Mathematica 11.3 Integration Test Results

Test results for the 178 problems in "7.6.1 u (a+b arccsch(c x))^n.m"

Problem 19: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(a + b\,\text{ArcCsch}[c\,x])^2}{x}\,dx$$

Optimal (type 4, 81 leaves, 6 steps):

$$\frac{(a + b\,\text{ArcCsch}[c\,x])^3}{3\,b} - (a + b\,\text{ArcCsch}[c\,x])^2\,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right] - b\,(a + b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right] + \frac{1}{2}\,b^2\,\text{PolyLog}\left[3, e^{2\,\text{ArcCsch}[c\,x]}\right]$$

Result (type 4, 121 leaves):

$$a^2\,\text{Log}[c\,x] + a\,b\,\left(-\text{ArcCsch}[c\,x]\,\left(\text{ArcCsch}[c\,x] + 2\,\text{Log}\left[1 - e^{-2\,\text{ArcCsch}[c\,x]}\right]\right) + \text{PolyLog}\left[2, e^{-2\,\text{ArcCsch}[c\,x]}\right]\right) + \frac{1}{24}\,b^2\,\left(-i\,\pi^3 + 8\,\text{ArcCsch}[c\,x]^3 - 24\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right] - 24\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right] + 12\,\text{PolyLog}\left[3, e^{2\,\text{ArcCsch}[c\,x]}\right]\right)$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int x^2\,(a + b\,\text{ArcCsch}[c\,x])^3\,dx$$

Optimal (type 4, 194 leaves, 11 steps):

$$\frac{b^2\,x\,(a + b\,\text{ArcCsch}[c\,x])}{c^2} + \frac{b\,\sqrt{1 + \frac{1}{c^2\,x^2}}\,x^2\,(a + b\,\text{ArcCsch}[c\,x])^2}{2\,c} + \frac{1}{3}\,x^3\,(a + b\,\text{ArcCsch}[c\,x])^3 - \frac{b\,(a + b\,\text{ArcCsch}[c\,x])^2\,\text{ArcTanh}\left[e^{\text{ArcCsch}[c\,x]}\right]}{c^3} + \frac{b^3\,\text{ArcTanh}\left[\sqrt{1 + \frac{1}{c^2\,x^2}}\right]}{c^3} - \frac{b^2\,(a + b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[2, -e^{\text{ArcCsch}[c\,x]}\right]}{c^3} + \frac{b^2\,(a + b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[2, e^{\text{ArcCsch}[c\,x]}\right]}{c^3} + \frac{b^3\,\text{PolyLog}\left[3, -e^{\text{ArcCsch}[c\,x]}\right]}{c^3} - \frac{b^3\,\text{PolyLog}\left[3, e^{\text{ArcCsch}[c\,x]}\right]}{c^3}$$

Result (type 4, 548 leaves):

$$\frac{a^3 x^3}{3}+\frac{a^2 b x^2 \sqrt{\frac{1+c^2 x^2}{c^2 x^2}}}{2 c}+a^2 b x^3 \text{ArcCsch}[c x]-\frac{a^2 b \text{Log}\left[x \left(1+\sqrt{\frac{1+c^2 x^2}{c^2 x^2}}\right)\right]}{2 c^3}+\frac{1}{8 c^3} a b^2 \left(8 \text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c x]}\right]+2 c^3 x^3 \left(-2+4 \text{ArcCsch}[c x]^2+2 \text{Cosh}[2 \text{ArcCsch}[c x]]-\frac{3 \text{ArcCsch}[c x] \text{Log}\left[1-e^{-\text{ArcCsch}[c x]}\right]}{c x}+\frac{3 \text{ArcCsch}[c x] \text{Log}\left[1+e^{-\text{ArcCsch}[c x]}\right]}{c x}-\frac{4 \text{PolyLog}\left[2,e^{-\text{ArcCsch}[c x]}\right]}{c^3 x^3}+2 \text{ArcCsch}[c x] \text{Sinh}[2 \text{ArcCsch}[c x]]+\text{ArcCsch}[c x] \text{Log}\left[1-e^{-\text{ArcCsch}[c x]}\right] \text{Sinh}[3 \text{ArcCsch}[c x]]-\text{ArcCsch}[c x] \text{Log}\left[1+e^{-\text{ArcCsch}[c x]}\right] \text{Sinh}[3 \text{ArcCsch}[c x]]\right)\right)+\frac{1}{48 c^3} b^3 \left(24 \text{ArcCsch}[c x] \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]-4 \text{ArcCsch}[c x]^3 \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]+6 \text{ArcCsch}[c x]^2 \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]^2+\frac{\text{ArcCsch}[c x]^3 \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]^4}{c x}+24 \text{ArcCsch}[c x]^2 \text{Log}\left[1-e^{-\text{ArcCsch}[c x]}\right]-24 \text{ArcCsch}[c x]^2 \text{Log}\left[1+e^{-\text{ArcCsch}[c x]}\right]-48 \text{Log}\left[\text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]\right]+48 \text{ArcCsch}[c x] \text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c x]}\right]-48 \text{ArcCsch}[c x] \text{PolyLog}\left[2,e^{-\text{ArcCsch}[c x]}\right]+48 \text{PolyLog}\left[3,-e^{-\text{ArcCsch}[c x]}\right]-48 \text{PolyLog}\left[3,e^{-\text{ArcCsch}[c x]}\right]+6 \text{ArcCsch}[c x]^2 \text{Sech}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]^2+16 c^3 x^3 \text{ArcCsch}[c x]^3 \text{Sinh}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]^4-24 \text{ArcCsch}[c x] \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]+4 \text{ArcCsch}[c x]^3 \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c x]\right]\right)$$

Problem 27: Result more than twice size of optimal antiderivative.

$$\int (a+b \text{ArcCsch}[c x])^3 \, dx$$

Optimal (type 4, 120 leaves, 9 steps):

$$x\,(a+b\,\text{ArcCsch}[c\,x])^3+\frac{6\,b\,(a+b\,\text{ArcCsch}[c\,x])^2\,\text{ArcTanh}\left[e^{\text{ArcCsch}[c\,x]}\right]}{c}+\frac{6\,b^2\,(a+b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[2,-e^{\text{ArcCsch}[c\,x]}\right]}{c}-\frac{6\,b^2\,(a+b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[2,e^{\text{ArcCsch}[c\,x]}\right]}{c}-\frac{6\,b^3\,\text{PolyLog}\left[3,-e^{\text{ArcCsch}[c\,x]}\right]}{c}+\frac{6\,b^3\,\text{PolyLog}\left[3,e^{\text{ArcCsch}[c\,x]}\right]}{c}$$

