} \left( \frac{2 a}{d + e x^2} + \frac{2 b \operatorname{ArcSech}[c x]}{d + e x^2} + \frac{2 b \operatorname{Log}[x]}{d} - \frac{2 b \operatorname{Log}\left[1 + \sqrt{\frac{1-c x}{1+c x}} + c x \sqrt{\frac{1-c x}{1+c x}}\right]}{d} + \frac{b \sqrt{e} \operatorname{Log}\left[\frac{4 \left(\frac{i d e + c^2 d^{3/2} \sqrt{e} x}{\sqrt{c^2 d+e} (\sqrt{d+i \sqrt{e} x})} + \frac{d e \sqrt{\frac{1-c x}{1+c x}} (1+c x)}{-i \sqrt{d} \sqrt{e+e x}}\right)}{b}\right]}{d \sqrt{c^2 d+e}} + \frac{b \sqrt{e} \operatorname{Log}\left[\frac{4 \left(\frac{d e + i c^2 d^{3/2} \sqrt{e} x}{\sqrt{c^2 d+e} (i \sqrt{d} + \sqrt{e} x)} + \frac{d e \sqrt{\frac{1-c x}{1+c x}} (1+c x)}{i \sqrt{d} \sqrt{e+e x}}\right)}{b}\right]}{d \sqrt{c^2 d+e}} \right)$$

**Problem 118: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + b \operatorname{ArcSech}[c x]}{x (d + e x^2)^2} dx$$

Optimal (type 4, 542 leaves, 25 steps):

$$\begin{aligned}
 & -\frac{e(a+b \operatorname{ArcSech}[c x])}{2 d^2\left(e+\frac{d}{x^2}\right)}+\frac{(a+b \operatorname{ArcSech}[c x])^2}{2 b d^2}+ \\
 & \frac{b \sqrt{e} \sqrt{-1+\frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d+e}}{c \sqrt{e} \sqrt{-1+\frac{1}{c^2 x^2}} x}\right](a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^2 \sqrt{c^2 d+e} \sqrt{-1+\frac{1}{c x}} \sqrt{1+\frac{1}{c x}}}-\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^2} \\
 & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^2}-\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d^2} \\
 & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d^2}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^2} \\
 & \frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{2 d^2}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d^2}-\frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{2 d^2}
 \end{aligned}$$

Result (type 4, 1189 leaves):

$$\begin{aligned}
 & \frac{1}{4 d^2} \left( \frac{2 a d}{d+e x^2} + \frac{b \sqrt{d} \operatorname{ArcSech}[c x]}{\sqrt{d}-i \sqrt{e} x} + \frac{b \sqrt{d} \operatorname{ArcSech}[c x]}{\sqrt{d}+i \sqrt{e} x} - 2 b \operatorname{ArcSech}[c x]^2 - \right. \\
 & 8 i b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d}+\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]- \\
 & 8 i b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d}+\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]- \\
 & 2 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+
 \end{aligned}$$

$$\begin{aligned}
 & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 4 i b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 4 a \operatorname{Log}[x]+2 b \operatorname{Log}[x]-2 a \operatorname{Log}[d+e x^2]-2 b \operatorname{Log}\left[1+\sqrt{\frac{1-c x}{1+c x}}+c x \sqrt{\frac{1-c x}{1+c x}}\right]+ \\
 & \frac{b \sqrt{e} \operatorname{Log}\left[\frac{2 i \sqrt{e}\left(\sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\sqrt{d} \sqrt{e+i c^2 d x}\right)}{i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{c^2 d+e}}+\frac{b \sqrt{e} \operatorname{Log}\left[\frac{2 \sqrt{e}\left(i \sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\frac{i \sqrt{d} \sqrt{e+c^2 d x}}{\sqrt{c^2 d+e}}\right)}{-i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{c^2 d+e}}+ \\
 & 2 b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 2 b \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & 2 b \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+ \\
 & \left. 2 b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\right)
 \end{aligned}$$

**Problem 119: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^4 (a+b \operatorname{ArcSech}[c x])}{(d+e x^2)^2} dx$$

Optimal (type 4, 840 leaves, 50 steps):

$$\begin{aligned}
 & -\frac{d(a+b \operatorname{ArcSech}[c x])}{4 e^2\left(\sqrt{-d} \sqrt{e-\frac{d}{x}}\right)}+\frac{d(a+b \operatorname{ArcSech}[c x])}{4 e^2\left(\sqrt{-d} \sqrt{e+\frac{d}{x}}\right)}+\frac{x(a+b \operatorname{ArcSech}[c x])}{e^2}+ \\
 & \frac{b d \operatorname{ArcTan}\left[\frac{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{2 \sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{c d+\sqrt{-d} \sqrt{e}} e^2}+\frac{b d \operatorname{ArcTan}\left[\frac{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{2 \sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{c d+\sqrt{-d} \sqrt{e}} e^2}- \\
 & \frac{b \operatorname{ArcTan}\left[\sqrt{-1+\frac{1}{c x}} \sqrt{1+\frac{1}{c x}}\right]}{c e^2}+\frac{3 \sqrt{-d}(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4 e^{5 / 2}}- \\
 & \frac{3 \sqrt{-d}(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4 e^{5 / 2}}+ \\
 & \frac{3 \sqrt{-d}(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4 e^{5 / 2}}- \\
 & \frac{3 \sqrt{-d}(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4 e^{5 / 2}}- \\
 & \frac{3 b \sqrt{-d} \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4 e^{5 / 2}}+\frac{3 b \sqrt{-d} \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4 e^{5 / 2}}- \\
 & \frac{3 b \sqrt{-d} \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4 e^{5 / 2}}+\frac{3 b \sqrt{-d} \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4 e^{5 / 2}}
 \end{aligned}$$

Result (type 4, 1270 leaves):

$$\begin{aligned}
 & \frac{1}{4 e^{5 / 2}}\left(4 a \sqrt{e} x+\frac{2 a d \sqrt{e} x}{d+e x^2}+4 b \sqrt{e} x \operatorname{ArcSech}[c x]+\frac{b d \operatorname{ArcSech}[c x]}{-i \sqrt{d}+\sqrt{e} x}+\right. \\
 & \frac{b d \operatorname{ArcSech}[c x]}{i \sqrt{d}+\sqrt{e} x}-6 a \sqrt{d} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]-\frac{8 b \sqrt{e} \operatorname{ArcTan}\left[\operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]\right]}{c}+ \\
 & 12 b \sqrt{d} \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d}+\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]- \\
 & 12 b \sqrt{d} \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d}+\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]+
 \end{aligned}$$

$$\begin{aligned}
 & 3 i b \sqrt{d} \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 6 b \sqrt{d} \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 3 i b \sqrt{d} \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 6 b \sqrt{d} \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 3 i b \sqrt{d} \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 - \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 6 b \sqrt{d} \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 3 i b \sqrt{d} \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 6 b \sqrt{d} \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & \frac{i b \sqrt{d} \sqrt{e} \operatorname{Log}\left[\frac{2 i \sqrt{e}\left(\sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\frac{\sqrt{d} \sqrt{e+i c^2 d x}}{\sqrt{c^2 d+e}}\right)}{i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{c^2 d+e}} + \\
 & \frac{i b \sqrt{d} \sqrt{e} \operatorname{Log}\left[\frac{2 \sqrt{e}\left(i \sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\frac{i \sqrt{d} \sqrt{e+i c^2 d x}}{\sqrt{c^2 d+e}}\right)}{-i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{c^2 d+e}} + \\
 & 3 i b \sqrt{d} \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 3 i b \sqrt{d} \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 3 i b \sqrt{d} \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +
 \end{aligned}$$

$$\left. 3 i b \sqrt{d} \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\right)$$

**Problem 120: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2(a+b \operatorname{ArcSech}[c x])}{(d+e x^2)^2} dx$$

Optimal (type 4, 786 leaves, 27 steps):

$$\begin{aligned} & \frac{a+b \operatorname{ArcSech}[c x]}{4 e\left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}-\frac{a+b \operatorname{ArcSech}[c x]}{4 e\left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}-\frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{2 \sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{c d+\sqrt{-d} \sqrt{e}} e} \\ & \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{2 \sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{c d+\sqrt{-d} \sqrt{e}} e}+\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4 \sqrt{-d} e^{3 / 2}} \\ & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4 \sqrt{-d} e^{3 / 2}}+\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4 \sqrt{-d} e^{3 / 2}} \\ & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4 \sqrt{-d} e^{3 / 2}}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4 \sqrt{-d} e^{3 / 2}}+ \\ & \frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4 \sqrt{-d} e^{3 / 2}}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4 \sqrt{-d} e^{3 / 2}}+\frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4 \sqrt{-d} e^{3 / 2}} \end{aligned}$$

Result (type 4, 1226 leaves):

$$\frac{1}{4 e^{3 / 2}}\left(-\frac{2 a \sqrt{e} x}{d+e x^2}+\frac{b \operatorname{ArcSech}[c x]}{i \sqrt{d}-\sqrt{e} x}-\frac{b \operatorname{ArcSech}[c x]}{i \sqrt{d}+\sqrt{e} x}+\right.$$



$$\frac{2 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] - 4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right]}{\sqrt{d}} +$$

$$\frac{4 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right]}{\sqrt{d}} -$$

$$\frac{i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} -$$

$$\frac{2 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} +$$

$$\frac{i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} +$$

$$\frac{2 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} +$$

$$\frac{i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} -$$

$$\frac{2 b \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} -$$

$$\frac{i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} +$$

$$\frac{2 b \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} +$$

$$\begin{aligned}
 & \frac{i b \sqrt{e} \operatorname{Log}\left[\frac{2 i \sqrt{e}\left(\sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\sqrt{d} \sqrt{e+i c^2 d x}\right)}{i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{d} \sqrt{c^2 d+e}} - \\
 & \frac{i b \sqrt{e} \operatorname{Log}\left[\frac{2 \sqrt{e}\left(i \sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\frac{i \sqrt{d} \sqrt{e+i c^2 d x}}{\sqrt{c^2 d+e}}\right)}{-i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{d} \sqrt{c^2 d+e}} - \\
 & \frac{i b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} + \frac{i b \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} + \\
 & \left. \frac{i b \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}} - \frac{i b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{d}}\right)
 \end{aligned}$$

**Problem 121: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcSech}[c x]}{(d + e x^2)^2} dx$$

Optimal (type 4, 786 leaves, 47 steps):

$$\begin{aligned}
 & -\frac{a+b \operatorname{ArcSech}[c x]}{4 d\left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}+\frac{a+b \operatorname{ArcSech}[c x]}{4 d\left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}+\frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{2 d \sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{c d+\sqrt{-d} \sqrt{e}}}+ \\
 & \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{2 d \sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{c d+\sqrt{-d} \sqrt{e}}}-\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4(-d)^{3 / 2} \sqrt{e}}+ \\
 & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4(-d)^{3 / 2} \sqrt{e}}-\frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4(-d)^{3 / 2} \sqrt{e}}+ \\
 & \frac{(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4(-d)^{3 / 2} \sqrt{e}}+\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4(-d)^{3 / 2} \sqrt{e}}- \\
 & \frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{4(-d)^{3 / 2} \sqrt{e}}+\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4(-d)^{3 / 2} \sqrt{e}}-\frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{4(-d)^{3 / 2} \sqrt{e}}
 \end{aligned}$$

Result (type 4, 1216 leaves):

$$\begin{aligned}
 & \frac{1}{4 d^{3 / 2}}\left(\frac{2 a \sqrt{d} x}{d+e x^2}+\frac{b \sqrt{d} \operatorname{ArcSech}[c x]}{-i \sqrt{d} \sqrt{e}+e x}+\frac{b \sqrt{d} \operatorname{ArcSech}[c x]}{i \sqrt{d} \sqrt{e}+e x}\right)+ \\
 & \frac{2 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{\sqrt{e}}-\frac{4 b \operatorname{ArcSin}\left[\sqrt{\frac{1-i \sqrt{e}}{c \sqrt{d}}}\right] \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d}+\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]}{\sqrt{e}}+ \\
 & \frac{4 b \operatorname{ArcSin}\left[\sqrt{\frac{1+i \sqrt{e}}{c \sqrt{d}}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d}+\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right]}{\sqrt{e}}- \\
 & \frac{i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}}-
 \end{aligned}$$

$$\begin{aligned}
 & \frac{2 b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} + \\
 & \frac{i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} + \\
 & \frac{2 b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} + \\
 & \frac{i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} - \\
 & \frac{2 b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} - \\
 & \frac{i b \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} + \\
 & \frac{2 b \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} - \\
 & \frac{i b \operatorname{Log}\left[\frac{2 i \sqrt{e}\left(\sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\frac{\sqrt{d} \sqrt{e+i c^2 d x}}{\sqrt{c^2 d+e}}\right)}{i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{c^2 d+e}} + \frac{i b \operatorname{Log}\left[\frac{2 \sqrt{e}\left(i \sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\frac{i \sqrt{d} \sqrt{e+c^2 d x}}{\sqrt{c^2 d+e}}\right)}{-i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{c^2 d+e}} - \\
 & \frac{i b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} + \frac{i b \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} + \\
 & \left. \frac{i b \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} - \frac{i b \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]}{\sqrt{e}} \right)
 \end{aligned}$$

**Problem 122: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcSech}[c x]}{x^2 (d + e x^2)^2} dx$$

Optimal (type 4, 844 leaves, 50 steps):

$$\begin{aligned} & \frac{b c \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{d^2} - \frac{a}{d^2 x} - \frac{b \operatorname{ArcSech}[c x]}{d^2 x} + \frac{e (a + b \operatorname{ArcSech}[c x])}{4 d^2 \left(\sqrt{-d} \sqrt{e - \frac{d}{x}}\right)} \\ & - \frac{e (a + b \operatorname{ArcSech}[c x])}{4 d^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x}\right)} - \frac{b e \operatorname{ArcTan}\left[\frac{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{2 d^2 \sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{c d + \sqrt{-d} \sqrt{e}}} \\ & - \frac{b e \operatorname{ArcTan}\left[\frac{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{2 d^2 \sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{c d + \sqrt{-d} \sqrt{e}}} - \frac{3 \sqrt{e} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{4 (-d)^{5/2}} + \\ & \frac{3 \sqrt{e} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{4 (-d)^{5/2}} - \\ & \frac{3 \sqrt{e} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{4 (-d)^{5/2}} + \\ & \frac{3 \sqrt{e} (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{4 (-d)^{5/2}} + \\ & \frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{4 (-d)^{5/2}} - \frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{4 (-d)^{5/2}} + \\ & \frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{4 (-d)^{5/2}} - \frac{3 b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{4 (-d)^{5/2}} \end{aligned}$$

Result (type 4, 1305 leaves):

$$\frac{1}{4 d^{5/2}} \left[ -\frac{4 a \sqrt{d}}{x} + 4 b c \sqrt{d} \sqrt{\frac{1-c x}{1+c x}} + \frac{4 b \sqrt{d} \sqrt{\frac{1-c x}{1+c x}}}{x} - \frac{2 a \sqrt{d} e x}{d+e x^2} - \frac{4 b \sqrt{d} \operatorname{ArcSech}[c x]}{x} - \right.$$

$$\frac{b \sqrt{d} e \operatorname{ArcSech}[c x]}{-i \sqrt{d} \sqrt{e}+e x} - \frac{b \sqrt{d} e \operatorname{ArcSech}[c x]}{i \sqrt{d} \sqrt{e}+e x} - 6 a \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right] +$$

$$12 b \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d}+\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right] -$$

$$12 b \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d}+\sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d+e}}\right] +$$

$$3 i b \sqrt{e} \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$6 b \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] -$$

$$3 i b \sqrt{e} \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] -$$

$$6 b \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] -$$

$$3 i b \sqrt{e} \operatorname{ArcSech}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$6 b \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$3 i b \sqrt{e} \operatorname{ArcSech}[c x] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] -$$

$$6 b \sqrt{e} \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] +$$

$$\begin{aligned}
 & \frac{i b e \operatorname{Log}\left[\frac{2 i \sqrt{e}\left(\sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\frac{\sqrt{d} \sqrt{e+i c^2 d x}}{\sqrt{c^2 d+e}}\right)}{i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{c^2 d+e}} - \frac{i b e \operatorname{Log}\left[\frac{2 \sqrt{e}\left(i \sqrt{d} \sqrt{\frac{1-c x}{1+c x}}(1+c x)+\frac{i \sqrt{d} \sqrt{e-c^2 d x}}{\sqrt{c^2 d+e}}\right)}{-i \sqrt{d}+\sqrt{e} x}\right]}{\sqrt{c^2 d+e}} + \\
 & 3 i b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 3 i b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 3 i b \sqrt{e} \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & \left. 3 i b \sqrt{e} \operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\right)
 \end{aligned}$$

**Problem 123:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^5 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 760 leaves, 35 steps):

$$\frac{b d \left( c^2 - \frac{1}{x^2} \right)}{8 c e^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} x} - \frac{a + b \operatorname{ArcSech}[c x]}{4 e \left( e + \frac{d}{x^2} \right)^2} - \frac{a + b \operatorname{ArcSech}[c x]}{2 e^2 \left( e + \frac{d}{x^2} \right)} +$$

$$\frac{b \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}}}\right]}{2 e^{5/2} \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} + \frac{b (c^2 d + 2 e) \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}}}\right]}{8 e^{5/2} (c^2 d + e)^{3/2} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} +$$

$$\frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} +$$

$$\frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} -$$

$$\frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + e^{2 \operatorname{ArcSech}[c x]}\right]}{e^3} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} +$$

$$\frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 e^3} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} +$$

$$\frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 e^3} - \frac{b \operatorname{PolyLog}\left[2, -e^{2 \operatorname{ArcSech}[c x]}\right]}{2 e^3}$$

Result (type 4, 2000 leaves):

$$-\frac{a d^2}{4 e^3 (d + e x^2)^2} + \frac{a d}{e^3 (d + e x^2)} + \frac{a \operatorname{Log}[d + e x^2]}{2 e^3} +$$

$$b \left( -\frac{1}{16 e^{5/2}} d \left( -\frac{i \sqrt{e} \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{\sqrt{d} (c^2 d + e) (-i \sqrt{d} + \sqrt{e} x)} - \frac{\operatorname{ArcSech}[c x]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \right. \right.$$

$$\left. \frac{\operatorname{Log}[x]}{d \sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \right.$$

$$\left. (2 c^2 d + e) \operatorname{Log}\left[-\left(4 d \sqrt{e} \sqrt{c^2 d + e} \left(\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{\frac{1-cx}{1+cx}}\right) + \right.\right. \right.$$



$$\left. \left. \left. c \sqrt{c^2 d + e} x \sqrt{\frac{1 - c x}{1 + c x}} \right) / \left( (2 c^2 d + e) (-i \sqrt{d} + \sqrt{e} x) \right) \right) \right] -$$

$$\frac{1}{16 e^{5/2}} d \left( \frac{i \sqrt{e} \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x)}{\sqrt{d} (c^2 d + e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\operatorname{ArcSech}[c x]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} + \frac{\operatorname{Log}[x]}{d \sqrt{e}} - \right.$$

$$\left. \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1 - c x}{1 + c x}} + c x \sqrt{\frac{1 - c x}{1 + c x}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \right.$$

$$\left. \left. \left. (2 c^2 d + e) \operatorname{Log}\left[-\left(\left(4 d \sqrt{e} \sqrt{c^2 d + e} \left(\sqrt{e} + i c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{\frac{1 - c x}{1 + c x}} + \right.\right.\right.\right. \right.$$

$$\left. \left. \left. \left. c \sqrt{c^2 d + e} x \sqrt{\frac{1 - c x}{1 + c x}} \right) / \left( (2 c^2 d + e) (i \sqrt{d} + \sqrt{e} x) \right) \right) \right) \right] - \right.$$

$$\left. \frac{1}{16 e^{5/2}} 7 i \sqrt{d} \left( -\frac{\operatorname{ArcSech}[c x]}{i \sqrt{d} \sqrt{e} + e x} + \frac{1}{\sqrt{d}} i \left( \frac{\operatorname{Log}[x]}{\sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1 - c x}{1 + c x}} + c x \sqrt{\frac{1 - c x}{1 + c x}}\right]}{\sqrt{e}} + \right. \right. \right.$$

$$\left. \left. \left. \left. \frac{\operatorname{Log}\left[\frac{2 i \sqrt{e} \left(\sqrt{d} \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x) + \frac{\sqrt{d} \sqrt{e} + i c^2 d x}{\sqrt{c^2 d + e}}\right)}{i \sqrt{d} + \sqrt{e} x}\right]}{\sqrt{c^2 d + e}} \right) \right) \right) + \frac{1}{16 e^{5/2}} \right.$$

$$\left. \left. \left. 7 i \sqrt{d} \left( -\frac{\operatorname{ArcSech}[c x]}{-i \sqrt{d} \sqrt{e} + e x} - \frac{1}{\sqrt{d}} i \left( \frac{\operatorname{Log}[x]}{\sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1 - c x}{1 + c x}} + c x \sqrt{\frac{1 - c x}{1 + c x}}\right]}{\sqrt{e}} + \right. \right. \right.$$

$$\begin{aligned}
 & \left. \left. \left. \frac{\operatorname{Log}\left[\frac{2\sqrt{e}\left(i\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx)+\frac{i\sqrt{d}\sqrt{e+c^2d}x}{\sqrt{c^2d+e}}\right)}{-i\sqrt{d}+\sqrt{e}x}\right]}{\sqrt{c^2d+e}}\right]}{\right)} + \frac{1}{4e^3} \left( \operatorname{PolyLog}\left[2, -e^{-2\operatorname{ArcSech}[cx]}\right] - \right. \\
 & 2 \left( -4i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(ic\sqrt{d}+\sqrt{e})\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcSech}[cx]\right]}{\sqrt{c^2d+e}}\right] + \operatorname{ArcSech}[ \right. \\
 & \left. cx] \operatorname{Log}\left[1+e^{-2\operatorname{ArcSech}[cx]}\right] - \operatorname{ArcSech}[cx] \operatorname{Log}\left[1+\frac{i(\sqrt{e}-\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \right. \\
 & 2i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i(\sqrt{e}-\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] - \\
 & \operatorname{ArcSech}[cx] \operatorname{Log}\left[1+\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] - \\
 & 2i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \\
 & \operatorname{PolyLog}\left[2, \frac{i(-\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \\
 & \left. \left. \left. \operatorname{PolyLog}\left[2, -\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] \right) \right) - \right. \\
 & \left. \frac{1}{4e^3} \left( -\operatorname{PolyLog}\left[2, -e^{-2\operatorname{ArcSech}[cx]}\right] + 2 \left( -4i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[ \right. \right. \right. \\
 & \left. \left. \left. \frac{(-ic\sqrt{d}+\sqrt{e})\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcSech}[cx]\right]}{\sqrt{c^2d+e}}\right] + \operatorname{ArcSech}[cx] \operatorname{Log}\left[1+e^{-2\operatorname{ArcSech}[cx]}\right] - \right. \right. \\
 & \left. \left. \operatorname{ArcSech}[cx] \operatorname{Log}\left[1+\frac{i(-\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2, \right. \\
 & \left. \frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2, \frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \left. \right) \left. \right) \left. \right)
 \end{aligned}$$

**Problem 124:** Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3 (a+b \operatorname{ArcSech}[c x])}{(d+e x^2)^3} dx$$

Optimal (type 3, 173 leaves, 6 steps):

$$\begin{aligned}
 & \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{8 e (c^2 d+e) (d+e x^2)} + \frac{x^4 (a+b \operatorname{ArcSech}[c x])}{4 d (d+e x^2)^2} - \\
 & \frac{b (c^2 d+2 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{\sqrt{c^2 d+e}}\right]}{8 d e^{3/2} (c^2 d+e)^{3/2}}
 \end{aligned}$$

Result (type 3, 486 leaves):

$$\begin{aligned}
 & -\frac{1}{16 e^2} \left( -\frac{4 a d}{(d+e x^2)^2} + \frac{8 a}{d+e x^2} - \frac{2 e \sqrt{\frac{1-c x}{1+c x}} (b+b c x)}{(c^2 d+e)(d+e x^2)} + \frac{4 b (d+2 e x^2) \operatorname{ArcSech}[c x]}{(d+e x^2)^2} + \right. \\
 & \frac{4 b \operatorname{Log}[x]}{d} - \frac{4 b \operatorname{Log}\left[1+\sqrt{\frac{1-c x}{1+c x}}+c x \sqrt{\frac{1-c x}{1+c x}}\right]}{d} + \frac{1}{d\left(c^2 d+e\right)^{3 / 2}} b \sqrt{e}\left(c^2 d+2 e\right) \\
 & \left. \operatorname{Log}\left[\left(16 d e^{3 / 2} \sqrt{c^2 d+e}\left(\sqrt{e}-i c^2 \sqrt{d} x+\sqrt{c^2 d+e} \sqrt{\frac{1-c x}{1+c x}}+c \sqrt{c^2 d+e} x \sqrt{\frac{1-c x}{1+c x}}\right)\right) / \right.\right. \\
 & \left. \left.(b\left(c^2 d+2 e\right)\left(-i \sqrt{d}+\sqrt{e} x\right)\right)\right] + \frac{1}{d\left(c^2 d+e\right)^{3 / 2}} b \sqrt{e}\left(c^2 d+2 e\right) \right. \\
 & \left. \operatorname{Log}\left[\left(16 d e^{3 / 2} \sqrt{c^2 d+e}\left(\sqrt{e}+i c^2 \sqrt{d} x+\sqrt{c^2 d+e} \sqrt{\frac{1-c x}{1+c x}}+c \sqrt{c^2 d+e} x \sqrt{\frac{1-c x}{1+c x}}\right)\right) / \right.\right. \\
 & \left. \left.(b\left(c^2 d+2 e\right)\left(i \sqrt{d}+\sqrt{e} x\right)\right)\right]\right)
 \end{aligned}$$

**Problem 125: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{x(a+b \operatorname{ArcSech}[c x])}{(d+e x^2)^3} dx$$

Optimal (type 3, 217 leaves, 9 steps):

$$\begin{aligned}
 & -\frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{8 d\left(c^2 d+e\right)\left(d+e x^2\right)} - \frac{a+b \operatorname{ArcSech}[c x]}{4 e\left(d+e x^2\right)^2} + \\
 & \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\sqrt{1-c^2 x^2}\right]}{4 d^2 e} - \frac{b\left(3 c^2 d+2 e\right) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{\sqrt{c^2 d+e}}\right]}{8 d^2 \sqrt{e}\left(c^2 d+e\right)^{3 / 2}}
 \end{aligned}$$

Result (type 3, 486 leaves):

$$\begin{aligned}
 & \frac{1}{16} \left( -\frac{4a}{e(d+ex^2)^2} - \frac{2\sqrt{\frac{1-cx}{1+cx}}(b+bcx)}{d(c^2d+e)(d+ex^2)} - \frac{4b \operatorname{ArcSech}[cx]}{e(d+ex^2)^2} - \right. \\
 & \frac{4b \operatorname{Log}[x]}{d^2e} + \frac{4b \operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + cx \sqrt{\frac{1-cx}{1+cx}}\right]}{d^2e} - \left( b(3c^2d+2e) \operatorname{Log}\left[ \right. \right. \\
 & \left. \left. \left( 16d^2\sqrt{e}\sqrt{c^2d+e} \left( \sqrt{e} - i c^2\sqrt{d}x + \sqrt{c^2d+e} \sqrt{\frac{1-cx}{1+cx}} + c\sqrt{c^2d+e}x \sqrt{\frac{1-cx}{1+cx}} \right) \right) \right] \right) / \\
 & \left. \left( b(3c^2d+2e) \left( -i\sqrt{d} + \sqrt{e}x \right) \right) \right] / \left( d^2\sqrt{e}(c^2d+e)^{3/2} \right) - \left( b(3c^2d+2e) \operatorname{Log}\left[ \right. \right. \\
 & \left. \left. \left( 16d^2\sqrt{e}\sqrt{c^2d+e} \left( \sqrt{e} + i c^2\sqrt{d}x + \sqrt{c^2d+e} \sqrt{\frac{1-cx}{1+cx}} + c\sqrt{c^2d+e}x \sqrt{\frac{1-cx}{1+cx}} \right) \right) \right] \right) / \\
 & \left. \left( b(3c^2d+2e) \left( i\sqrt{d} + \sqrt{e}x \right) \right) \right] / \left( d^2\sqrt{e}(c^2d+e)^{3/2} \right) \right)
 \end{aligned}$$

**Problem 126: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a + b \operatorname{ArcSech}[cx]}{x(d+ex^2)^3} dx$$

Optimal (type 4, 741 leaves, 30 steps):

$$\begin{aligned}
 & - \frac{b e \left( c^2 - \frac{1}{x^2} \right)}{8 c d^2 (c^2 d + e) \left( e + \frac{d}{x^2} \right) \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}} x} + \frac{e^2 (a + b \operatorname{ArcSech}[c x])}{4 d^3 \left( e + \frac{d}{x^2} \right)^2} - \\
 & \frac{e (a + b \operatorname{ArcSech}[c x])}{d^3 \left( e + \frac{d}{x^2} \right)} + \frac{(a + b \operatorname{ArcSech}[c x])^2}{2 b d^3} + \frac{b \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x}\right]}{d^3 \sqrt{c^2 d + e} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} - \\
 & \frac{b \sqrt{e} (c^2 d + 2 e) \sqrt{-1 + \frac{1}{c^2 x^2}} \operatorname{ArcTanh}\left[\frac{\sqrt{c^2 d + e}}{c \sqrt{e} \sqrt{-1 + \frac{1}{c^2 x^2}} x}\right]}{8 d^3 (c^2 d + e)^{3/2} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}} - \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 d^3} - \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 d^3} - \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 d^3} - \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 d^3} - \\
 & \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 d^3} - \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{2 d^3} - \\
 & \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 d^3} - \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{2 d^3}
 \end{aligned}$$