Result (type 4, 246 leaves):

$$a^3\,x+3\,a^2\,b\,x\,\text{ArcCsch}[c\,x]+\frac{3\,a^2\,b\,\text{Log}\left[c\,x\left(1+\sqrt{\frac{1+c^2\,x^2}{c^2\,x^2}}\right)\right]}{c}+\frac{1}{c}3\,a\,b^2\left(\text{ArcCsch}[c\,x]\left(c\,x\,\text{ArcCsch}[c\,x]-2\,\text{Log}\left[1-e^{-\text{ArcCsch}[c\,x]}\right]+2\,\text{Log}\left[1+e^{-\text{ArcCsch}[c\,x]}\right]\right)-2\,\text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c\,x]}\right]+2\,\text{PolyLog}\left[2,e^{-\text{ArcCsch}[c\,x]}\right]\right)+\frac{1}{c}b^3\left(c\,x\,\text{ArcCsch}[c\,x]^3-3\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1-e^{-\text{ArcCsch}[c\,x]}\right]+3\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1+e^{-\text{ArcCsch}[c\,x]}\right]-6\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2,-e^{-\text{ArcCsch}[c\,x]}\right]+6\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2,e^{-\text{ArcCsch}[c\,x]}\right]-6\,\text{PolyLog}\left[3,-e^{-\text{ArcCsch}[c\,x]}\right]+6\,\text{PolyLog}\left[3,e^{-\text{ArcCsch}[c\,x]}\right]\right)$$

Problem 28: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{(a+b\,\text{ArcCsch}[c\,x])^3}{x}\,dx$$

Optimal (type 4, 110 leaves, 7 steps):

$$\frac{(a+b\,\text{ArcCsch}[c\,x])^4}{4\,b}-(a+b\,\text{ArcCsch}[c\,x])^3\,\text{Log}\left[1-e^{2\,\text{ArcCsch}[c\,x]}\right]-\frac{3}{2}\,b\,(a+b\,\text{ArcCsch}[c\,x])^2\,\text{PolyLog}\left[2,e^{2\,\text{ArcCsch}[c\,x]}\right]+\frac{3}{2}\,b^2\,(a+b\,\text{ArcCsch}[c\,x])\,\text{PolyLog}\left[3,e^{2\,\text{ArcCsch}[c\,x]}\right]-\frac{3}{4}\,b^3\,\text{PolyLog}\left[4,e^{2\,\text{ArcCsch}[c\,x]}\right]$$

Result (type 4, 213 leaves):

$$a^3\,\text{Log}[c\,x]+\frac{3}{2}\,a^2\,b\left(-\text{ArcCsch}[c\,x]\left(\text{ArcCsch}[c\,x]+2\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]\right)+\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]\right)+\frac{1}{8}\,a\,b^2\left(-i\,\pi^3+8\,\text{ArcCsch}[c\,x]^3-24\,\text{ArcCsch}[c\,x]^2\,\text{Log}\left[1-e^{2\,\text{ArcCsch}[c\,x]}\right]-24\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[2,e^{2\,\text{ArcCsch}[c\,x]}\right]+12\,\text{PolyLog}\left[3,e^{2\,\text{ArcCsch}[c\,x]}\right]\right)-\frac{1}{64}\,b^3\left(\pi^4-16\,\text{ArcCsch}[c\,x]^4+64\,\text{ArcCsch}[c\,x]^3\,\text{Log}\left[1-e^{2\,\text{ArcCsch}[c\,x]}\right]+96\,\text{ArcCsch}[c\,x]^2\,\text{PolyLog}\left[2,e^{2\,\text{ArcCsch}[c\,x]}\right]-96\,\text{ArcCsch}[c\,x]\,\text{PolyLog}\left[3,e^{2\,\text{ArcCsch}[c\,x]}\right]+48\,\text{PolyLog}\left[4,e^{2\,\text{ArcCsch}[c\,x]}\right]\right)$$

Problem 48: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a + b \,\text{ArcCsch}[c\, x]}{d + e\, x} \, dx$$

Optimal (type 4, 215 leaves, 4 steps):

$$\frac{\left(a + b\,\text{ArcCsch}[c\,x]\right) \text{Log}\left[1 - \frac{\left(e - \sqrt{c^2\,d^2 + e^2}\right) e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]}{e} + \frac{\left(a + b\,\text{ArcCsch}[c\,x]\right) \text{Log}\left[1 - \frac{\left(e + \sqrt{c^2\,d^2 + e^2}\right) e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]}{e} - \frac{\left(a + b\,\text{ArcCsch}[c\,x]\right) \text{Log}\left[1 - e^{2\,\text{ArcCsch}[c\,x]}\right]}{e} + \frac{b\,\text{PolyLog}\left[2, \frac{\left(e - \sqrt{c^2\,d^2 + e^2}\right) e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]}{e} + \frac{b\,\text{PolyLog}\left[2, \frac{\left(e + \sqrt{c^2\,d^2 + e^2}\right) e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]}{e} - \frac{b\,\text{PolyLog}\left[2, e^{2\,\text{ArcCsch}[c\,x]}\right]}{2\,e}$$

Result (type 4, 506 leaves):

$$\frac{a\,\text{Log}[d+e\,x]}{e}+\frac{1}{8\,e}b\left(\pi^2-4\,\mathrm{i}\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\,e}{c\,d}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{(\mathrm{i}\,c\,d+e)\,\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\mathrm{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{c^2\,d^2+e^2}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,\mathrm{i}\,\pi\,\text{Log}\left[1+\frac{\left(-e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{\left(-e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+16\,\mathrm{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\,e}{c\,d}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\left(-e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+4\,\mathrm{i}\,\pi\,\text{Log}\left[1-\frac{\left(e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{\left(e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]-16\,\mathrm{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\mathrm{i}\,e}{c\,d}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{\left(e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]-4\,\mathrm{i}\,\pi\,\text{Log}\left[e+\frac{d}{x}\right]+4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\,\frac{\left(e-\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]+8\,\text{PolyLog}\left[2,\,\frac{\left(e+\sqrt{c^2\,d^2+e^2}\right)e^{\text{ArcCsch}[c\,x]}}{c\,d}\right]\right)$$

Problem 51: Result unnecessarily involves imaginary or complex numbers.

$$\int x^2\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 4, 913 leaves, 31 steps):

$$\frac{4\,b\,\sqrt{d+e\,x}\,\left(1+c^2\,x^2\right)}{35\,c^3\,\sqrt{1+\frac{1}{c^2\,x^2}}}+\frac{8\,b\,d\,\sqrt{d+e\,x}\,\left(1+c^2\,x^2\right)}{105\,c^3\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}+\frac{2\,d^2\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^3}-$$

$$\frac{4\,d\,\left(d+e\,x\right)^{5/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5\,e^3}+\frac{2\,\left(d+e\,x\right)^{7/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{7\,e^3}-$$

$$\left(4\,b\,c\,d^2\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Bigg/$$

$$\left(35\,\left(-c^2\right)^{3/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\right)+$$

$$\left(4\,b\,c\,\left(2\,c^2\,d^2+9\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Bigg/$$

$$\left(105\,\left(-c^2\right)^{5/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\right)+$$

$$\left(32\,b\,c\,d^3\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Bigg/$$

$$\left(105\,\left(-c^2\right)^{3/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)-$$

$$\left(4\,b\,c\,d\,\left(c^2\,d^2+e^2\right)\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\right.\right.$$

$$\left.\left.-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Bigg/\left(105\,\left(-c^2\right)^{5/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)-$$

$$\left(32\,b\,d^4\,\sqrt{\frac{\sqrt{-c^2}\,\left(d+e\,x\right)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]\right)\Bigg/$$

$$\left(105\,c\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)$$

Result (type 4, 483 leaves) :