Result (type 4, 2054 leaves):

$$\begin{aligned}
 & \frac{a}{4 d (d + e x^2)^2} + \frac{a}{2 d^2 (d + e x^2)} + \frac{a \operatorname{Log}[x]}{d^3} - \\
 & \frac{a \operatorname{Log}[d + e x^2]}{2 d^3} + b \left( \frac{1}{16 d^2} \sqrt{e} \left( -\frac{i \sqrt{e} \sqrt{\frac{1-c x}{1+c x}} (1+c x)}{\sqrt{d} (c^2 d + e) (-i \sqrt{d} + \sqrt{e} x)} - \right. \right. \\
 & \left. \left. \frac{\operatorname{ArcSech}[c x]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{\operatorname{Log}[x]}{d \sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-c x}{1+c x}} + c x \sqrt{\frac{1-c x}{1+c x}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & (2c^2d+e) \operatorname{Log}\left[-\left(\left(4d\sqrt{e}\sqrt{c^2d+e}\left(\sqrt{e}-ic^2\sqrt{d}x+\sqrt{c^2d+e}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{c^2d+e}x\sqrt{\frac{1-cx}{1+cx}}\right)\right)/\left((2c^2d+e)(-i\sqrt{d}+\sqrt{e}x)\right)\right)\right]+ \\
 & \frac{1}{16d^2}\sqrt{e}\left(\frac{i\sqrt{e}\sqrt{\frac{1-cx}{1+cx}}(1+cx)}{\sqrt{d}(c^2d+e)(i\sqrt{d}+\sqrt{e}x)}-\frac{\operatorname{ArcSech}[cx]}{\sqrt{e}(i\sqrt{d}+\sqrt{e}x)^2}+\frac{\operatorname{Log}[x]}{d\sqrt{e}}-\frac{\operatorname{Log}\left[1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right]}{d\sqrt{e}}+\frac{1}{d(c^2d+e)^{3/2}}\right) \\
 & (2c^2d+e) \operatorname{Log}\left[-\left(\left(4d\sqrt{e}\sqrt{c^2d+e}\left(\sqrt{e}+ic^2\sqrt{d}x+\sqrt{c^2d+e}\sqrt{\frac{1-cx}{1+cx}}+c\sqrt{c^2d+e}x\sqrt{\frac{1-cx}{1+cx}}\right)\right)/\left((2c^2d+e)(i\sqrt{d}+\sqrt{e}x)\right)\right)\right]- \\
 & \frac{1}{16d^{5/2}}5i\sqrt{e}\left(-\frac{\operatorname{ArcSech}[cx]}{i\sqrt{d}\sqrt{e+ex}}+\frac{1}{\sqrt{d}}i\left(\frac{\operatorname{Log}[x]}{\sqrt{e}}-\frac{\operatorname{Log}\left[1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}}+\frac{\operatorname{Log}\left[\frac{2i\sqrt{e}\left(\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx)+\frac{\sqrt{d}\sqrt{e+ic^2dx}}{\sqrt{c^2d+e}}\right)}{i\sqrt{d}+\sqrt{e}x}\right]}{\sqrt{c^2d+e}}\right)\right)+\frac{1}{16d^{5/2}}5i\sqrt{e}\left(-\frac{\operatorname{ArcSech}[cx]}{-i\sqrt{d}\sqrt{e+ex}}-\frac{1}{\sqrt{d}}\right) \\
 & i\left(\frac{\operatorname{Log}[x]}{\sqrt{e}}-\frac{\operatorname{Log}\left[1+\sqrt{\frac{1-cx}{1+cx}}+cx\sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}}+\frac{\operatorname{Log}\left[\frac{2\sqrt{e}\left(i\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx)+\frac{i\sqrt{d}\sqrt{e+c^2dx}}{\sqrt{c^2d+e}}\right)}{-i\sqrt{d}+\sqrt{e}x}\right]}{\sqrt{c^2d+e}}\right)+
 \end{aligned}$$

$$\begin{aligned}
 & \frac{1}{2 d^3} \left( -\operatorname{ArcSech}[c x] \left( \operatorname{ArcSech}[c x] + 2 \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right]\right) + \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right] \right) - \\
 & \frac{1}{4 d^3} \left( \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right] - \right. \\
 & 2 \left( -4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(i c \sqrt{d} + \sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] - \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \right) + \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} - \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & \operatorname{PolyLog}\left[2, \frac{i\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & \left. \left. \operatorname{PolyLog}\left[2, -\frac{i\left(\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \right) \right) + \\
 & \frac{1}{4 d^3} \left( -\operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[c x]}\right] + 2 \left( -4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{\left(-i c \sqrt{d} + \sqrt{e}\right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] - \right. \\
 & \left. \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i\left(-\sqrt{e} + \sqrt{c^2 d + e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \right) + \right.
 \end{aligned}$$



$$\begin{aligned}
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]- \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2,\right. \\
 & \left.\left.\frac{i\left(\sqrt{e}-\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]+\operatorname{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{c^2 d+e}\right) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right]\right)
 \end{aligned}$$

**Problem 127: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^4 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 1272 leaves, 35 steps):

$$\begin{aligned}
 & \frac{b c \sqrt{-d} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{16 e^{3/2} (c^2 d + e) \left(\sqrt{-d} \sqrt{e} - \frac{d}{x}\right)} + \frac{b c \sqrt{-d} \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{16 e^{3/2} (c^2 d + e) \left(\sqrt{-d} \sqrt{e} + \frac{d}{x}\right)} + \\
 & \frac{\sqrt{-d} (a + b \operatorname{ArcSech}[c x])}{16 e^{3/2} \left(\sqrt{-d} \sqrt{e} - \frac{d}{x}\right)^2} + \frac{3 (a + b \operatorname{ArcSech}[c x])}{16 e^2 \left(\sqrt{-d} \sqrt{e} - \frac{d}{x}\right)} - \frac{\sqrt{-d} (a + b \operatorname{ArcSech}[c x])}{16 e^{3/2} \left(\sqrt{-d} \sqrt{e} + \frac{d}{x}\right)^2} - \\
 & \frac{3 (a + b \operatorname{ArcSech}[c x])}{16 e^2 \left(\sqrt{-d} \sqrt{e} + \frac{d}{x}\right)} - \frac{3 b \operatorname{ArcTan}\left[\frac{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{8 \sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{c d + \sqrt{-d} \sqrt{e}} e^2} - \\
 & \frac{b d \operatorname{ArcTan}\left[\frac{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{8 (c d - \sqrt{-d} \sqrt{e})^{3/2} (c d + \sqrt{-d} \sqrt{e})^{3/2} e} - \frac{3 b \operatorname{ArcTan}\left[\frac{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{8 \sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{c d + \sqrt{-d} \sqrt{e}} e^2} - \\
 & \frac{b d \operatorname{ArcTan}\left[\frac{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{8 (c d - \sqrt{-d} \sqrt{e})^{3/2} (c d + \sqrt{-d} \sqrt{e})^{3/2} e} + \frac{3 (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{16 \sqrt{-d} e^{5/2}} - \\
 & \frac{3 (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{16 \sqrt{-d} e^{5/2}} + \frac{3 (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{16 \sqrt{-d} e^{5/2}} - \\
 & \frac{3 (a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{16 \sqrt{-d} e^{5/2}} - \frac{3 b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{16 \sqrt{-d} e^{5/2}} + \\
 & \frac{3 b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{16 \sqrt{-d} e^{5/2}} - \frac{3 b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{16 \sqrt{-d} e^{5/2}} + \frac{3 b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{16 \sqrt{-d} e^{5/2}}
 \end{aligned}$$

Result (type 4, 2022 leaves):

$$\begin{aligned}
 & \frac{a d x}{4 e^2 (d + e x^2)^2} - \frac{5 a x}{8 e^2 (d + e x^2)} + \frac{3 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{8 \sqrt{d} e^{5/2}} + \\
 & b \left( \frac{1}{16 e^2} i \sqrt{d} \left( -\frac{i \sqrt{e} \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x)}{\sqrt{d} (c^2 d + e) (-i \sqrt{d} + \sqrt{e} x)} \right) - \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\text{ArcSech}[c x]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{\text{Log}[x]}{d \sqrt{e}} - \frac{\text{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \\
 & (2 c^2 d + e) \text{Log}\left[-\left(\left(4 d \sqrt{e} \sqrt{c^2 d + e} \left(\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{\frac{1-cx}{1+cx}} + \right.\right.\right.\right. \\
 & \quad \left.\left.\left.\left. c \sqrt{c^2 d + e} x \sqrt{\frac{1-cx}{1+cx}}\right)\right) / \left((2 c^2 d + e) (-i \sqrt{d} + \sqrt{e} x)\right)\right)\right] - \\
 & \frac{1}{16 e^2} i \sqrt{d} \left( \frac{i \sqrt{e} \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{\sqrt{d} (c^2 d + e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\text{ArcSech}[c x]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} + \frac{\text{Log}[x]}{d \sqrt{e}} - \right. \\
 & \frac{\text{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \\
 & \left. (2 c^2 d + e) \text{Log}\left[-\left(\left(4 d \sqrt{e} \sqrt{c^2 d + e} \left(\sqrt{e} + i c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{\frac{1-cx}{1+cx}} + \right.\right.\right.\right. \right. \\
 & \quad \left.\left.\left.\left. c \sqrt{c^2 d + e} x \sqrt{\frac{1-cx}{1+cx}}\right)\right) / \left((2 c^2 d + e) (i \sqrt{d} + \sqrt{e} x)\right)\right)\right] + \right. \\
 & \frac{1}{16 e^2} 5 \left( -\frac{\text{ArcSech}[c x]}{i \sqrt{d} \sqrt{e} + e x} + \frac{1}{\sqrt{d}} i \left( \frac{\text{Log}[x]}{\sqrt{e}} - \frac{\text{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}} + \right. \right. \\
 & \quad \left. \left. \frac{\text{Log}\left[\frac{2 i \sqrt{e} \left(\sqrt{d} \sqrt{\frac{1-cx}{1+cx}} (1+cx) + \frac{\sqrt{d} \sqrt{e} + i c^2 d x}{\sqrt{c^2 d + e}}\right)}{i \sqrt{d} + \sqrt{e} x}\right]}{\sqrt{c^2 d + e}} \right) \right) + \frac{1}{16 e^2} 5 \left( -\frac{\text{ArcSech}[c x]}{-i \sqrt{d} \sqrt{e} + e x} - \frac{1}{\sqrt{d}} \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{i}{\sqrt{e}} \operatorname{Log}[x] - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + cx \sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}} + \frac{\operatorname{Log}\left[\frac{2\sqrt{e}\left(i\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx) + \frac{i\sqrt{d}\sqrt{e+c^2dx}}{\sqrt{c^2d+e}}\right)}{-i\sqrt{d}+\sqrt{e}x}\right]}{\sqrt{c^2d+e}} \right) - \\
 & \frac{1}{32\sqrt{d}e^{5/2}} 3i \left( \operatorname{PolyLog}\left[2, -e^{-2\operatorname{ArcSech}[cx]}\right] - \right. \\
 & 2 \left( -4i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(ic\sqrt{d}+\sqrt{e})\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcSech}[cx]\right]}{\sqrt{c^2d+e}}\right] + \operatorname{ArcSech}[cx] \right. \\
 & \left. \operatorname{Log}\left[1+e^{-2\operatorname{ArcSech}[cx]}\right] - \operatorname{ArcSech}[cx] \operatorname{Log}\left[1+\frac{i(\sqrt{e}-\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \right. \\
 & 2i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i(\sqrt{e}-\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] - \\
 & \operatorname{ArcSech}[cx] \operatorname{Log}\left[1+\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] - \\
 & 2i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \\
 & \operatorname{PolyLog}\left[2, \frac{i(-\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \\
 & \left. \left. \operatorname{PolyLog}\left[2, -\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] \right) \right) - \\
 & \frac{1}{32\sqrt{d}e^{5/2}} 3i \left( -\operatorname{PolyLog}\left[2, -e^{-2\operatorname{ArcSech}[cx]}\right] + 2 \left( -4i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \operatorname{ArcSech}[c x] \\
 & \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] - \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \right. \\
 & \left. \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \left. \right) \left. \right) \left. \right)
 \end{aligned}$$

**Problem 128: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^2 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^3} dx$$

Optimal (type 4, 1276 leaves, 63 steps):

$$\begin{aligned}
 & \frac{b c \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{16 \sqrt{-d} \sqrt{e} (c^2 d + e) \left(\sqrt{-d} \sqrt{e} - \frac{d}{x}\right)} + \frac{b c \sqrt{-1 + \frac{1}{c x}} \sqrt{1 + \frac{1}{c x}}}{16 \sqrt{-d} \sqrt{e} (c^2 d + e) \left(\sqrt{-d} \sqrt{e} + \frac{d}{x}\right)} + \\
 & \frac{a + b \operatorname{ArcSech}[c x]}{16 \sqrt{-d} \sqrt{e} \left(\sqrt{-d} \sqrt{e} - \frac{d}{x}\right)^2} + \frac{a + b \operatorname{ArcSech}[c x]}{16 d e \left(\sqrt{-d} \sqrt{e} - \frac{d}{x}\right)} - \frac{a + b \operatorname{ArcSech}[c x]}{16 \sqrt{-d} \sqrt{e} \left(\sqrt{-d} \sqrt{e} + \frac{d}{x}\right)^2} - \\
 & \frac{a + b \operatorname{ArcSech}[c x]}{16 d e \left(\sqrt{-d} \sqrt{e} + \frac{d}{x}\right)} + \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{8 (c d - \sqrt{-d} \sqrt{e})^{3/2} (c d + \sqrt{-d} \sqrt{e})^{3/2}} - \\
 & \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{8 d \sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{c d + \sqrt{-d} \sqrt{e}} e} + \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{8 (c d - \sqrt{-d} \sqrt{e})^{3/2} (c d + \sqrt{-d} \sqrt{e})^{3/2}} - \\
 & \frac{b \operatorname{ArcTan}\left[\frac{\sqrt{c d + \sqrt{-d} \sqrt{e}} \sqrt{1 + \frac{1}{c x}}}{\sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{-1 + \frac{1}{c x}}}\right]}{8 d \sqrt{c d - \sqrt{-d} \sqrt{e}} \sqrt{c d + \sqrt{-d} \sqrt{e}} e} - \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{16 (-d)^{3/2} e^{3/2}} + \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{16 (-d)^{3/2} e^{3/2}} - \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 - \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{16 (-d)^{3/2} e^{3/2}} + \\
 & \frac{(a + b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1 + \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{16 (-d)^{3/2} e^{3/2}} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{16 (-d)^{3/2} e^{3/2}} - \\
 & \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} - \sqrt{c^2 d + e}}\right]}{16 (-d)^{3/2} e^{3/2}} + \frac{b \operatorname{PolyLog}\left[2, -\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{16 (-d)^{3/2} e^{3/2}} - \frac{b \operatorname{PolyLog}\left[2, \frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e} + \sqrt{c^2 d + e}}\right]}{16 (-d)^{3/2} e^{3/2}}
 \end{aligned}$$