$$\frac{1}{105\,e^3}\,2\left(\frac{2\,b\,e^2\sqrt{1+\frac{1}{c^2 x^2}}\;x\,\sqrt{d+e\,x}\;\left(2\,d+3\,e\,x\right)}{c}+a\,\sqrt{d+e\,x}\;\left(8\,d^3-4\,d^2\,e\,x+3\,d\,e^2\,x^2+15\,e^3\,x^3\right)+\right.$$

$$b\,\sqrt{d+e\,x}\;\left(8\,d^3-4\,d^2\,e\,x+3\,d\,e^2\,x^2+15\,e^3\,x^3\right)\text{ArcCsch}[c\,x]+\frac{1}{c^4\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{1+\frac{1}{c^2 x^2}}\;x}$$

$$2\,b\,\sqrt{-\frac{e\,\left(-i+c\,x\right)}{c\,d+i\,e}}\;\sqrt{-\frac{e\,\left(i+c\,x\right)}{c\,d-i\,e}}\;\left(\left(-5\,i\,c^3\,d^3+5\,c^2\,d^2\,e-9\,i\,c\,d\,e^2+9\,e^3\right)\text{EllipticE}\left[\right.\right.$$

$$i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\left(-4\,i\,c^3\,d^3-5\,c^2\,d^2\,e+8\,i\,c\,d\,e^2-9\,e^3\right)$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+$$

$$\left.\left.8\,i\,c^3\,d^3\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)$$

Problem 52: Result unnecessarily involves imaginary or complex numbers.

$$\int x\,\sqrt{d+e\,x}\;\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 4, 679 leaves, 24 steps):

$$\frac{4\,b\,\sqrt{d+e\,x}\,\left(1+c^2\,x^2\right)}{15\,c^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}-\frac{2\,d\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{3\,e^2}+\frac{2\,\left(d+e\,x\right)^{5/2}\,\left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{5\,e^2}+$$

$$\left(8\,b\,c\,d\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(15\,\left(-c^2\right)^{3/2}\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\right)-$$

$$\left(8\,b\,c\,d^2\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(15\,\left(-c^2\right)^{3/2}\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)+$$

$$\left(4\,b\,c\,\left(c^2\,d^2+e^2\right)\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\right.\right.$$

$$\left.\left.-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Big/\left(15\,\left(-c^2\right)^{5/2}\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)+$$

$$\left(8\,b\,d^3\,\sqrt{\frac{\sqrt{-c^2}\,\left(d+e\,x\right)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\mathrm{EllipticPi}\left[2,\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]\right)\Big/$$

$$\left(15\,c\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)$$

Result (type 4, 418 leaves):

$$\frac{1}{15}\left(\frac{4\,b\,\sqrt{1+\frac{1}{c^2 x^2}}\,x\,\sqrt{d+e\,x}}{c}+\frac{2\,a\,\sqrt{d+e\,x}\,\left(-2\,d^2+d\,e\,x+3\,e^2\,x^2\right)}{e^2}+\right.$$

$$\frac{2\,b\,\sqrt{d+e\,x}\,\left(-2\,d^2+d\,e\,x+3\,e^2\,x^2\right)\,\text{ArcCsch}[c\,x]}{e^2}+\left(4\,i\,b\,\sqrt{-\frac{e\,(-i+c\,x)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,(i+c\,x)}{c\,d-i\,e}}\right.$$

$$\left(2\,c\,d\,(c\,d+i\,e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\right.$$

$$\left(c^2\,d^2-2\,i\,c\,d\,e+e^2\right)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-$$

$$\left.\left.2\,c^2\,d^2\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)\Bigg/$$

$$\left.\left(c^3\,\sqrt{-\frac{c}{c\,d-i\,e}}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 53: Result unnecessarily involves imaginary or complex numbers.

$$\int\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 4, 429 leaves, 15 steps):

$$\frac{2\,(d+e\,x)^{3/2}\,(a+b\,\text{ArcCsch}[c\,x])}{3\,e}+$$

$$\left(4\,b\,c\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(3\,(-c^2)^{3/2}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}\right)+$$

$$\left(4\,b\,c\,d\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(3\,(-c^2)^{3/2}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)-$$

$$\left(4\,b\,d^2\,\sqrt{\frac{\sqrt{-c^2}\,(d+e\,x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]\right)\Big/$$

$$\left(3\,c\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)$$

Result (type 4, 329 leaves):

$$\frac{1}{3\,e}2\left(a\,(d+e\,x)^{3/2}+b\,(d+e\,x)^{3/2}\,\text{ArcCsch}[c\,x]+\right.$$

$$\left(2\,b\,\sqrt{-\frac{e\,(-i+c\,x)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,(i+c\,x)}{c\,d-i\,e}}\,\left((i\,c\,d-e)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\right.\right.\right.$$

$$\left.\frac{c\,d-i\,e}{c\,d+i\,e}\right]+(-2\,i\,c\,d+e)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+$$

$$\left.\left.i\,c\,d\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)\Big/$$

$$\left.\left(c^2\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 56: Result unnecessarily involves imaginary or complex numbers.

$$\int (d+e x)^{3/2} \left(a+b \text{ArcCsch}[c x]\right) dx$$

Optimal (type 4, 486 leaves, 22 steps):

$$\frac{4 b e \sqrt{d+e x} \left(1+c^2 x^2\right)}{15 c^3 \sqrt{1+\frac{1}{c^2 x^2}}\, x}+\frac{2 (d+e x)^{5/2} \left(a+b \text{ArcCsch}[c x]\right)}{5 e}+$$

$$\left(28 b c d \sqrt{d+e x} \sqrt{1+c^2 x^2} \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right], -\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right) \Big/$$

$$\left(15 \left(-c^2\right)^{3/2} \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{\frac{d+e x}{d+\frac{e}{\sqrt{-c^2}}}}\right) -$$

$$\left(4 b c \left(2 c^2 d^2-e^2\right) \sqrt{\frac{d+e x}{d+\frac{e}{\sqrt{-c^2}}}} \sqrt{1+c^2 x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right], -\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right) \Big/$$

$$\left(15 \left(-c^2\right)^{5/2} \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right) -$$

$$\left(4 b d^3 \sqrt{\frac{\sqrt{-c^2}\, (d+e x)}{\sqrt{-c^2}\, d+e}} \sqrt{1+c^2 x^2} \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right], \frac{2 e}{\sqrt{-c^2}\, d+e}\right]\right) \Big/$$

$$\left(5 c e \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)$$

Result (type 4, 380 leaves):

$$\frac{1}{15 e}2\left(\frac{2 b e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}{c}+3 a (d+e x)^{5/2}+3 b (d+e x)^{5/2} \text{ArcCsch}[c x]+\left(2 i b \sqrt{-\frac{e (-i+c x)}{c d+i e}} \sqrt{-\frac{e (i+c x)}{c d-i e}}\left(7 c d (c d+i e) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right]+\left(-9 c^2 d^2-7 i c d e+e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right]+3 c^2 d^2 \text{EllipticPi}\left[1-\frac{i e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right]\right)\right)\Big/\left(c^3 \sqrt{-\frac{c}{c d-i e}} \sqrt{1+\frac{1}{c^2 x^2}}\, x\right)\right)$$