Result (type 4, 2030 leaves):

$$\begin{aligned}
 & -\frac{a x}{4 e (d + e x^2)^2} + \frac{a x}{8 d e (d + e x^2)} + \frac{a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{8 d^{3/2} e^{3/2}} + \\
 & b \left( -\frac{1}{16 \sqrt{d} e} i \left( -\frac{i \sqrt{e} \sqrt{\frac{1-c x}{1+c x}} (1+c x)}{\sqrt{d} (c^2 d + e) (-i \sqrt{d} + \sqrt{e} x)} \right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\operatorname{ArcSech}[c x]}{\sqrt{e} (-i \sqrt{d} + \sqrt{e} x)^2} + \frac{\operatorname{Log}[x]}{d \sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \\
 & (2 c^2 d + e) \operatorname{Log}\left[-\left(\left(4 d \sqrt{e} \sqrt{c^2 d + e} \left(\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{\frac{1-cx}{1+cx}} + \right.\right.\right.\right. \\
 & \left.\left.\left.\left. c \sqrt{c^2 d + e} x \sqrt{\frac{1-cx}{1+cx}}\right)\right) / \left((2 c^2 d + e) (-i \sqrt{d} + \sqrt{e} x)\right)\right)\right] + \\
 & \frac{1}{16 \sqrt{d} e} i \left( \frac{i \sqrt{e} \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{\sqrt{d} (c^2 d + e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\operatorname{ArcSech}[c x]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} + \frac{\operatorname{Log}[x]}{d \sqrt{e}} - \right. \\
 & \left. \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \right. \\
 & \left. (2 c^2 d + e) \operatorname{Log}\left[-\left(\left(4 d \sqrt{e} \sqrt{c^2 d + e} \left(\sqrt{e} + i c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{\frac{1-cx}{1+cx}} + \right.\right.\right.\right. \right. \\
 & \left.\left.\left.\left. c \sqrt{c^2 d + e} x \sqrt{\frac{1-cx}{1+cx}}\right)\right) / \left((2 c^2 d + e) (i \sqrt{d} + \sqrt{e} x)\right)\right)\right] - \\
 & \frac{1}{16 d e} \left( -\frac{\operatorname{ArcSech}[c x]}{i \sqrt{d} \sqrt{e} + e x} + \frac{1}{\sqrt{d}} i \left( \frac{\operatorname{Log}[x]}{\sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + c x \sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}} + \right. \right. \\
 & \left. \left. \frac{\operatorname{Log}\left[\frac{2 i \sqrt{e} \left(\sqrt{d} \sqrt{\frac{1-cx}{1+cx}} (1+cx) + \frac{\sqrt{d} \sqrt{e} + i c^2 d x}{\sqrt{c^2 d + e}}\right)}{i \sqrt{d} + \sqrt{e} x}\right]}{\sqrt{c^2 d + e}} \right) \right) - \frac{1}{16 d e} \left( -\frac{\operatorname{ArcSech}[c x]}{-i \sqrt{d} \sqrt{e} + e x} - \frac{1}{\sqrt{d}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left( \frac{i}{\sqrt{e}} \operatorname{Log}[x] - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + cx \sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}} + \frac{\operatorname{Log}\left[\frac{2\sqrt{e}\left(i\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx) + \frac{i\sqrt{d}\sqrt{e-c^2dx}}{\sqrt{c^2d+e}}\right)}{-i\sqrt{d}+\sqrt{e}x}\right]}{\sqrt{c^2d+e}} \right) - \\
 & \frac{1}{32d^{3/2}e^{3/2}} i \left( \operatorname{PolyLog}\left[2, -e^{-2\operatorname{ArcSech}[cx]}\right] - \right. \\
 & 2 \left( -4i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(ic\sqrt{d}+\sqrt{e})\operatorname{Tanh}\left[\frac{1}{2}\operatorname{ArcSech}[cx]\right]}{\sqrt{c^2d+e}}\right] + \operatorname{ArcSech}[cx] \right. \\
 & \left. \operatorname{Log}\left[1+e^{-2\operatorname{ArcSech}[cx]}\right] - \operatorname{ArcSech}[cx] \operatorname{Log}\left[1+\frac{i(\sqrt{e}-\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \right. \\
 & 2i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i(\sqrt{e}-\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] - \\
 & \operatorname{ArcSech}[cx] \operatorname{Log}\left[1+\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] - \\
 & 2i \operatorname{ArcSin}\left[\frac{\sqrt{1+\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1+\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \\
 & \operatorname{PolyLog}\left[2, \frac{i(-\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] + \\
 & \left. \left. \operatorname{PolyLog}\left[2, -\frac{i(\sqrt{e}+\sqrt{c^2d+e})e^{-\operatorname{ArcSech}[cx]}}{c\sqrt{d}}\right] \right) \right) - \\
 & \frac{1}{32d^{3/2}e^{3/2}} i \left( -\operatorname{PolyLog}\left[2, -e^{-2\operatorname{ArcSech}[cx]}\right] + 2 \left( -4i \operatorname{ArcSin}\left[\frac{\sqrt{1-\frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \right. \right.
 \end{aligned}$$



$$\begin{aligned}
 & \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \operatorname{ArcSech}[c x] \\
 & \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] - \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \right. \\
 & \left. \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \left. \right) \left. \right) \left. \right)
 \end{aligned}$$

**Problem 129: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{a + b \operatorname{ArcSech}[c x]}{(d + e x^2)^3} dx$$

Optimal (type 4, 1272 leaves, 81 steps):

$$\begin{aligned}
 & \frac{b c \sqrt{e} \sqrt{-1+\frac{1}{c x}} \sqrt{1+\frac{1}{c x}}}{16(-d)^{3/2}(c^2 d+e)\left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)} + \frac{b c \sqrt{e} \sqrt{-1+\frac{1}{c x}} \sqrt{1+\frac{1}{c x}}}{16(-d)^{3/2}(c^2 d+e)\left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)} + \\
 & \frac{\sqrt{e}(a+b \operatorname{ArcSech}[c x])}{16(-d)^{3/2}\left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)^2} - \frac{5(a+b \operatorname{ArcSech}[c x])}{16 d^2\left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)} - \frac{\sqrt{e}(a+b \operatorname{ArcSech}[c x])}{16(-d)^{3/2}\left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)^2} + \\
 & \frac{5(a+b \operatorname{ArcSech}[c x])}{16 d^2\left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)} + \frac{5 b \operatorname{ArcTan}\left[\frac{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{8 d^2 \sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{c d+\sqrt{-d} \sqrt{e}}} - \\
 & \frac{b e \operatorname{ArcTan}\left[\frac{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{8 d\left(c d-\sqrt{-d} \sqrt{e}\right)^{3/2}\left(c d+\sqrt{-d} \sqrt{e}\right)^{3/2}} + \frac{5 b \operatorname{ArcTan}\left[\frac{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{8 d^2 \sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{c d+\sqrt{-d} \sqrt{e}}} - \\
 & \frac{b e \operatorname{ArcTan}\left[\frac{\sqrt{c d+\sqrt{-d} \sqrt{e}} \sqrt{1+\frac{1}{c x}}}{\sqrt{c d-\sqrt{-d} \sqrt{e}} \sqrt{-1+\frac{1}{c x}}}\right]}{8 d\left(c d-\sqrt{-d} \sqrt{e}\right)^{3/2}\left(c d+\sqrt{-d} \sqrt{e}\right)^{3/2}} + \frac{3(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{16(-d)^{5/2} \sqrt{e}} - \\
 & \frac{3(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{16(-d)^{5/2} \sqrt{e}} + \frac{3(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{16(-d)^{5/2} \sqrt{e}} - \\
 & \frac{3(a+b \operatorname{ArcSech}[c x]) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{16(-d)^{5/2} \sqrt{e}} - \frac{3 b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{16(-d)^{5/2} \sqrt{e}} + \\
 & \frac{3 b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}-\sqrt{c^2 d+e}}\right]}{16(-d)^{5/2} \sqrt{e}} - \frac{3 b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{16(-d)^{5/2} \sqrt{e}} + \frac{3 b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcSech}[c x]}}{\sqrt{e}+\sqrt{c^2 d+e}}\right]}{16(-d)^{5/2} \sqrt{e}}
 \end{aligned}$$

Result (type 4, 2015 leaves):

$$\begin{aligned}
 & \frac{a x}{4 d\left(d+e x^2\right)^2} + \frac{3 a x}{8 d^2\left(d+e x^2\right)} + \frac{3 a \operatorname{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{8 d^{5/2} \sqrt{e}} + \\
 & b \left( \frac{1}{16 d^{3/2}} \left( -\frac{i \sqrt{e} \sqrt{\frac{1-c x}{1+c x}}(1+c x)}{\sqrt{d}\left(c^2 d+e\right)\left(-i \sqrt{d}+\sqrt{e} x\right)} - \frac{\operatorname{ArcSech}[c x]}{\sqrt{e}\left(-i \sqrt{d}+\sqrt{e} x\right)^2} + \right. \right.
 \end{aligned}$$

$$\begin{aligned}
 & \frac{\operatorname{Log}[x]}{d \sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + cx \sqrt{\frac{1-cx}{1+cx}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \\
 & (2 c^2 d + e) \operatorname{Log}\left[-\left(\left(4 d \sqrt{e} \sqrt{c^2 d + e} \left(\sqrt{e} - i c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{\frac{1-cx}{1+cx}} + \right.\right.\right.\right. \\
 & \quad \left.\left.\left.\left. c \sqrt{c^2 d + e} x \sqrt{\frac{1-cx}{1+cx}}\right)\right) / \left(\left(2 c^2 d + e\right) \left(-i \sqrt{d} + \sqrt{e} x\right)\right)\right]\right) - \\
 & \frac{1}{16 d^{3/2}} i \left( \frac{i \sqrt{e} \sqrt{\frac{1-cx}{1+cx}} (1+cx)}{\sqrt{d} (c^2 d + e) (i \sqrt{d} + \sqrt{e} x)} - \frac{\operatorname{ArcSech}[cx]}{\sqrt{e} (i \sqrt{d} + \sqrt{e} x)^2} + \frac{\operatorname{Log}[x]}{d \sqrt{e}} - \right. \\
 & \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + cx \sqrt{\frac{1-cx}{1+cx}}\right]}{d \sqrt{e}} + \frac{1}{d (c^2 d + e)^{3/2}} \\
 & (2 c^2 d + e) \operatorname{Log}\left[-\left(\left(4 d \sqrt{e} \sqrt{c^2 d + e} \left(\sqrt{e} + i c^2 \sqrt{d} x + \sqrt{c^2 d + e} \sqrt{\frac{1-cx}{1+cx}} + \right.\right.\right.\right. \\
 & \quad \left.\left.\left.\left. c \sqrt{c^2 d + e} x \sqrt{\frac{1-cx}{1+cx}}\right)\right) / \left(\left(2 c^2 d + e\right) \left(i \sqrt{d} + \sqrt{e} x\right)\right)\right]\right) - \\
 & \frac{1}{16 d^2} 3 \left( -\frac{\operatorname{ArcSech}[cx]}{i \sqrt{d} \sqrt{e} + ex} + \frac{1}{\sqrt{d}} i \left( \frac{\operatorname{Log}[x]}{\sqrt{e}} - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + cx \sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}} + \right. \right. \\
 & \quad \left. \left. \frac{\operatorname{Log}\left[\frac{2 i \sqrt{e} \left(\sqrt{d} \sqrt{\frac{1-cx}{1+cx}} (1+cx) + \sqrt{d} \sqrt{e} - i c^2 d x\right)}{\sqrt{c^2 d + e}}\right]}{i \sqrt{d} + \sqrt{e} x} \right) \right) - \frac{1}{16 d^2} 3 \left( -\frac{\operatorname{ArcSech}[cx]}{-i \sqrt{d} \sqrt{e} + ex} - \frac{1}{\sqrt{d}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left. \left( \frac{i}{\sqrt{e}} \operatorname{Log}[x] - \frac{\operatorname{Log}\left[1 + \sqrt{\frac{1-cx}{1+cx}} + cx \sqrt{\frac{1-cx}{1+cx}}\right]}{\sqrt{e}} + \frac{\operatorname{Log}\left[\frac{2\sqrt{e}\left(i\sqrt{d}\sqrt{\frac{1-cx}{1+cx}}(1+cx) + \frac{i\sqrt{d}\sqrt{e+c^2dx}}{\sqrt{c^2d+e}}\right)}{-i\sqrt{d}+\sqrt{e}x}\right]}{\sqrt{c^2d+e}} \right)}{32 d^{5/2} \sqrt{e}} \right) - \\
 & \frac{1}{32 d^{5/2} \sqrt{e}} \left( 3 i \operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[cx]}\right] - \right. \\
 & 2 \left( -4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[cx]\right]}{\sqrt{c^2 d + e}}\right] + \operatorname{ArcSech}[cx] \right. \\
 & \left. \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[cx]}\right] - \operatorname{ArcSech}[cx] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[cx]}}{c \sqrt{d}}\right] + \right. \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[cx]}}{c \sqrt{d}}\right] - \\
 & \operatorname{ArcSech}[cx] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[cx]}}{c \sqrt{d}}\right] - \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 + \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[cx]}}{c \sqrt{d}}\right] + \\
 & \operatorname{PolyLog}\left[2, \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[cx]}}{c \sqrt{d}}\right] + \\
 & \left. \left. \operatorname{PolyLog}\left[2, -\frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[cx]}}{c \sqrt{d}}\right] \right) \right) - \\
 & \frac{1}{32 d^{5/2} \sqrt{e}} \left( 3 i \left( -\operatorname{PolyLog}\left[2, -e^{-2 \operatorname{ArcSech}[cx]}\right] + 2 \left( -4 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right] \right. \right. \right)
 \end{aligned}$$

$$\begin{aligned}
 & \operatorname{ArcTanh}\left[\frac{(-i c \sqrt{d} + \sqrt{e}) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcSech}[c x]\right]}{\sqrt{c^2 d + e}}\right] + \operatorname{ArcSech}[c x] \\
 & \operatorname{Log}\left[1 + e^{-2 \operatorname{ArcSech}[c x]}\right] - \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{i(-\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & \operatorname{ArcSech}[c x] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] - \\
 & 2 i \operatorname{ArcSin}\left[\frac{\sqrt{1 - \frac{i \sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \right. \\
 & \left. \frac{i(\sqrt{e} - \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] + \operatorname{PolyLog}\left[2, \frac{i(\sqrt{e} + \sqrt{c^2 d + e}) e^{-\operatorname{ArcSech}[c x]}}{c \sqrt{d}}\right] \left. \right] \left. \right) \left. \right)
 \end{aligned}$$