Problem 57: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^3 (a+b \text{ArcCsch}[c x])}{\sqrt{d+e x}} \, dx$$

Optimal (type 4, 939 leaves, 27 steps):

$$\frac{4 b \sqrt{d+e x}\,\left(1+c^2 x^2\right)}{35 c^3 e \sqrt{1+\frac{1}{c^2 x^2}}}-\frac{4 b d \sqrt{d+e x}\,\left(1+c^2 x^2\right)}{21 c^3 e^2 \sqrt{1+\frac{1}{c^2 x^2}}\,x}-$$

$$\frac{2 d^3 \sqrt{d+e x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4}+\frac{2 d^2 \left(d+e x\right)^{3/2} \left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^4}-$$

$$\frac{6 d \left(d+e x\right)^{5/2} \left(a+b\,\text{ArcCsch}[c\,x]\right)}{5 e^4}+\frac{2 \left(d+e x\right)^{7/2} \left(a+b\,\text{ArcCsch}[c\,x]\right)}{7 e^4}+$$

$$\left(24 b c d^2 \sqrt{d+e x}\,\sqrt{1+c^2 x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(35 \left(-c^2\right)^{3/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{\frac{c^2 \left(d+e x\right)}{c^2 d-\sqrt{-c^2}\,e}}\right)+$$

$$\left(4 b c \left(2 c^2 d^2+9 e^2\right) \sqrt{d+e x}\,\sqrt{1+c^2 x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(105 \left(-c^2\right)^{5/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{\frac{c^2 \left(d+e x\right)}{c^2 d-\sqrt{-c^2}\,e}}\right)-$$

$$\left(64 b c d^3 \sqrt{\frac{c^2 \left(d+e x\right)}{c^2 d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(35 \left(-c^2\right)^{3/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{d+e x}\right)-$$

$$\left(32 b c d \left(c^2 d^2+e^2\right) \sqrt{\frac{c^2 \left(d+e x\right)}{c^2 d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\right.\right.$$

$$\left.\left.-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/\left(105 \left(-c^2\right)^{5/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{d+e x}\right)+$$

$$\left(64 b d^4 \sqrt{\frac{\sqrt{-c^2}\,\left(d+e x\right)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\,d+e}\right]\right)\Big/$$

$$\left(35 c e^4 \sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{d+e x}\right)$$

Result (type 4, 485 leaves):

$$\frac{1}{105\,e^4}\,2\left(\frac{2\,b\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}\,\left(-5\,d+3\,e\,x\right)}{c}+3\,a\,\sqrt{d+e\,x}\,\left(-16\,d^3+8\,d^2\,e\,x-6\,d\,e^2\,x^2+5\,e^3\,x^3\right)+\right.$$

$$3\,b\,\sqrt{d+e\,x}\,\left(-16\,d^3+8\,d^2\,e\,x-6\,d\,e^2\,x^2+5\,e^3\,x^3\right)\,\text{ArcCsch}[c\,x]+\frac{1}{c^4\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x}$$

$$2\,b\,\sqrt{-\frac{e\,\left(-i+c\,x\right)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,\left(i+c\,x\right)}{c\,d-i\,e}}\,\left(\left(16\,i\,c^3\,d^3-16\,c^2\,d^2\,e-9\,i\,c\,d\,e^2+9\,e^3\right)\,\text{EllipticE}\right[$$

$$i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\,\frac{c\,d-i\,e}{c\,d+i\,e}\Big]+\left(24\,i\,c^3\,d^3+16\,c^2\,d^2\,e+i\,c\,d\,e^2-9\,e^3\right)$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\,\frac{c\,d-i\,e}{c\,d+i\,e}\right]-$$

$$\left.\left.48\,i\,c^3\,d^3\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\,\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)$$

Problem 58: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\sqrt{d+e\,x}}\,dx$$

Optimal (type 4, 707 leaves, 20 steps):

$$\frac{4 b \sqrt{d+e x}\left(1+c^2 x^2\right)}{15 c^3 e \sqrt{1+\frac{1}{c^2 x^2}}\, x}+\frac{2 d^2 \sqrt{d+e x}\left(a+b \,\text{ArcCsch}[c x]\right)}{e^3}-$$

$$\frac{4 d (d+e x)^{3/2}\left(a+b \,\text{ArcCsch}[c x]\right)}{3 e^3}+\frac{2 (d+e x)^{5/2}\left(a+b \,\text{ArcCsch}[c x]\right)}{5 e^3}-$$

$$\left(4 b c d \sqrt{d+e x} \sqrt{1+c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right)\Big/$$

$$\left(5 \left(-c^2\right)^{3/2} e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}}\right)+$$

$$\left(32 b c d^2 \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}} \sqrt{1+c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right)\Big/$$

$$\left(15 \left(-c^2\right)^{3/2} e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)+$$

$$\left(4 b c \left(c^2 d^2+e^2\right) \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}} \sqrt{1+c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],\right.\right.$$

$$\left.\left.-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right)\Big/\left(15 \left(-c^2\right)^{5/2} e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)-$$

$$\left(32 b d^3 \sqrt{\frac{\sqrt{-c^2}\, (d+e x)}{\sqrt{-c^2}\, d+e}} \sqrt{1+c^2 x^2}\, \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right], \frac{2 e}{\sqrt{-c^2}\, d+e}\right]\right)\Big/$$

$$\left(15 c e^3 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)$$

Result (type 4, 419 leaves) :

$$\frac{1}{15 e^3} 2 \left(\frac{2 b e^2 \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}}{c} + a \sqrt{d+e x} \left(8 d^2 - 4 d e x + 3 e^2 x^2\right) + b \sqrt{d+e x} \left(8 d^2 - 4 d e x + 3 e^2 x^2\right) \text{ArcCsch}[c x] + \left(2 b \sqrt{-\frac{e (-i + c x)}{c d + i e}} \sqrt{-\frac{e (i + c x)}{c d - i e}} \left(3 c d (-i c d + e) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d - i e}} \sqrt{d+e x}\right], \frac{c d - i e}{c d + i e}\right] + \left(-4 i c^2 d^2 - 3 c d e + i e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d - i e}} \sqrt{d+e x}\right], \frac{c d - i e}{c d + i e}\right] + 8 i c^2 d^2 \text{EllipticPi}\left[1 - \frac{i e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d - i e}} \sqrt{d+e x}\right], \frac{c d - i e}{c d + i e}\right]\right)\right) \Big/ \left(c^3 \sqrt{-\frac{c}{c d - i e}} \sqrt{1+\frac{1}{c^2 x^2}} x\right)\right)$$

Problem 59: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x (a + b \text{ArcCsch}[c x])}{\sqrt{d+e x}} \, dx$$

Optimal (type 4, 474 leaves, 14 steps):

$$-\frac{2 d \sqrt{d+e x}\,(a+b\,\mathrm{ArcCsch}[c x])}{e^2}+\frac{2 (d+e x)^{3/2}(a+b\,\mathrm{ArcCsch}[c x])}{3 e^2}+$$

$$\left(4 b c \sqrt{d+e x}\sqrt{1+c^2 x^2}\,\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(3\left(-c^2\right)^{3/2} e\sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\,e}}\right)-$$

$$\left(8 b c d\sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\,e}}\sqrt{1+c^2 x^2}\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(3\left(-c^2\right)^{3/2} e\sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{d+e x}\right)+$$

$$\left(8 b d^2\sqrt{\frac{\sqrt{-c^2}\,(d+e x)}{\sqrt{-c^2}\,d+e}}\sqrt{1+c^2 x^2}\,\mathrm{EllipticPi}\left[2,\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\,d+e}\right]\right)\Big/$$

$$\left(3 c e^2\sqrt{1+\frac{1}{c^2 x^2}}\,x\sqrt{d+e x}\right)$$

Result (type 4, 343 leaves):

$$\frac{1}{3 e^2}2\left(a(-2 d+e x)\sqrt{d+e x}+\right.$$

$$b(-2 d+e x)\sqrt{d+e x}\,\mathrm{ArcCsch}[c x]+\left(2 b\sqrt{-\frac{e(-i+c x)}{c d+i e}}\sqrt{-\frac{e(i+c x)}{c d-i e}}\right.$$

$$\left((i c d-e)\,\mathrm{EllipticE}\left[i\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}}\sqrt{d+e x}\right],\frac{c d-i e}{c d+i e}\right]+(i c d+e)\right.$$

$$\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}}\sqrt{d+e x}\right],\frac{c d-i e}{c d+i e}\right]-2 i c d\,\mathrm{EllipticPi}\left[1-\frac{i e}{c d},\right.$$

$$\left.\left.\left.i\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}}\sqrt{d+e x}\right],\frac{c d-i e}{c d+i e}\right]\right)\right)\Big/\left(c^2\sqrt{-\frac{c}{c d-i e}}\sqrt{1+\frac{1}{c^2 x^2}}\,x\right)\right)$$

Problem 60: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{a+b\,\mathrm{ArcCsch}[c x]}{\sqrt{d+e x}}\,dx$$

Optimal (type 4, 284 leaves, 9 steps):

$$\frac{2\sqrt{d+e x}\,(a+b\,\text{ArcCsch}[c x])}{e}+\left(4 b c\sqrt{\frac{d+e x}{d+\frac{e}{\sqrt{-c^2}}}}\sqrt{1+c^2x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Bigg/\left(\left(-c^2\right)^{3/2}\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+e x}\right)-\left(4 b d\sqrt{\frac{\sqrt{-c^2}\,(d+e x)}{\sqrt{-c^2}\,d+e}}\sqrt{1+c^2x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\,d+e}\right]\right)\Bigg/\left(c e\sqrt{1+\frac{1}{c^2x^2}}\,x\sqrt{d+e x}\right)$$

Result (type 4, 250 leaves):

$$\frac{1}{e}2\left(a\sqrt{d+e x}+b\sqrt{d+e x}\,\text{ArcCsch}[c x]-\left(2\,\text{i}\,b\sqrt{-\frac{e\,(-\text{i}+c x)}{c d+\text{i} e}}\sqrt{-\frac{e\,(\text{i}+c x)}{c d-\text{i} e}}\left(\text{EllipticF}\left[\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c d-\text{i} e}}\sqrt{d+e x}\right],\frac{c d-\text{i} e}{c d+\text{i} e}\right]-\text{EllipticPi}\left[1-\frac{\text{i} e}{c d},\text{i}\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c d-\text{i} e}}\sqrt{d+e x}\right],\frac{c d-\text{i} e}{c d+\text{i} e}\right]\right)\right)\Bigg/\left(c\sqrt{-\frac{c}{c d-\text{i} e}}\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\right)$$

Problem 63: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,(a+b\,\text{ArcCsch}[c x])}{(d+e x)^{3/2}}\,dx$$

Optimal (type 4, 731 leaves, 23 steps):

$$\frac{4 b \sqrt{d+e x}\,\left(1+c^2 x^2\right)}{15 c^3 e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x}+\frac{2 d^3\,(a+b\,\text{ArcCsch}[c\,x])}{e^4 \sqrt{d+e x}}+\frac{6 d^2 \sqrt{d+e x}\,(a+b\,\text{ArcCsch}[c\,x])}{e^4}-$$

$$\frac{2 d\,(d+e x)^{3/2}\,(a+b\,\text{ArcCsch}[c\,x])}{e^4}+\frac{2\,(d+e x)^{5/2}\,(a+b\,\text{ArcCsch}[c\,x])}{5 e^4}-$$

$$\left(32 b c d \sqrt{d+e x}\,\sqrt{1+c^2 x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(15\,(-c^2)^{3/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\,x \sqrt{\frac{c^2\,(d+e x)}{c^2 d-\sqrt{-c^2}\,e}}\right)+$$

$$\left(8 b c d^2 \sqrt{\frac{c^2\,(d+e x)}{c^2 d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left((-c^2)^{3/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\,x \sqrt{d+e x}\right)-$$

$$\left(4 b c\,(2 c^2 d^2-e^2) \sqrt{\frac{c^2\,(d+e x)}{c^2 d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\right.\right.$$

$$\left.\left.-\frac{2\sqrt{-c^2}\,e}{c^2 d-\sqrt{-c^2}\,e}\right]\right)\Big/\left(15\,(-c^2)^{5/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\,x \sqrt{d+e x}\right)-$$

$$\left(64 b d^3 \sqrt{\frac{\sqrt{-c^2}\,(d+e x)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2 x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\,d+e}\right]\right)\Big/$$

$$\left(5 c e^4 \sqrt{1+\frac{1}{c^2 x^2}}\,x \sqrt{d+e x}\right)$$

Result (type 4, 441 leaves) :

$$\frac{1}{15 e^4} 2 \left(\frac{2 b e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}{c} + \frac{3 a \left(16 d^3+8 d^2 e x-2 d e^2 x^2+e^3 x^3\right)}{\sqrt{d+e x}} + \right.$$

$$\frac{3 b \left(16 d^3+8 d^2 e x-2 d e^2 x^2+e^3 x^3\right) \text{ArcCsch}[c x]}{\sqrt{d+e x}} + \left(2 b \sqrt{-\frac{e (-i+c x)}{c d+i e}} \sqrt{-\frac{e (i+c x)}{c d-i e}} \right.$$

$$\left(8 c d (-i c d+e) \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right] + \right.$$

$$\left(-24 i c^2 d^2-8 c d e+i e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right] +$$

$$\left.\left. 48 i c^2 d^2 \text{EllipticPi}\left[1-\frac{i e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right]\right)\right) \Big/$$

$$\left.\left(c^3 \sqrt{-\frac{c}{c d-i e}} \sqrt{1+\frac{1}{c^2 x^2}}\, x \right)\right)$$

Problem 64: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a+b \text{ArcCsch}[c x])}{(d+e x)^{3/2}} \, dx$$

Optimal (type 4, 499 leaves, 16 steps):

$$-\frac{2\,d^2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^3\,\sqrt{d+e\,x}}-\frac{4\,d\,\sqrt{d+e\,x}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^3}+\frac{2\,\left(d+e\,x\right)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^3}+$$

$$\left(4\,b\,c\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\,\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(3\,\left(-c^2\right)^{3/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\right)-$$

$$\left(20\,b\,c\,d\,\sqrt{\frac{c^2\,\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\,e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(3\,\left(-c^2\right)^{3/2}\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)+$$

$$\left(32\,b\,d^2\,\sqrt{\frac{\sqrt{-c^2}\,\left(d+e\,x\right)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\,\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]\right)\Big/$$

$$\left(3\,c\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\sqrt{d+e\,x}\right)$$