**Problem 130: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^5 \sqrt{d + e x^2} (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 447 leaves, 12 steps):

$$\begin{aligned}
& \frac{b \left( 23 c^4 d^2 + 12 c^2 d e - 75 e^2 \right) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{1680 c^6 e^2} + \\
& \frac{b \left( 29 c^2 d - 25 e \right) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} (d+e x^2)^{3/2}}{840 c^4 e^2} - \\
& \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} (d+e x^2)^{5/2}}{42 c^2 e^2} + \frac{d^2 (d+e x^2)^{3/2} (a+b \operatorname{ArcSech}[c x])}{3 e^3} - \\
& \frac{2 d (d+e x^2)^{5/2} (a+b \operatorname{ArcSech}[c x])}{5 e^3} + \frac{(d+e x^2)^{7/2} (a+b \operatorname{ArcSech}[c x])}{7 e^3} - \frac{1}{1680 c^7 e^{5/2}} \\
& \frac{b \left( 105 c^6 d^3 - 35 c^4 d^2 e + 63 c^2 d e^2 + 75 e^3 \right) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{105 e^3} - \\
& \frac{8 b d^{7/2} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{105 e^3}
\end{aligned}$$

Result (type 3, 396 leaves):

$$\begin{aligned}
& \frac{1}{1680 c^6 e^3} \sqrt{d+e x^2} \left( 16 a c^6 \left( 8 d^3 - 4 d^2 e x^2 + 3 d e^2 x^4 + 15 e^3 x^6 \right) - \right. \\
& \left. b e \sqrt{\frac{1-c x}{1+c x}} \left( 1+c x \right) \left( 75 e^2 + 2 c^2 e \left( 19 d + 25 e x^2 \right) + c^4 \left( -41 d^2 + 22 d e x^2 + 40 e^2 x^4 \right) \right) + \right. \\
& \left. 16 b c^6 \left( 8 d^3 - 4 d^2 e x^2 + 3 d e^2 x^4 + 15 e^3 x^6 \right) \operatorname{ArcSech}[c x] \right) - \left( b \sqrt{\frac{1-c x}{1+c x}} \sqrt{-1+c^2 x^2} \right. \\
& \left. \left( -128 i c^7 d^{7/2} \operatorname{Log}\left[\frac{-i e x^2 + i d \left( -2 + c^2 x^2 \right) + 2 \sqrt{d} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{128 c^6 d^{9/2} x^2}\right] + \right. \right. \\
& \left. \left. \sqrt{e} \left( 105 c^6 d^3 - 35 c^4 d^2 e + 63 c^2 d e^2 + 75 e^3 \right) \right. \right. \\
& \left. \left. \operatorname{Log}\left[-e + 2 c \sqrt{e} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2} + c^2 \left( d + 2 e x^2 \right) \right] \right) \right) / \left( 3360 c^7 e^3 \left( -1+c x \right) \right)
\end{aligned}$$

**Problem 131: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3 \sqrt{d+e x^2} (a+b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 329 leaves, 11 steps):

$$\begin{aligned}
 & - \frac{b (c^2 d + 9 e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+ex^2}}{120 c^4 e} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} (d+ex^2)^{3/2}}{20 c^2 e} \\
 & \frac{d (d+ex^2)^{3/2} (a+b \operatorname{ArcSech}[cx])}{3 e^2} + \frac{(d+ex^2)^{5/2} (a+b \operatorname{ArcSech}[cx])}{5 e^2} + \\
 & \frac{b (15 c^4 d^2 - 10 c^2 d e - 9 e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+ex^2}}\right]}{120 c^5 e^{3/2}} + \\
 & \frac{2 b d^{5/2} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{15 e^2}
 \end{aligned}$$

Result (type 3, 333 leaves):

$$\begin{aligned}
 & - \frac{1}{120 c^4 e^2} \sqrt{d+ex^2} \left( 8 a c^4 (2 d^2 - d e x^2 - 3 e^2 x^4) + \right. \\
 & \left. b e \sqrt{\frac{1-cx}{1+cx}} (1+cx) (9 e + c^2 (7 d + 6 e x^2)) + 8 b c^4 (2 d^2 - d e x^2 - 3 e^2 x^4) \operatorname{ArcSech}[cx] \right) - \\
 & \left( b \sqrt{\frac{1-cx}{1+cx}} \sqrt{-1+c^2 x^2} \left( 16 i c^5 d^{5/2} \operatorname{Log}\left[ \frac{-i e x^2 + i d (-2 + c^2 x^2) + 2 \sqrt{d} \sqrt{-1+c^2 x^2} \sqrt{d+ex^2}}{16 c^4 d^{7/2} x^2} \right] + \right. \right. \\
 & \left. \left. \sqrt{e} (-15 c^4 d^2 + 10 c^2 d e + 9 e^2) \right. \right. \\
 & \left. \left. \operatorname{Log}\left[ -e + 2 c \sqrt{e} \sqrt{-1+c^2 x^2} \sqrt{d+ex^2} + c^2 (d + 2 e x^2) \right] \right) \right) / (240 c^5 e^2 (-1+cx))
 \end{aligned}$$

**Problem 132: Result unnecessarily involves imaginary or complex numbers.**

$$\int x \sqrt{d+ex^2} (a+b \operatorname{ArcSech}[cx]) dx$$

Optimal (type 3, 221 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+ex^2}}{6 c^2} + \frac{(d+ex^2)^{3/2} (a+b \operatorname{ArcSech}[cx])}{3 e} - \\
 & \frac{b (3 c^2 d + e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+ex^2}}\right]}{6 c^3 \sqrt{e}} - \frac{b d^{3/2} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{3 e}
 \end{aligned}$$

Result (type 3, 275 leaves):

$$\frac{1}{6 c^2 e} \sqrt{d+e x^2} \left( -b e \sqrt{\frac{1-c x}{1+c x}} (1+c x) + 2 a c^2 (d+e x^2) + 2 b c^2 (d+e x^2) \text{ArcSech}[c x] \right) -$$

$$\frac{1}{12 c^3 e (-1+c x)}$$

$$b \sqrt{\frac{1-c x}{1+c x}} \sqrt{-1+c^2 x^2} \left( -2 i c^3 d^{3/2} \text{Log} \left[ \frac{-i e x^2 + i d (-2+c^2 x^2) + 2 \sqrt{d} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{2 c^2 d^{5/2} x^2} \right] + \right.$$

$$\left. \sqrt{e} (3 c^2 d+e) \text{Log} \left[ -e+2 c \sqrt{e} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2} + c^2 (d+2 e x^2) \right] \right)$$

**Problem 138: Unable to integrate problem.**

$$\int \frac{\sqrt{d+e x^2} (a+b \text{ArcSech}[c x])}{x^4} dx$$

Optimal (type 4, 312 leaves, 9 steps):

$$\frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{9 x^3} + \frac{2 b (c^2 d+2 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{9 d x} -$$

$$\frac{(d+e x^2)^{3/2} (a+b \text{ArcSech}[c x])}{3 d x^3} + \frac{1}{9 d \sqrt{1+\frac{e x^2}{d}}}$$

$$2 b c (c^2 d+2 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x^2} \text{EllipticE}[\text{ArcSin}[c x], -\frac{e}{c^2 d}] - \frac{1}{9 c d \sqrt{d+e x^2}}$$

$$b (c^2 d+e) (2 c^2 d+3 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1+\frac{e x^2}{d}} \text{EllipticF}[\text{ArcSin}[c x], -\frac{e}{c^2 d}]$$

Result (type 8, 25 leaves):

$$\int \frac{\sqrt{d+e x^2} (a+b \text{ArcSech}[c x])}{x^4} dx$$

**Problem 139: Unable to integrate problem.**

$$\int \frac{\sqrt{d+e x^2} (a+b \text{ArcSech}[c x])}{x^6} dx$$

Optimal (type 4, 446 leaves, 10 steps):



$$\begin{aligned}
 & \frac{b (12 c^2 d - e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{225 d x^3} + \\
 & \frac{b (24 c^4 d^2 + 19 c^2 d e - 31 e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{225 d^2 x} + \\
 & \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} (d+e x^2)^{3/2}}{25 d x^5} - \frac{(d+e x^2)^{3/2} (a+b \operatorname{ArcSech}[c x])}{5 d x^5} + \\
 & \frac{2 e (d+e x^2)^{3/2} (a+b \operatorname{ArcSech}[c x])}{15 d^2 x^3} + \frac{1}{225 d^2 \sqrt{1+\frac{e x^2}{d}}} \\
 & b c (24 c^4 d^2 + 19 c^2 d e - 31 e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{d+e x^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}\right] - \\
 & \left( b (c^2 d + e) (24 c^4 d^2 + 7 c^2 d e - 30 e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right. \\
 & \left. \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}\right] \right) / \left( 225 c d^2 \sqrt{d+e x^2} \right)
 \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{\sqrt{d+e x^2} (a+b \operatorname{ArcSech}[c x])}{x^6} dx$$

**Problem 140: Result unnecessarily involves imaginary or complex numbers.**

$$\int x^3 (d+e x^2)^{3/2} (a+b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 3, 418 leaves, 12 steps):

$$\frac{b (3 c^4 d^2 - 38 c^2 d e - 25 e^2) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{560 c^6 e} -$$

$$\frac{b (13 c^2 d + 25 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} (d+e x^2)^{3/2}}{840 c^4 e} -$$

$$\frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} (d+e x^2)^{5/2}}{42 c^2 e} - \frac{d (d+e x^2)^{5/2} (a+b \text{ArcSech}[c x])}{5 e^2} +$$

$$\frac{(d+e x^2)^{7/2} (a+b \text{ArcSech}[c x])}{7 e^2} + \frac{1}{560 c^7 e^{3/2}}$$

$$b (35 c^6 d^3 - 35 c^4 d^2 e - 63 c^2 d e^2 - 25 e^3) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+e x^2}}\right] +$$

$$\frac{2 b d^{7/2} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \text{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{35 e^2}$$

Result (type 3, 369 leaves):

$$-\frac{1}{1680 c^6 e^2} \sqrt{d+e x^2} \left( 48 a c^6 (2 d - 5 e x^2) (d+e x^2)^2 + \right.$$

$$b e \sqrt{\frac{1-c x}{1+c x}} (1+c x) (75 e^2 + 2 c^2 e (82 d + 25 e x^2) + c^4 (57 d^2 + 106 d e x^2 + 40 e^2 x^4)) +$$

$$\left. 48 b c^6 (2 d - 5 e x^2) (d+e x^2)^2 \text{ArcSech}[c x] \right) -$$

$$\left( b \sqrt{\frac{1-c x}{1+c x}} \sqrt{-1+c^2 x^2} \left( 32 i c^7 d^{7/2} \text{Log}\left[ \frac{-i e x^2 + i d (-2 + c^2 x^2) + 2 \sqrt{d} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{32 c^6 d^{9/2} x^2} \right] + \right.$$

$$\left. \sqrt{e} (-35 c^6 d^3 + 35 c^4 d^2 e + 63 c^2 d e^2 + 25 e^3) \right.$$

$$\left. \left. \text{Log}[-e + 2 c \sqrt{e} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2} + c^2 (d+2 e x^2)] \right) \right) / (1120 c^7 e^2 (-1+c x))$$

**Problem 141: Result unnecessarily involves imaginary or complex numbers.**

$$\int x (d+e x^2)^{3/2} (a+b \text{ArcSech}[c x]) dx$$

Optimal (type 3, 297 leaves, 11 steps):

$$\begin{aligned}
 & \frac{b (7 c^2 d + 3 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{40 c^4} \\
 & \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} (d+e x^2)^{3/2}}{20 c^2} + \frac{(d+e x^2)^{5/2} (a+b \operatorname{ArcSech}[c x])}{5 e} \\
 & \frac{b (15 c^4 d^2 + 10 c^2 d e + 3 e^2) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{40 c^5 \sqrt{e}} \\
 & \frac{b d^{5/2} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{5 e}
 \end{aligned}$$

Result (type 3, 313 leaves):

$$\begin{aligned}
 & \frac{1}{40 c^4 e} \sqrt{d+e x^2} \\
 & \left( 8 a c^4 (d+e x^2)^2 - b e \sqrt{\frac{1-c x}{1+c x}} (1+c x) (3 e + c^2 (9 d + 2 e x^2)) + 8 b c^4 (d+e x^2)^2 \operatorname{ArcSech}[c x] \right) + \\
 & \frac{1}{80 c^5 e (-1+c x)} \operatorname{I} b \sqrt{\frac{1-c x}{1+c x}} \sqrt{-1+c^2 x^2} \\
 & \left( 8 c^5 d^{5/2} \operatorname{Log}\left[\frac{-\operatorname{I} e x^2 + \operatorname{I} d (-2+c^2 x^2) + 2 \sqrt{d} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{8 c^4 d^{7/2} x^2}\right] + \right. \\
 & \left. \operatorname{I} \sqrt{e} (15 c^4 d^2 + 10 c^2 d e + 3 e^2) \operatorname{Log}\left[-e + 2 c \sqrt{e} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2} + c^2 (d+2 e x^2)\right] \right)
 \end{aligned}$$

**Problem 148: Unable to integrate problem.**

$$\int \frac{(d+e x^2)^{3/2} (a+b \operatorname{ArcSech}[c x])}{x^6} dx$$

Optimal (type 4, 409 leaves, 10 steps):

$$\begin{aligned}
& \frac{4 b (c^2 d + 2 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{75 x^3} + \\
& \frac{b (8 c^4 d^2 + 23 c^2 d e + 23 e^2) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{75 d x} + \\
& \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} (d+e x^2)^{3/2}}{25 x^5} - \frac{(d+e x^2)^{5/2} (a+b \operatorname{ArcSech}[c x])}{5 d x^5} + \frac{1}{75 d \sqrt{1+\frac{e x^2}{d}}} \\
& b c (8 c^4 d^2 + 23 c^2 d e + 23 e^2) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}] - \\
& \frac{1}{75 c d \sqrt{d+e x^2}} b (c^2 d + e) (8 c^4 d^2 + 19 c^2 d e + 15 e^2) \\
& \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{(d+e x^2)^{3/2} (a+b \operatorname{ArcSech}[c x])}{x^6} dx$$

**Problem 149: Unable to integrate problem.**

$$\int \frac{(d+e x^2)^{3/2} (a+b \operatorname{ArcSech}[c x])}{x^8} dx$$

Optimal (type 4, 556 leaves, 11 steps):

$$\begin{aligned}
 & \frac{b (120 c^4 d^2 + 159 c^2 d e - 37 e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{3675 d x^3} + \frac{1}{3675 d^2 x} \\
 & b (240 c^6 d^3 + 528 c^4 d^2 e + 193 c^2 d e^2 - 247 e^3) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+e x^2} + \\
 & \frac{b (30 c^2 d + 11 e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} (d+e x^2)^{3/2}}{1225 d x^5} + \\
 & \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} (d+e x^2)^{5/2}}{49 d x^7} - \frac{(d+e x^2)^{5/2} (a+b \operatorname{ArcSech}[c x])}{7 d x^7} + \\
 & \frac{2 e (d+e x^2)^{5/2} (a+b \operatorname{ArcSech}[c x])}{35 d^2 x^5} + \left( b c (240 c^6 d^3 + 528 c^4 d^2 e + 193 c^2 d e^2 - 247 e^3) \right. \\
 & \left. \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}] \right) / \left( 3675 d^2 \sqrt{1+\frac{e x^2}{d}} \right) - \\
 & \left( 2 b (c^2 d + e) (120 c^6 d^3 + 204 c^4 d^2 e + 17 c^2 d e^2 - 105 e^3) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \right. \\
 & \left. \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}] \right) / \left( 3675 c d^2 \sqrt{d+e x^2} \right)
 \end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{(d+e x^2)^{3/2} (a+b \operatorname{ArcSech}[c x])}{x^8} dx$$