Result (type 4, 365 leaves):

$$\frac{1}{3\,e^3}2\left(\frac{a\,\left(-8\,d^2-4\,d\,e\,x+e^2\,x^2\right)}{\sqrt{d+e\,x}}+\right.$$

$$\frac{b\,\left(-8\,d^2-4\,d\,e\,x+e^2\,x^2\right)\,\text{ArcCsch}[c\,x]}{\sqrt{d+e\,x}}+\left(2\,b\,\sqrt{-\frac{e\,\left(-i+c\,x\right)}{c\,d+i\,e}}\,\sqrt{-\frac{e\,\left(i+c\,x\right)}{c\,d-i\,e}}\right.$$

$$\left(\left(i\,c\,d-e\right)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\left(4\,i\,c\,d+e\right)\right.$$

$$\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-8\,i\,c\,d\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\right.$$

$$\left.\left.\left.i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)\Big/\left(c^2\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 65: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x\right)^{3/2}}\,dx$$

Optimal (type 4, 318 leaves, 11 steps):

$$\frac{2 d (a+b \text{ArcCsch}[c x])}{e^2 \sqrt{d+e x}}+\frac{2 \sqrt{d+e x} (a+b \text{ArcCsch}[c x])}{e^2}+\left(4 b c \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2} e}} \sqrt{1+c^2 x^2} \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2} e}{c^2 d-\sqrt{-c^2} e}\right]\right) \Big/ \left((-c^2)^{3/2} e \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}\right)-\left(8 b d \sqrt{\frac{\sqrt{-c^2} (d+e x)}{\sqrt{-c^2} d+e}} \sqrt{1+c^2 x^2} \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right], \frac{2 e}{\sqrt{-c^2} d+e}\right]\right) \Big/ \left(c e^2 \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}\right)$$

Result (type 4, 264 leaves):

$$\frac{1}{e^2}2 \left(\frac{a (2 d+e x)}{\sqrt{d+e x}}+\frac{b (2 d+e x) \text{ArcCsch}[c x]}{\sqrt{d+e x}}-\left(2 i b \sqrt{-\frac{e (-i+c x)}{c d+i e}} \sqrt{-\frac{e (i+c x)}{c d-i e}} \left(\text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right]-2 \text{EllipticPi}\left[1-\frac{i e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right]\right)\right) \Big/ \left(c \sqrt{-\frac{c}{c d-i e}} \sqrt{1+\frac{1}{c^2 x^2}} x\right)\right)$$

Problem 66: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b \text{ArcCsch}[c x]}{(d+e x)^{3/2}} \, dx$$

Optimal (type 4, 149 leaves, 6 steps):

$$-\frac{2 (a+b \text{ArcCsch}[c x])}{e \sqrt{d+e x}}+\left(4 b \sqrt{\frac{\sqrt{-c^2} (d+e x)}{\sqrt{-c^2} d+e}} \sqrt{1+c^2 x^2} \text{EllipticPi}\left[2, \text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2} x}}{\sqrt{2}}\right], \frac{2 e}{\sqrt{-c^2} d+e}\right]\right) \Big/ \left(c e \sqrt{1+\frac{1}{c^2 x^2}} x \sqrt{d+e x}\right)$$

Result (type 4, 166 leaves):

$$\left(-2\,e\,\left(1+c^2\,x^2\right)\,\left(a+b\,\text{ArcCsch}[c\,x]\right)+2\,b\,c\,\left(\mathrm{i}\,c\,d+e\right)\,\sqrt{2+\frac{2}{c^2\,x^2}}\,x\,\sqrt{1+\mathrm{i}\,c\,x}\,\sqrt{\frac{c\,e\,\left(\mathrm{i}+c\,x\right)\,\left(d+e\,x\right)}{\left(\mathrm{i}\,c\,d+e\right)^2}}\,\text{EllipticPi}\left[1+\frac{\mathrm{i}\,c\,d}{e},\,\text{ArcSin}\left[\sqrt{-\frac{e\,\left(\mathrm{i}+c\,x\right)}{c\,d-\mathrm{i}\,e}}\right],\,\frac{\mathrm{i}\,c\,d+e}{2\,e}\right]\right)\Big/\left(e^2\,\sqrt{d+e\,x}\,\left(1+c^2\,x^2\right)\right)$$

Problem 69: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^3\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x\right)^{5/2}}\,dx$$

Optimal (type 4, 777 leaves, 31 steps):

$$\frac{4 b d^2 \left(1+c^2 x^2\right)}{3 c e^2 \left(c^2 d^2+e^2\right) \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}+\frac{2 d^3 \left(a+b \text{ArcCsch}[c x]\right)}{3 e^4 (d+e x)^{3/2}}-\frac{6 d^2 \left(a+b \text{ArcCsch}[c x]\right)}{e^4 \sqrt{d+e x}}-$$

$$\frac{6 d \sqrt{d+e x} \left(a+b \text{ArcCsch}[c x]\right)}{e^4}+\frac{2 (d+e x)^{3/2} \left(a+b \text{ArcCsch}[c x]\right)}{3 e^4}-$$

$$\left(8 b \sqrt{-c^2}\, d^2 \sqrt{d+e x} \sqrt{1+c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right)\Big/$$

$$\left(3 c e^3 \left(c^2 d^2+e^2\right) \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}}\right)+$$

$$\left(4 b c \left(2 c^2 d^2+e^2\right) \sqrt{d+e x} \sqrt{1+c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right)\Big/$$

$$\left(3 \left(-c^2\right)^{3/2} e^3 \left(c^2 d^2+e^2\right) \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}}\right)-$$

$$\left(32 b c d \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}} \sqrt{1+c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right)\Big/$$

$$\left(3 \left(-c^2\right)^{3/2} e^3 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)+$$

$$\left(64 b d^2 \sqrt{\frac{\sqrt{-c^2}\, (d+e x)}{\sqrt{-c^2}\, d+e}} \sqrt{1+c^2 x^2}\, \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\, d+e}\right]\right)\Big/$$

$$\left(3 c e^4 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)$$

Result (type 4, 448 leaves) :

$$\frac{1}{3 e^4} 2 \left(\frac{2 b c d^2 e^2 \sqrt{1+\frac{1}{c^2 x^2}}\ x}{\left(c^2 d^2+e^2\right) \sqrt{d+e x}} + \frac{a \left(-16 d^3-24 d^2 e x-6 d e^2 x^2+e^3 x^3\right)}{(d+e x)^{3/2}} + \frac{b \left(-16 d^3-24 d^2 e x-6 d e^2 x^2+e^3 x^3\right) \text{ArcCsch}[c x]}{(d+e x)^{3/2}} - \frac{1}{c^3 \sqrt{1+\frac{1}{c^2 x^2}}\ x} 2 i b \sqrt{-\frac{c}{c d-i e}} \sqrt{-\frac{e (-i+c x)}{c d+i e}} \sqrt{-\frac{e (i+c x)}{c d-i e}} \left(e^2 \text{EllipticE}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right] + \left(8 c^2 d^2-8 i c d e-e^2\right) \text{EllipticF}\left[i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right] - 16 c d (c d-i e) \text{EllipticPi}\left[1-\frac{i e}{c d}, i \text{ArcSinh}\left[\sqrt{-\frac{c}{c d-i e}} \sqrt{d+e x}\right], \frac{c d-i e}{c d+i e}\right] \right) \right)$$