**Problem 150: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^5 (a+b \operatorname{ArcSech}[c x])}{\sqrt{d+e x^2}} dx$$

Optimal (type 3, 356 leaves, 11 steps):

$$\frac{b (19 c^2 d - 9 e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+ex^2}}{120 c^4 e^2} -$$

$$\frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} (d+ex^2)^{3/2}}{20 c^2 e^2} + \frac{d^2 \sqrt{d+ex^2} (a+b \text{ArcSech}[cx])}{e^3} -$$

$$\frac{2 d (d+ex^2)^{3/2} (a+b \text{ArcSech}[cx])}{3 e^3} + \frac{(d+ex^2)^{5/2} (a+b \text{ArcSech}[cx])}{5 e^3} -$$

$$\frac{b (45 c^4 d^2 - 10 c^2 d e + 9 e^2) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+ex^2}}\right]}{120 c^5 e^{5/2}} -$$

$$\frac{8 b d^{5/2} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \text{ArcTanh}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{15 e^3}$$

Result (type 3, 334 leaves):

$$\frac{1}{120 c^4 e^3} \sqrt{d+ex^2} \left( 8 a c^4 (8 d^2 - 4 d e x^2 + 3 e^2 x^4) - b e \sqrt{\frac{1-cx}{1+cx}} (1+cx) (9 e + c^2 (-13 d + 6 e x^2)) + \right.$$

$$\left. 8 b c^4 (8 d^2 - 4 d e x^2 + 3 e^2 x^4) \text{ArcSech}[cx] \right) - \left( b \sqrt{\frac{1-cx}{1+cx}} \sqrt{-1+c^2 x^2} \right.$$

$$\left. \left( -64 i c^5 d^{5/2} \text{Log}\left[\frac{-i e x^2 + i d (-2+c^2 x^2) + 2 \sqrt{d} \sqrt{-1+c^2 x^2} \sqrt{d+ex^2}}{64 c^4 d^{7/2} x^2}\right] + \right.$$

$$\left. \sqrt{e} (45 c^4 d^2 - 10 c^2 d e + 9 e^2) \right.$$

$$\left. \left. \text{Log}\left[-e + 2 c \sqrt{e} \sqrt{-1+c^2 x^2} \sqrt{d+ex^2} + c^2 (d+2 e x^2)\right] \right) \right) / (240 c^5 e^3 (-1+cx))$$

**Problem 151: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^3 (a + b \text{ArcSech}[cx])}{\sqrt{d+ex^2}} dx$$

Optimal (type 3, 251 leaves, 10 steps):

$$\begin{aligned}
 & - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2e} - \\
 & \frac{d\sqrt{d+ex^2} (a+b \operatorname{ArcSech}[cx])}{e^2} + \frac{(d+ex^2)^{3/2} (a+b \operatorname{ArcSech}[cx])}{3e^2} + \\
 & \frac{b(3c^2d-e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTan}\left[\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right]}{6c^3e^{3/2}} + \frac{2bd^{3/2} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right]}{3e^2}
 \end{aligned}$$

Result (type 3, 280 leaves):

$$\begin{aligned}
 & - \frac{1}{6c^2e^2} \sqrt{d+ex^2} \left( b e \sqrt{\frac{1-cx}{1+cx}} (1+cx) + 2ac^2(2d-ex^2) + 2bc^2(2d-ex^2) \operatorname{ArcSech}[cx] \right) - \\
 & \left( b \sqrt{\frac{1-cx}{1+cx}} \sqrt{-1+c^2x^2} \right. \\
 & \left. \left( 4i c^3 d^{3/2} \operatorname{Log}\left[ \frac{-i ex^2 + i d (-2+c^2x^2) + 2\sqrt{d}\sqrt{-1+c^2x^2}\sqrt{d+ex^2}}{4c^2d^{5/2}x^2} \right] + \sqrt{e}(-3c^2d+e) \right. \right. \\
 & \left. \left. \operatorname{Log}\left[ -e + 2c\sqrt{e}\sqrt{-1+c^2x^2}\sqrt{d+ex^2} + c^2(d+2ex^2) \right] \right) \right) / (12c^3e^2(-1+cx))
 \end{aligned}$$

**Problem 152: Unable to integrate problem.**

$$\int \frac{x (a + b \operatorname{ArcSech}[cx])}{\sqrt{d+ex^2}} dx$$

Optimal (type 3, 153 leaves, 10 steps):

$$\begin{aligned}
 & \frac{\sqrt{d+ex^2} (a + b \operatorname{ArcSech}[cx])}{e} - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTan}\left[\frac{\sqrt{e}\sqrt{1-c^2x^2}}{c\sqrt{d+ex^2}}\right]}{c\sqrt{e}} - \\
 & \frac{b\sqrt{d} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right]}{e}
 \end{aligned}$$

Result (type 8, 23 leaves):

$$\int \frac{x (a + b \operatorname{ArcSech}[cx])}{\sqrt{d+ex^2}} dx$$

### Problem 157: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcSech}[c x]}{x^2 \sqrt{d + e x^2}} dx$$

Optimal (type 4, 221 leaves, 9 steps):

$$\frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+ex^2}}{dx} - \frac{\sqrt{d+ex^2} (a + b \operatorname{ArcSech}[c x])}{dx} +$$

$$\frac{bc \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{d+ex^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{d \sqrt{1 + \frac{ex^2}{d}}} - \frac{1}{cd \sqrt{d+ex^2}}$$

$$b (c^2 d + e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 + \frac{ex^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]$$

Result (type 8, 25 leaves):

$$\int \frac{a + b \operatorname{ArcSech}[c x]}{x^2 \sqrt{d + e x^2}} dx$$

### Problem 158: Unable to integrate problem.

$$\int \frac{a + b \operatorname{ArcSech}[c x]}{x^4 \sqrt{d + e x^2}} dx$$

Optimal (type 4, 346 leaves, 9 steps):

$$\frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+ex^2}}{9 d x^3} + \frac{b (2 c^2 d - 5 e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2 x^2} \sqrt{d+ex^2}}{9 d^2 x} -$$

$$\frac{\sqrt{d+ex^2} (a + b \operatorname{ArcSech}[c x])}{3 d x^3} + \frac{2 e \sqrt{d+ex^2} (a + b \operatorname{ArcSech}[c x])}{3 d^2 x} + \frac{1}{9 d^2 \sqrt{1 + \frac{ex^2}{d}}}$$

$$bc (2 c^2 d - 5 e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{d+ex^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}] - \frac{1}{9 c d^2 \sqrt{d+ex^2}}$$

$$2 b (c^2 d - 3 e) (c^2 d + e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1 + \frac{ex^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]$$

Result (type 8, 25 leaves):



$$\int \frac{a + b \operatorname{ArcSech}[c x]}{x^4 \sqrt{d + e x^2}} dx$$

**Problem 159: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^5 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^{3/2}} dx$$

Optimal (type 3, 278 leaves, 10 steps):

$$\begin{aligned} & - \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2} \sqrt{d+ex^2}}{6c^2e^2} - \frac{d^2 (a + b \operatorname{ArcSech}[c x])}{e^3 \sqrt{d+ex^2}} - \\ & \frac{2d \sqrt{d+ex^2} (a + b \operatorname{ArcSech}[c x])}{e^3} + \frac{(d+ex^2)^{3/2} (a + b \operatorname{ArcSech}[c x])}{3e^3} + \\ & \frac{b(9c^2d - e) \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2x^2}}{c \sqrt{d+ex^2}}\right]}{6c^3e^{5/2}} + \frac{8bd^{3/2} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d} \sqrt{1-c^2x^2}}\right]}{3e^3} \end{aligned}$$

Result (type 3, 310 leaves):

$$\begin{aligned} & \frac{1}{6c^2e^3 \sqrt{d+ex^2}} \left( -be \sqrt{\frac{1-cx}{1+cx}} (1+cx) (d+ex^2) - 2ac^2 (8d^2 + 4dex^2 - e^2x^4) - \right. \\ & \left. 2bc^2 (8d^2 + 4dex^2 - e^2x^4) \operatorname{ArcSech}[c x] \right) - \left( b \sqrt{\frac{1-cx}{1+cx}} \sqrt{-1+c^2x^2} \right. \\ & \left. \left( 16i c^3 d^{3/2} \operatorname{Log}\left[ \frac{-i e x^2 + i d (-2 + c^2 x^2) + 2 \sqrt{d} \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{16 c^2 d^{5/2} x^2} \right] + \sqrt{e} (-9 c^2 d + e) \right. \right. \\ & \left. \left. \operatorname{Log}\left[ -e + 2c \sqrt{e} \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2} + c^2 (d + 2e x^2) \right] \right) \right) / (12 c^3 e^3 (-1 + c x)) \end{aligned}$$

**Problem 160: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^3 (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^{3/2}} dx$$

Optimal (type 3, 177 leaves, 9 steps):

$$\frac{d (a + b \operatorname{ArcSech}[c x])}{e^2 \sqrt{d + e x^2}} + \frac{\sqrt{d + e x^2} (a + b \operatorname{ArcSech}[c x])}{e^2} -$$

$$b \frac{\sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{c e^{3/2}} - \frac{2 b \sqrt{d} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{e^2}$$

Result (type 3, 213 leaves):

$$\frac{(2 d + e x^2) (a + b \operatorname{ArcSech}[c x])}{e^2 \sqrt{d + e x^2}} - \frac{1}{2 c e^2 (-1 + c x)}$$

$$b \sqrt{\frac{1 - c x}{1 + c x}} \sqrt{-1 + c^2 x^2} \left( \sqrt{e} \operatorname{Log}\left[-e + 2 c \sqrt{e} \sqrt{-1 + c^2 x^2} \sqrt{d + e x^2} + c^2 (d + 2 e x^2)\right] - \right.$$

$$\left. 2 i c \sqrt{d} \operatorname{Log}\left[\frac{\sqrt{-1 + c^2 x^2} \sqrt{d + e x^2}}{d x^2} + \frac{i (-e x^2 + d (-2 + c^2 x^2))}{2 d^{3/2} x^2}\right] \right)$$

**Problem 161: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{x (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^{3/2}} dx$$

Optimal (type 3, 87 leaves, 5 steps):

$$-\frac{a + b \operatorname{ArcSech}[c x]}{e \sqrt{d + e x^2}} + \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{\sqrt{d} e}$$

Result (type 4, 573 leaves):

$$\begin{aligned}
 & -\frac{a+b \operatorname{ArcSech}[c x]}{e \sqrt{d+e x^2}} + \left( 2 b (-1+c x) \sqrt{\frac{1-c x}{1+c x}} \sqrt{\frac{(-i c \sqrt{d} + \sqrt{e}) \left(-1 + \frac{2}{1-c x}\right)}{i c \sqrt{d} + \sqrt{e}}} \right. \\
 & \left. \left( -\frac{1}{-1+c x} i c (c \sqrt{d} - i \sqrt{e}) (-i \sqrt{d} + \sqrt{e} x) \sqrt{-\frac{-1 + \frac{i \sqrt{e} x}{\sqrt{d}} + c \left(\frac{i \sqrt{d}}{\sqrt{e}} + x\right)}{1-c x}} \right. \right. \\
 & \left. \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 + \frac{i c \sqrt{d}}{\sqrt{e}} - c x + \frac{i \sqrt{e} x}{\sqrt{d}}}}{2-2 c x}}\right], -\frac{4 i c \sqrt{d} \sqrt{e}}{(c \sqrt{d} - i \sqrt{e})^2}\right] + \right. \right. \\
 & \left. \left. (i c \sqrt{d} - \sqrt{e}) \sqrt{e} \sqrt{\frac{1 + \frac{i c \sqrt{d}}{\sqrt{e}} - c x + \frac{i \sqrt{e} x}{\sqrt{d}}}}{1-c x}} \sqrt{\frac{(c^2 d + e)(d + e x^2)}{d e (-1+c x)^2}} \right. \right. \\
 & \left. \left. \operatorname{EllipticPi}\left[-\frac{2 i \sqrt{e}}{c \sqrt{d} - i \sqrt{e}}, \operatorname{ArcSin}\left[\sqrt{\frac{1 + \frac{i c \sqrt{d}}{\sqrt{e}} - c x + \frac{i \sqrt{e} x}{\sqrt{d}}}}{2-2 c x}}\right], -\frac{4 i c \sqrt{d} \sqrt{e}}{(c \sqrt{d} - i \sqrt{e})^2}\right] \right) \right) / \\
 & \left( e (c^2 d + e) \sqrt{\frac{1 + \frac{i c \sqrt{d}}{\sqrt{e}} - c x + \frac{i \sqrt{e} x}{\sqrt{d}}}}{1-c x}} \sqrt{d + e x^2} \right)
 \end{aligned}$$

**Problem 166: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{(d+e x^2)^{3/2}} dx$$

Optimal (type 4, 92 leaves, 4 steps):

$$\frac{x (a+b \operatorname{ArcSech}[c x])}{d \sqrt{d+e x^2}} + \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}\right]}{c d \sqrt{d+e x^2}}$$

Result (type 4, 334 leaves):

$$\frac{x(a+b \operatorname{ArcSech}[c x])}{d \sqrt{d+e x^2}} + \left( 2 i b \sqrt{\frac{1-c x}{1+c x}} \sqrt{\frac{(c \sqrt{d}+i \sqrt{e})(1+c x)}{(c \sqrt{d}-i \sqrt{e})(-1+c x)}}} (-i \sqrt{d}+\sqrt{e} x) \sqrt{-\frac{-1+\frac{i \sqrt{e} x}{\sqrt{d}}+c\left(\frac{i \sqrt{d}}{\sqrt{e}}+x\right)}{1-c x}} \right. \\ \left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1+\frac{i c \sqrt{d}}{\sqrt{e}}-c x+\frac{i \sqrt{e} x}{\sqrt{d}}}}{2-2 c x}}\right],-\frac{4 i c \sqrt{d} \sqrt{e}}{(c \sqrt{d}-i \sqrt{e})^2}\right] \right) / \\ \left( d(c \sqrt{d}+i \sqrt{e}) \sqrt{\frac{1+\frac{i c \sqrt{d}}{\sqrt{e}}-c x+\frac{i \sqrt{e} x}{\sqrt{d}}}{1-c x}} \sqrt{d+e x^2} \right)$$

**Problem 167: Unable to integrate problem.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{x^2(d+e x^2)^{3/2}} dx$$

Optimal (type 4, 249 leaves, 8 steps):

$$\frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \sqrt{d+e x^2}}{d^2 x} - \frac{a+b \operatorname{ArcSech}[c x]}{d x \sqrt{d+e x^2}} - \frac{2 e x(a+b \operatorname{ArcSech}[c x])}{d^2 \sqrt{d+e x^2}} + \\ \frac{b c \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x^2} \operatorname{EllipticE}\left[\operatorname{ArcSin}[c x],-\frac{e}{c^2 d}\right]}{d^2 \sqrt{1+\frac{e x^2}{d}}} - \frac{1}{c d^2 \sqrt{d+e x^2}} \\ b(c^2 d+2 e) \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}\left[\operatorname{ArcSin}[c x],-\frac{e}{c^2 d}\right]$$