Problem 70: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^2 (a+b \text{ArcCsch}[c x])}{(d+e x)^{5/2}} \, dx$$

Optimal (type 4, 569 leaves, 25 steps):

$$-\frac{4 b d \left(1+c^2 x^2\right)}{3 c e \left(c^2 d^2+e^2\right) \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}}-\frac{2 d^2 \left(a+b \text{ArcCsch}[c x]\right)}{3 e^3 (d+e x)^{3/2}}+$$

$$\frac{4 d \left(a+b \text{ArcCsch}[c x]\right)}{e^3 \sqrt{d+e x}}+\frac{2 \sqrt{d+e x} \left(a+b \text{ArcCsch}[c x]\right)}{e^3}+$$

$$\left(4 b \sqrt{-c^2}\, d \sqrt{d+e x} \sqrt{1+c^2 x^2}\, \text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right)\Big/$$

$$\left(3 c e^2 \left(c^2 d^2+e^2\right) \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}}\right)+$$

$$\left(4 b c \sqrt{\frac{c^2 (d+e x)}{c^2 d-\sqrt{-c^2}\, e}} \sqrt{1+c^2 x^2}\, \text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],-\frac{2 \sqrt{-c^2}\, e}{c^2 d-\sqrt{-c^2}\, e}\right]\right)\Big/$$

$$\left(\left(-c^2\right)^{3/2} e^2 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)-$$

$$\left(32 b d \sqrt{\frac{\sqrt{-c^2}\, (d+e x)}{\sqrt{-c^2}\, d+e}} \sqrt{1+c^2 x^2}\, \text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\, x}}{\sqrt{2}}\right],\frac{2 e}{\sqrt{-c^2}\, d+e}\right]\right)\Big/$$

$$\left(3 c e^3 \sqrt{1+\frac{1}{c^2 x^2}}\, x \sqrt{d+e x}\right)$$

Result (type 4, 416 leaves) :

$$\frac{2}{3}\left(-\frac{2\,b\,c\,d\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x}{\left(c^2\,d^2\,e+e^3\right)\sqrt{d+e\,x}}+\frac{a\left(8\,d^2+12\,d\,e\,x+3\,e^2\,x^2\right)}{e^3\left(d+e\,x\right)^{3/2}}+\right.$$

$$\frac{b\left(8\,d^2+12\,d\,e\,x+3\,e^2\,x^2\right)\mathrm{ArcCsch}[c\,x]}{e^3\left(d+e\,x\right)^{3/2}}-\left(2\,b\,\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{-\frac{e\left(-i+c\,x\right)}{c\,d+i\,e}}\,\sqrt{-\frac{e\left(i+c\,x\right)}{c\,d-i\,e}}\right.$$

$$\left(i\,c\,d\,\mathrm{EllipticE}\left[i\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+\left(-4\,i\,c\,d-3\,e\right)\right.$$

$$\mathrm{EllipticF}\left[i\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+8\left(i\,c\,d+e\right)\mathrm{EllipticPi}\left[\right.$$

$$\left.\left.\left.1-\frac{i\,e}{c\,d},\,i\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right/\left(c^2\,e^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\right)\right)$$

Problem 71: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{\left(d+e\,x\right)^{5/2}}\,dx$$

Optimal (type 4, 393 leaves, 19 steps):

$$\frac{4\,b\left(1+c^2\,x^2\right)}{3\,c\left(c^2\,d^2+e^2\right)\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}+\frac{2\,d\left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{3\,e^2\left(d+e\,x\right)^{3/2}}-\frac{2\left(a+b\,\mathrm{ArcCsch}[c\,x]\right)}{e^2\,\sqrt{d+e\,x}}-$$

$$\left(4\,b\,\sqrt{-c^2}\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\;\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\;x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\;e}{c^2\,d-\sqrt{-c^2}\;e}\right]\right)\Big/$$

$$\left(3\,c\,e\left(c^2\,d^2+e^2\right)\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{\frac{c^2\left(d+e\,x\right)}{c^2\,d-\sqrt{-c^2}\;e}}\right)+$$

$$\left(8\,b\,\sqrt{\frac{\sqrt{-c^2}\,\left(d+e\,x\right)}{\sqrt{-c^2}\;d+e}}\,\sqrt{1+c^2\,x^2}\;\mathrm{EllipticPi}\left[2,\mathrm{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\;x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\;d+e}\right]\right)\Big/$$

$$\left(3\,c\,e^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}\right)$$

Result (type 4, 390 leaves):

$$\frac{2}{3}\left(\frac{2\,b\,c\,\sqrt{1+\frac{1}{c^2 x^2}}\;x}{\left(c^2 d^2+e^2\right)\sqrt{d+e\,x}}-\frac{a\,\left(2\,d+3\,e\,x\right)}{e^2\,\left(d+e\,x\right)^{3/2}}-\frac{b\,\left(2\,d+3\,e\,x\right)\,\text{ArcCsch}[c\,x]}{e^2\,\left(d+e\,x\right)^{3/2}}+\left(2\,i\,b\,\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{-\frac{e\,\left(-i+c\,x\right)}{c\,d+i\,e}}\;\sqrt{-\frac{e\,\left(i+c\,x\right)}{c\,d-i\,e}}\;\left(c\,d\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]-c\,d\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]+2\,\left(c\,d-i\,e\right)\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\;\sqrt{d+e\,x}\right],\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)\Big/\left(c^2\,d\,e^2\,\sqrt{1+\frac{1}{c^2 x^2}}\;x\right)\right)$$

Problem 72: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x\right)^{5/2}}\,dx$$

Optimal (type 4, 369 leaves, 12 steps):

$$-\frac{4\,b\,e\,\left(1+c^2 x^2\right)}{3\,c\,d\,\left(c^2 d^2+e^2\right)\sqrt{1+\frac{1}{c^2 x^2}}\;x\,\sqrt{d+e\,x}}-\frac{2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e\,\left(d+e\,x\right)^{3/2}}+\left(4\,b\,\sqrt{-c^2}\;\sqrt{d+e\,x}\;\sqrt{1+c^2 x^2}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\;x}}{\sqrt{2}}\right],-\frac{2\,\sqrt{-c^2}\;e}{c^2\,d-\sqrt{-c^2}\;e}\right]\right)\Big/\left(3\,c\,d\,\left(c^2 d^2+e^2\right)\sqrt{1+\frac{1}{c^2 x^2}}\;x\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}\right)+\left(4\,b\,\sqrt{\frac{\sqrt{-c^2}\;\left(d+e\,x\right)}{\sqrt{-c^2}\;d+e}}\;\sqrt{1+c^2 x^2}\;\text{EllipticPi}\left[2,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\;x}}{\sqrt{2}}\right],\frac{2\,e}{\sqrt{-c^2}\;d+e}\right]\right)\Big/\left(3\,c\,d\,e\,\sqrt{1+\frac{1}{c^2 x^2}}\;x\,\sqrt{d+e\,x}\right)$$