Result (type 8, 25 leaves):

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{x^2(d+e x^2)^{3/2}} dx$$

**Problem 168: Result unnecessarily involves imaginary or complex numbers.**

$$\int \frac{x^5(a+b \operatorname{ArcSech}[c x])}{(d+e x^2)^{5/2}} dx$$

Optimal (type 3, 272 leaves, 10 steps):

$$\begin{aligned}
 & -\frac{b d \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{3 e^2 (c^2 d+e) \sqrt{d+e x^2}} - \frac{d^2 (a+b \operatorname{ArcSech}[c x])}{3 e^3 (d+e x^2)^{3/2}} + \\
 & \frac{2 d (a+b \operatorname{ArcSech}[c x])}{e^3 \sqrt{d+e x^2}} + \frac{\sqrt{d+e x^2} (a+b \operatorname{ArcSech}[c x])}{e^3} - \\
 & \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTan}\left[\frac{\sqrt{e} \sqrt{1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{c e^{5/2}} - \frac{8 b \sqrt{d} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{3 e^3}
 \end{aligned}$$

Result (type 3, 313 leaves):

$$\begin{aligned}
 & \left( -b d e \sqrt{\frac{1-c x}{1+c x}} (1+c x) (d+e x^2) + \right. \\
 & \left. a (c^2 d+e) (8 d^2+12 d e x^2+3 e^2 x^4) + b (c^2 d+e) (8 d^2+12 d e x^2+3 e^2 x^4) \operatorname{ArcSech}[c x] \right) / \\
 & \left( 3 e^3 (c^2 d+e) (d+e x^2)^{3/2} \right) + \frac{1}{6 c e^3 (-1+c x)} i b \sqrt{\frac{1-c x}{1+c x}} \sqrt{-1+c^2 x^2} \\
 & \left( 8 c \sqrt{d} \operatorname{Log}\left[\frac{-i e x^2+i d (-2+c^2 x^2)+2 \sqrt{d} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}}{8 d^{3/2} x^2}\right] + \right. \\
 & \left. 3 i \sqrt{e} \operatorname{Log}\left[-e+2 c \sqrt{e} \sqrt{-1+c^2 x^2} \sqrt{d+e x^2}+c^2 (d+2 e x^2)\right] \right)
 \end{aligned}$$

**Problem 169: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int \frac{x^3 (a+b \operatorname{ArcSech}[c x])}{(d+e x^2)^{5/2}} dx$$

Optimal (type 3, 179 leaves, 7 steps):

$$\begin{aligned}
 & \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{3 e (c^2 d+e) \sqrt{d+e x^2}} + \frac{d (a+b \operatorname{ArcSech}[c x])}{3 e^2 (d+e x^2)^{3/2}} - \\
 & \frac{a+b \operatorname{ArcSech}[c x]}{e^2 \sqrt{d+e x^2}} + \frac{2 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{ArcTanh}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{1-c^2 x^2}}\right]}{3 \sqrt{d} e^2}
 \end{aligned}$$

Result (type 4, 656 leaves):

$$\left( b e \sqrt{\frac{1-c x}{1+c x}} (1+c x) (d+e x^2) - a (c^2 d+e) (2 d+3 e x^2) - \right.$$

$$\left. b (c^2 d+e) (2 d+3 e x^2) \operatorname{ArcSech}[c x] \right) / \left( 3 e^2 (c^2 d+e) (d+e x^2)^{3/2} \right) +$$

$$\left( 4 b (-1+c x) \sqrt{\frac{1-c x}{1+c x}} \sqrt{\frac{(-i c \sqrt{d} + \sqrt{e}) (-1 + \frac{2}{1-c x})}{i c \sqrt{d} + \sqrt{e}}} \right.$$

$$\left( -\frac{1}{-1+c x} i c (c \sqrt{d} - i \sqrt{e}) (-i \sqrt{d} + \sqrt{e} x) \sqrt{-\frac{-1 + \frac{i \sqrt{e} x}{\sqrt{d}} + c \left( \frac{i \sqrt{d}}{\sqrt{e}} + x \right)}{1-c x}} \right.$$

$$\left. \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 + \frac{i c \sqrt{d}}{\sqrt{e}} - c x + \frac{i \sqrt{e} x}{\sqrt{d}}}{2 - 2 c x}}\right], -\frac{4 i c \sqrt{d} \sqrt{e}}{(c \sqrt{d} - i \sqrt{e})^2}\right] + \right.$$

$$\left. (i c \sqrt{d} - \sqrt{e}) \sqrt{e} \sqrt{\frac{1 + \frac{i c \sqrt{d}}{\sqrt{e}} - c x + \frac{i \sqrt{e} x}{\sqrt{d}}}{1-c x}} \sqrt{\frac{(c^2 d+e) (d+e x^2)}{d e (-1+c x)^2}} \right.$$

$$\left. \operatorname{EllipticPi}\left[-\frac{2 i \sqrt{e}}{c \sqrt{d} - i \sqrt{e}}, \operatorname{ArcSin}\left[\sqrt{\frac{1 + \frac{i c \sqrt{d}}{\sqrt{e}} - c x + \frac{i \sqrt{e} x}{\sqrt{d}}}{2 - 2 c x}}\right], -\frac{4 i c \sqrt{d} \sqrt{e}}{(c \sqrt{d} - i \sqrt{e})^2}\right] \right) /$$

$$\left( 3 e^2 (c^2 d+e) \sqrt{\frac{1 + \frac{i c \sqrt{d}}{\sqrt{e}} - c x + \frac{i \sqrt{e} x}{\sqrt{d}}}{1-c x}} \sqrt{d+e x^2} \right)$$

Problem 170: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x (a + b \operatorname{ArcSech}[c x])}{(d + e x^2)^{5/2}} dx$$

Optimal (type 3, 154 leaves, 6 steps):

$$-\frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{3d(c^2d+e)\sqrt{d+ex^2}} - \frac{a+b \operatorname{ArcSech}[cx]}{3e(d+ex^2)^{3/2}} + \frac{b \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \operatorname{ArcTanh}\left[\frac{\sqrt{d+ex^2}}{\sqrt{d}\sqrt{1-c^2x^2}}\right]}{3d^{3/2}e}$$

Result (type 4, 645 leaves):

$$\left( -ad(c^2d+e) - be \sqrt{\frac{1-cx}{1+cx}} (1+cx)(d+ex^2) - bd(c^2d+e) \operatorname{ArcSech}[cx] \right) /$$

$$(3de(c^2d+e)(d+ex^2)^{3/2}) + \left( 2b(-1+cx) \sqrt{\frac{1-cx}{1+cx}} \sqrt{\frac{(-ic\sqrt{d} + \sqrt{e})(-1 + \frac{2}{1-cx})}{ic\sqrt{d} + \sqrt{e}}} \right.$$

$$\left( -\frac{1}{-1+cx} ic(c\sqrt{d} - i\sqrt{e})(-i\sqrt{d} + \sqrt{e}x) \sqrt{-\frac{-1 + \frac{i\sqrt{e}x}{\sqrt{d}} + c\left(\frac{i\sqrt{d}}{\sqrt{e}} + x\right)}{1-cx}} \right.$$

$$\operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\sqrt{\frac{1 + \frac{ic\sqrt{d}}{\sqrt{e}} - cx + \frac{i\sqrt{e}x}{\sqrt{d}}}{2-2cx}}\right], -\frac{4ic\sqrt{d}\sqrt{e}}{(c\sqrt{d} - i\sqrt{e})^2}\right] +$$

$$(ic\sqrt{d} - \sqrt{e})\sqrt{e} \sqrt{\frac{1 + \frac{ic\sqrt{d}}{\sqrt{e}} - cx + \frac{i\sqrt{e}x}{\sqrt{d}}}{1-cx}} \sqrt{\frac{(c^2d+e)(d+ex^2)}{de(-1+cx)^2}}$$

$$\operatorname{EllipticPi}\left[-\frac{2i\sqrt{e}}{c\sqrt{d} - i\sqrt{e}}, \operatorname{ArcSin}\left[\sqrt{\frac{1 + \frac{ic\sqrt{d}}{\sqrt{e}} - cx + \frac{i\sqrt{e}x}{\sqrt{d}}}{2-2cx}}\right], -\frac{4ic\sqrt{d}\sqrt{e}}{(c\sqrt{d} - i\sqrt{e})^2}\right] \right) /$$

$$\left( 3de(c^2d+e) \sqrt{\frac{1 + \frac{ic\sqrt{d}}{\sqrt{e}} - cx + \frac{i\sqrt{e}x}{\sqrt{d}}}{1-cx}} \sqrt{d+ex^2} \right)$$

**Problem 175: Unable to integrate problem.**

$$\int \frac{x^2 (a + b \operatorname{ArcSech}[cx])}{(d + ex^2)^{5/2}} dx$$

Optimal (type 4, 246 leaves, 8 steps):

$$\begin{aligned}
& - \frac{b x \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{3 d (c^2 d+e) \sqrt{d+e x^2}} + \frac{x^3 (a+b \operatorname{ArcSech}[c x])}{3 d (d+e x^2)^{3/2}} - \\
& \frac{b c \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3 d e (c^2 d+e) \sqrt{1+\frac{e x^2}{d}}} + \\
& \frac{b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3 c d e \sqrt{d+e x^2}}
\end{aligned}$$

Result (type 8, 25 leaves):

$$\int \frac{x^2 (a+b \operatorname{ArcSech}[c x])}{(d+e x^2)^{5/2}} dx$$

**Problem 176: Unable to integrate problem.**

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{(d+e x^2)^{5/2}} dx$$

Optimal (type 4, 266 leaves, 8 steps):

$$\begin{aligned}
& \frac{b e x \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{3 d^2 (c^2 d+e) \sqrt{d+e x^2}} + \frac{x (a+b \operatorname{ArcSech}[c x])}{3 d (d+e x^2)^{3/2}} + \\
& \frac{2 x (a+b \operatorname{ArcSech}[c x])}{3 d^2 \sqrt{d+e x^2}} + \frac{b c \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{d+e x^2} \operatorname{EllipticE}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3 d^2 (c^2 d+e) \sqrt{1+\frac{e x^2}{d}}} + \\
& \frac{2 b \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1+\frac{e x^2}{d}} \operatorname{EllipticF}[\operatorname{ArcSin}[c x], -\frac{e}{c^2 d}]}{3 c d^2 \sqrt{d+e x^2}}
\end{aligned}$$

Result (type 8, 22 leaves):

$$\int \frac{a+b \operatorname{ArcSech}[c x]}{(d+e x^2)^{5/2}} dx$$



**Problem 177: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (f x)^m (d + e x^2)^3 (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 5, 596 leaves, 5 steps):

$$\begin{aligned} & - \left( \left( b e \left( e^2 (15 + 8 m + m^2)^2 + 3 c^2 d e (3 + m)^2 (42 + 13 m + m^2) + 3 c^4 d^2 (840 + 638 m + 179 m^2 + 22 m^3 + m^4) \right) \right. \right. \\ & \quad \left. \left. (f x)^{1+m} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \right) / \right. \\ & \quad \left. (c^6 f (2+m) (3+m) (4+m) (5+m) (6+m) (7+m)) \right) - \\ & \left( b e^2 \left( e (5+m)^2 + 3 c^2 d (42 + 13 m + m^2) \right) (f x)^{3+m} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \right) / \\ & \quad (c^4 f^3 (4+m) (5+m) (6+m) (7+m)) - \\ & \frac{b e^3 (f x)^{5+m} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{c^2 f^5 (6+m) (7+m)} + \\ & \frac{d^3 (f x)^{1+m} (a + b \operatorname{ArcSech}[c x])}{f (1+m)} + \\ & \frac{3 d^2 e (f x)^{3+m} (a + b \operatorname{ArcSech}[c x])}{f^3 (3+m)} + \\ & \frac{3 d e^2 (f x)^{5+m} (a + b \operatorname{ArcSech}[c x])}{f^5 (5+m)} + \\ & \frac{e^3 (f x)^{7+m} (a + b \operatorname{ArcSech}[c x])}{f^7 (7+m)} + \\ & \left( b \left( \frac{c^6 d^3 (2+m) (4+m) (6+m)}{1+m} + (e (1+m) \left( e^2 (15 + 8 m + m^2)^2 + 3 c^2 d e (3+m)^2 (42 + 13 m + m^2) \right) \right. \right. \\ & \quad \left. \left. + 3 c^4 d^2 (840 + 638 m + 179 m^2 + 22 m^3 + m^4) \right) \right) / ((3+m) (5+m) (7+m)) (f x)^{1+m} \sqrt{\frac{1}{1+c x}} \\ & \quad \left. \sqrt{1+c x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] \right) / (c^6 f (1+m) (2+m) (4+m) (6+m)) \end{aligned}$$

Result (type 6, 2335 leaves):

$$\frac{a d^3 x (f x)^m}{1+m} + \frac{3 a d^2 e x^3 (f x)^m}{3+m} + \frac{3 a d e^2 x^5 (f x)^m}{5+m} + \frac{a e^3 x^7 (f x)^m}{7+m} + \frac{1}{c} b d^3 (c x)^{-m} (f x)^m$$

$$\left( - \left( \left( 12 (c x)^m \sqrt{\frac{1-c x}{1+c x}} (1+c x) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \right. \right.$$

$$\left. \left( (1+m) (-1+c x) \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \right. \right. \right.$$

$$\left. \left. (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \right. \right. \right.$$

$$\left. \left. \left. \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) \right) \right) +$$

$$\left. \frac{(c x)^{1+m} \operatorname{ArcSech}[c x]}{1+m} \right) + \frac{1}{c} 3 b d^2 e x^2 (c x)^{-2-m} (f x)^m$$

$$\left( - \left( \left( 4 (c x)^m \sqrt{\frac{1-c x}{1+c x}} (1+c x) \left( \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] + (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) \right) + \right. \right.$$

$$\left. \left( 5 (-1+c^2 x^2) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] - 3 (1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) \right) \right) /$$

$$\left. \left( (3+m) (-1+c x) \right) \right) + \frac{(c x)^{3+m} \operatorname{ArcSech}[c x]}{3+m} + \frac{1}{c} 3 b d e^2 x^4 (c x)^{-4-m}$$

$$(f x)^m \left( - \frac{1}{7 (5+m) (-1+c x)} 4 (c x)^m \sqrt{\frac{1-c x}{1+c x}} (1+c x) \right.$$

$$\left. \left( \left( 21 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \right. \right.$$

$$\left. \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] + (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) \right) +$$

$$\begin{aligned}
 & \left( 70 (-1 + c x) (1 + c x) \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] \right) / \\
 & \left( 30 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] - \right. \\
 & \quad 3 (1 + c x) \left( 4 m \operatorname{AppellF1} \left[ \frac{5}{2}, -\frac{1}{2}, 1 - m, \frac{7}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] + \right. \\
 & \quad \quad \left. \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] \right) \left. \right) - \\
 & \left( 98 (-1 + c x) (1 + c x)^2 \operatorname{AppellF1} \left[ \frac{5}{2}, -\frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] \right) / \\
 & \left( 70 \operatorname{AppellF1} \left[ \frac{5}{2}, -\frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] - \right. \\
 & \quad 5 (1 + c x) \left( 4 m \operatorname{AppellF1} \left[ \frac{7}{2}, -\frac{1}{2}, 1 - m, \frac{9}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] + \right. \\
 & \quad \quad \left. \operatorname{AppellF1} \left[ \frac{7}{2}, \frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] \right) \left. \right) - \\
 & \left( 9 (-1 + c x) (1 + c x)^3 \operatorname{AppellF1} \left[ \frac{7}{2}, -\frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] \right) / \\
 & \left( -18 \operatorname{AppellF1} \left[ \frac{7}{2}, -\frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] + \right. \\
 & \quad (1 + c x) \left( 4 m \operatorname{AppellF1} \left[ \frac{9}{2}, -\frac{1}{2}, 1 - m, \frac{11}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] + \right. \\
 & \quad \quad \left. \operatorname{AppellF1} \left[ \frac{9}{2}, \frac{1}{2}, -m, \frac{11}{2}, \frac{1}{2} (1 + c x), 1 + c x \right] \right) \left. \right) + \\
 & \left. \frac{(c x)^{5+m} \operatorname{ArcSech}[c x]}{5+m} \right) + \frac{1}{c} b e^3 x^6 (c x)^{-6-m} (f x)^m \\
 & \left( -\frac{1}{7+m} \left( \left( 12 (c x)^m \sqrt{\frac{1-c x}{1+c x}} (1+c x) \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x \right] \right) / \right. \right. \\
 & \quad \left( (-1+c x) \left( 6 \operatorname{AppellF1} \left[ \frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x \right] + \right. \right. \\
 & \quad \quad (1+c x) \left( -4 m \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x \right] + \right. \\
 & \quad \quad \quad \left. \left. \operatorname{AppellF1} \left[ \frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x \right] \right) \right) \right) \left. \right) + \\
 & \left( 60 (c x)^m (1-c x) \sqrt{\frac{1-c x}{1+c x}} (1+c x)^2 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x \right] \right) / \\
 & \left( (-1+c x) \left( -30 \operatorname{AppellF1} \left[ \frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x \right] + \right. \right. \\
 & \quad 3 (1+c x) \left( 4 m \operatorname{AppellF1} \left[ \frac{5}{2}, -\frac{1}{2}, 1-m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x \right] + \right. \\
 & \quad \quad \left. \left. \operatorname{AppellF1} \left[ \frac{5}{2}, \frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x \right] \right) \right) \left. \right) -
 \end{aligned}$$

$$\begin{aligned}
 & \left( 168 (c x)^m (1 - c x) \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x)^3 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1 + c x), \right. \right. \\
 & \left. \left. 1 + c x\right] \right) / \left( (-1 + c x) \left( -70 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] + \right. \right. \\
 & \left. \left. 5 (1 + c x) \left( 4 m \operatorname{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, 1 - m, \frac{9}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] + \right. \right. \right. \\
 & \left. \left. \left. \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1 + c x), 1 + c x\right]\right)\right)\right) + \\
 & \left( 36 (c x)^m (1 - c x) \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x)^4 \operatorname{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] \right) / \\
 & \left( (-1 + c x) \left( -18 \operatorname{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] + \right. \right. \\
 & \left. \left. (1 + c x) \left( 4 m \operatorname{AppellF1}\left[\frac{9}{2}, -\frac{1}{2}, 1 - m, \frac{11}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] + \right. \right. \right. \\
 & \left. \left. \left. \operatorname{AppellF1}\left[\frac{9}{2}, \frac{1}{2}, -m, \frac{11}{2}, \frac{1}{2} (1 + c x), 1 + c x\right]\right)\right)\right) - \left( 176 (c x)^m (1 - c x) \right. \\
 & \left. \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x)^5 \operatorname{AppellF1}\left[\frac{9}{2}, -\frac{1}{2}, -m, \frac{11}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] \right) / \\
 & \left( (-1 + c x) \left( -22 \operatorname{AppellF1}\left[\frac{9}{2}, -\frac{1}{2}, -m, \frac{11}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] + \right. \right. \\
 & \left. \left. (1 + c x) \left( 4 m \operatorname{AppellF1}\left[\frac{11}{2}, -\frac{1}{2}, 1 - m, \frac{13}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] + \right. \right. \right. \\
 & \left. \left. \left. \operatorname{AppellF1}\left[\frac{11}{2}, \frac{1}{2}, -m, \frac{13}{2}, \frac{1}{2} (1 + c x), 1 + c x\right]\right)\right)\right) + \left( 52 (c x)^m (1 - c x) \right. \\
 & \left. \sqrt{\frac{1 - c x}{1 + c x}} (1 + c x)^6 \operatorname{AppellF1}\left[\frac{11}{2}, -\frac{1}{2}, -m, \frac{13}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] \right) / \\
 & \left( (-1 + c x) \left( -26 \operatorname{AppellF1}\left[\frac{11}{2}, -\frac{1}{2}, -m, \frac{13}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] + \right. \right. \\
 & \left. \left. (1 + c x) \left( 4 m \operatorname{AppellF1}\left[\frac{13}{2}, -\frac{1}{2}, 1 - m, \frac{15}{2}, \frac{1}{2} (1 + c x), 1 + c x\right] + \right. \right. \right. \\
 & \left. \left. \left. \operatorname{AppellF1}\left[\frac{13}{2}, \frac{1}{2}, -m, \frac{15}{2}, \frac{1}{2} (1 + c x), 1 + c x\right]\right)\right)\right) + \frac{(c x)^{7+m} \operatorname{ArcSech}[c x]}{7 + m} \right)
 \end{aligned}$$

Problem 178: Result unnecessarily involves higher level functions and more

than twice size of optimal antiderivative.

$$\int (f x)^m (d + e x^2)^2 (a + b \operatorname{ArcSech}[c x]) dx$$

Optimal (type 5, 372 leaves, 5 steps):

$$\begin{aligned} & - \left( \left( b e (e (3+m)^2 + 2 c^2 d (2\theta + 9 m + m^2)) (f x)^{1+m} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2} \right) / \right. \\ & \quad \left. (c^4 f (2+m) (3+m) (4+m) (5+m)) \right) - \frac{b e^2 (f x)^{3+m} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \sqrt{1-c^2 x^2}}{c^2 f^3 (4+m) (5+m)} + \\ & \quad \frac{d^2 (f x)^{1+m} (a + b \operatorname{ArcSech}[c x])}{f (1+m)} + \frac{2 d e (f x)^{3+m} (a + b \operatorname{ArcSech}[c x])}{f^3 (3+m)} + \\ & \quad \frac{e^2 (f x)^{5+m} (a + b \operatorname{ArcSech}[c x])}{f^5 (5+m)} + \\ & \quad \left( b (c^4 d^2 (2+m) (3+m) (4+m) (5+m) + e (1+m)^2 (e (3+m)^2 + 2 c^2 d (2\theta + 9 m + m^2))) \right) \\ & \quad (f x)^{1+m} \sqrt{\frac{1}{1+c x}} \sqrt{1+c x} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2 x^2\right] / \\ & \quad (c^4 f (1+m)^2 (2+m) (3+m) (4+m) (5+m)) \end{aligned}$$

Result (type 6, 1240 leaves):

$$\begin{aligned} & \frac{a d^2 x (f x)^m}{1+m} + \frac{2 a d e x^3 (f x)^m}{3+m} + \frac{a e^2 x^5 (f x)^m}{5+m} + \frac{1}{c} b d^2 (c x)^{-m} (f x)^m \\ & - \left( \left( \left( 12 (c x)^m \sqrt{\frac{1-c x}{1+c x}} (1+c x) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \right. \right. \\ & \quad \left( (1+m) (-1+c x) \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \right. \right. \\ & \quad \left. \left. (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \right. \right. \right. \right. \\ & \quad \left. \left. \left. \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) \right) \right) + \frac{(c x)^{1+m} \operatorname{ArcSech}[c x]}{1+m} \left. \right) + \frac{1}{c} 2 b d e x^2 (c x)^{-2-m} (f x)^m \\ & - \left( \left( \left( 4 (c x)^m \sqrt{\frac{1-c x}{1+c x}} (1+c x) \left( \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \left( 6 \operatorname{AppellF1}\left[ \right. \right. \right. \right. \right. \\ & \quad \left. \left. \left. \frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] + (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \right. \right. \right. \right. \right. \end{aligned}$$

$$\begin{aligned}
 & \left. \left( \frac{1}{2} (1+c x), 1+c x \right) + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) + \\
 & \left( 5 (-1+c^2 x^2) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] - 3 (1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) \right) / \\
 & \left( (3+m) (-1+c x) \right) + \frac{(c x)^{3+m} \operatorname{ArcSech}[c x]}{3+m} + \frac{1}{c} b e^{2 x^4} (c x)^{-4-m} \\
 (f x)^m & \left( -\frac{1}{7(5+m)(-1+c x)} 4(c x)^m \sqrt{\frac{1-c x}{1+c x}} (1+c x) \right. \\
 & \left( \left( 21 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \right. \\
 & \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2} (1+c x), 1+c x\right] + (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) \right) + \\
 & \left( 70 (-1+c x) (1+c x) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \\
 & \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2} (1+c x), 1+c x\right] - \right. \\
 & \left. 3 (1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) - \\
 & \left( 98 (-1+c x) (1+c x)^2 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \\
 & \left( 70 \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2} (1+c x), 1+c x\right] - \right. \\
 & \left. 5 (1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, 1-m, \frac{9}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{7}{2}, \frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) \right) - \\
 & \left( 9 (-1+c x) (1+c x)^3 \operatorname{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1+c x), 1+c x\right] \right) / \\
 & \left( -18 \operatorname{AppellF1}\left[\frac{7}{2}, -\frac{1}{2}, -m, \frac{9}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \right. \\
 & \left. (1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{9}{2}, -\frac{1}{2}, 1-m, \frac{11}{2}, \frac{1}{2} (1+c x), 1+c x\right] + \right. \right.
 \end{aligned}$$

$$\text{AppellF1}\left[\frac{9}{2}, \frac{1}{2}, -m, \frac{11}{2}, \frac{1}{2}(1+cx), 1+cx\right] + \frac{(cx)^{5+m} \text{ArcSech}[cx]}{5+m}$$

**Problem 179: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.**

$$\int (fx)^m (d+ex^2) (a+b \text{ArcSech}[cx]) dx$$

Optimal (type 5, 206 leaves, 4 steps):

$$\begin{aligned} & -\frac{be(fx)^{1+m} \sqrt{\frac{1}{1+cx}} \sqrt{1+cx} \sqrt{1-c^2x^2}}{c^2 f (2+m) (3+m)} + \frac{d(fx)^{1+m} (a+b \text{ArcSech}[cx])}{f (1+m)} + \\ & \frac{e(fx)^{3+m} (a+b \text{ArcSech}[cx])}{f^3 (3+m)} + \left( b \left( e(1+m)^2 + c^2 d (2+m) (3+m) \right) (fx)^{1+m} \sqrt{\frac{1}{1+cx}} \right. \\ & \left. \sqrt{1+cx} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{1+m}{2}, \frac{3+m}{2}, c^2x^2\right] \right) / \left( c^2 f (1+m)^2 (2+m) (3+m) \right) \end{aligned}$$

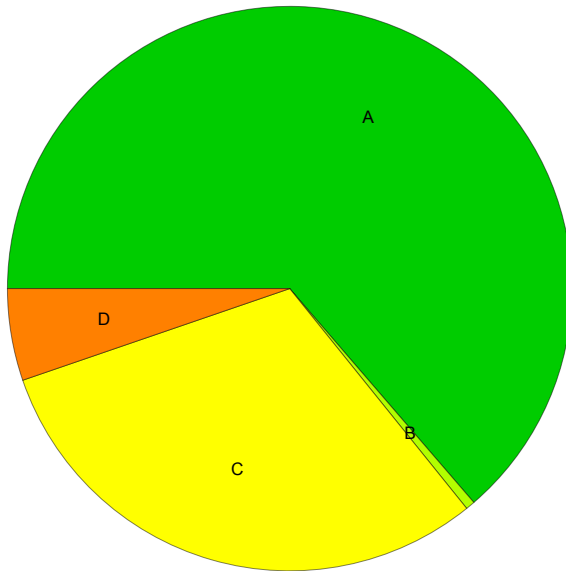
Result (type 6, 529 leaves):

$$\begin{aligned}
 & (f x)^m \left( \frac{a d x}{1+m} + \frac{a e x^3}{3+m} - \right. \\
 & \left. \left( 12 b d \sqrt{\frac{1-c x}{1+c x}} (1+c x) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1+c x), 1+c x\right] \right) / (c(1+m)(-1+c x)) \right. \\
 & \left. \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1+c x), 1+c x\right] + (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{5}{2}, \frac{1}{2}(1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2}(1+c x), 1+c x\right] \right) \right) \right) - \\
 & \left( 4 b e \sqrt{\frac{1-c x}{1+c x}} (1+c x) \left( \left( 3 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1+c x), 1+c x\right] \right) / \right. \right. \\
 & \left. \left( 6 \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -m, \frac{3}{2}, \frac{1}{2}(1+c x), 1+c x\right] + (1+c x) \left( -4 m \operatorname{AppellF1}\left[\frac{3}{2}, \frac{1}{2}, 1-m, \right. \right. \right. \right. \\
 & \left. \left. \left. \frac{5}{2}, \frac{1}{2}(1+c x), 1+c x\right] + \operatorname{AppellF1}\left[\frac{3}{2}, \frac{3}{2}, -m, \frac{5}{2}, \frac{1}{2}(1+c x), 1+c x\right] \right) \right) \right) + \\
 & \left( 5(-1+c^2 x^2) \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2}(1+c x), 1+c x\right] \right) / \\
 & \left( 30 \operatorname{AppellF1}\left[\frac{3}{2}, -\frac{1}{2}, -m, \frac{5}{2}, \frac{1}{2}(1+c x), 1+c x\right] - \right. \\
 & \left. 3(1+c x) \left( 4 m \operatorname{AppellF1}\left[\frac{5}{2}, -\frac{1}{2}, 1-m, \frac{7}{2}, \frac{1}{2}(1+c x), 1+c x\right] + \right. \right. \\
 & \left. \left. \operatorname{AppellF1}\left[\frac{5}{2}, \frac{1}{2}, -m, \frac{7}{2}, \frac{1}{2}(1+c x), 1+c x\right] \right) \right) \right) / \\
 & \left. \left( c^3(3+m)(-1+c x) \right) + \frac{b d x \operatorname{ArcSech}[c x]}{1+m} + \frac{b e x^3 \operatorname{ArcSech}[c x]}{3+m} \right)
 \end{aligned}$$



## Summary of Integration Test Results

190 integration problems



- A - 121 optimal antiderivatives
- B - 1 more than twice size of optimal antiderivatives
- C - 58 unnecessarily complex antiderivatives
- D - 10 unable to integrate problems
- E - 0 integration timeouts