Result (type 4, 375 leaves):

$$\frac{1}{3\,e}2\left(-\frac{a}{(d+e\,x)^{3/2}}-\frac{2\,b\,c\,e^2\sqrt{1+\frac{1}{c^2\,x^2}}\;x}{d\,(c^2\,d^2+e^2)\,\sqrt{d+e\,x}}-\frac{b\,\mathrm{ArcCsch}[c\,x]}{(d+e\,x)^{3/2}}+\left(2\,b\,\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\,\sqrt{-\frac{e\,(-\mathrm{i}+c\,x)}{c\,d+\mathrm{i}\,e}}\,\sqrt{-\frac{e\,(\mathrm{i}+c\,x)}{c\,d-\mathrm{i}\,e}}\left(-\mathrm{i}\,c\,d\,\mathrm{EllipticE}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]+\mathrm{i}\,c\,d\,\mathrm{EllipticF}\left[\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]+(\mathrm{i}\,c\,d+e)\,\mathrm{EllipticPi}\left[1-\frac{\mathrm{i}\,e}{c\,d},\mathrm{i}\,\mathrm{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-\mathrm{i}\,e}}\,\sqrt{d+e\,x}\right],\frac{c\,d-\mathrm{i}\,e}{c\,d+\mathrm{i}\,e}\right]\right)\right)\Big/\left(c^2\,d^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\right)\right)$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\mathrm{ArcCsch}[c\,x]}{(d+e\,x)^{7/2}}\,dx$$

Optimal (type 4, 648 leaves, 19 steps):

$$-\frac{4\,b\,e\,\left(1+c^2\,x^2\right)}{15\,c\,d\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\left(d+e\,x\right)^{3/2}}-\frac{16\,b\,c\,e\,\left(1+c^2\,x^2\right)}{15\,\left(c^2\,d^2+e^2\right)^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}-$$

$$\frac{4\,b\,e\,\left(1+c^2\,x^2\right)}{5\,c\,d^2\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}}-\frac{2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5\,e\,\left(d+e\,x\right)^{5/2}}-$$

$$\left(4\,b\,c\,\left(7\,c^2\,d^2+3\,e^2\right)\,\sqrt{d+e\,x}\,\sqrt{1+c^2\,x^2}\;\text{EllipticE}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\,\frac{2\,\sqrt{-c^2}\,e}{-c^2\,d+\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(15\,\sqrt{-c^2}\,d^2\,\left(c^2\,d^2+e^2\right)^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}\right)-$$

$$\left(4\,b\,\sqrt{-c^2}\,\sqrt{\frac{d+e\,x}{d+\frac{e}{\sqrt{-c^2}}}}\,\sqrt{1+c^2\,x^2}\;\text{EllipticF}\left[\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\,-\frac{2\,\sqrt{-c^2}\,e}{c^2\,d-\sqrt{-c^2}\,e}\right]\right)\Big/$$

$$\left(15\,c\,d\,\left(c^2\,d^2+e^2\right)\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}\right)+$$

$$\left(4\,b\,\sqrt{\frac{\sqrt{-c^2}\,\left(d+e\,x\right)}{\sqrt{-c^2}\,d+e}}\,\sqrt{1+c^2\,x^2}\;\text{EllipticPi}\left[2,\,\text{ArcSin}\left[\frac{\sqrt{1-\sqrt{-c^2}\,x}}{\sqrt{2}}\right],\,\frac{2\,e}{\sqrt{-c^2}\,d+e}\right]\right)\Big/$$

$$\left(5\,c\,d^2\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\,\sqrt{d+e\,x}\right)$$

Result (type 4, 472 leaves) :

$$\frac{1}{15}\left(-\frac{6\,a}{e\,(d+e\,x)^{5/2}}-\frac{4\,b\,c\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\left(e^2\,(4\,d+3\,e\,x)+c^2\,d^2\,(8\,d+7\,e\,x)\right)}{d^2\,(c^2\,d^2+e^2)^2\,(d+e\,x)^{3/2}}-\right.$$

$$\frac{6\,b\,\text{ArcCsch}[c\,x]}{e\,(d+e\,x)^{5/2}}+\left(4\,i\,b\,(c\,d+i\,e)\,\sqrt{\frac{e\,(1-i\,c\,x)}{i\,c\,d+e}}\,\sqrt{\frac{e\,(1+i\,c\,x)}{-i\,c\,d+e}}\right.$$

$$\left(c\,d\,(7\,c^2\,d^2+3\,e^2)\,\text{EllipticE}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\,\frac{c\,d-i\,e}{c\,d+i\,e}\right]-\right.$$

$$c\,d\,(6\,c^2\,d^2+i\,c\,d\,e+3\,e^2)\,\text{EllipticF}\left[i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\,\frac{c\,d-i\,e}{c\,d+i\,e}\right]-$$

$$3\,(c\,d-i\,e)^2\,(c\,d+i\,e)\,\text{EllipticPi}\left[1-\frac{i\,e}{c\,d},\,i\,\text{ArcSinh}\left[\sqrt{-\frac{c}{c\,d-i\,e}}\,\sqrt{d+e\,x}\right],\right.$$

$$\left.\left.\left.\frac{c\,d-i\,e}{c\,d+i\,e}\right]\right)\right)\Bigg/\left(c\,d^3\,\sqrt{-\frac{c}{c\,d-i\,e}}\,e\,(c^2\,d^2+e^2)^2\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\right)\right)$$

Problem 98: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^2\,(a+b\,\text{ArcCsch}[c\,x])}{d+e\,x^2}\,dx$$

Optimal (type 4, 512 leaves, 25 steps):

$$\frac{x\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e}+\frac{b\,\text{ArcTanh}\left[\sqrt{1+\frac{1}{c^2\,x^2}}\right]}{c\,e}+$$
$$\frac{\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,e^{3/2}}-$$
$$\frac{\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,e^{3/2}}+$$
$$\frac{\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,e^{3/2}}-$$
$$\frac{\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,e^{3/2}}-$$
$$\frac{b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,e^{3/2}}+\frac{b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{2\,e^{3/2}}-$$
$$\frac{b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,e^{3/2}}+\frac{b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{2\,e^{3/2}}$$

Result (type 4, 1239 leaves):

$$\frac{1}{4\,c\,e^{3/2}}\left(4\,a\,c\,\sqrt{e}\,x+4\,b\,c\,\sqrt{e}\,x\,\text{ArcCsch}[c\,x]-4\,a\,c\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]-\right.$$
$$8\,i\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]-$$
$$8\,i\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]+$$
$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1-\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$
$$2\,i\,b\,c\,\sqrt{d}\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$4\,b\,c\,\sqrt{d}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]-b\,c\,\sqrt{d}\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\,\sqrt{d}}{x}\right]+$$

$$4\,b\,\sqrt{e}\,\text{Log}\left[\text{Cosh}\left[\frac{1}{2}\,\text{ArcCsch}[c\,x]\right]\right]-4\,b\,\sqrt{e}\,\text{Log}\left[\text{Sinh}\left[\frac{1}{2}\,\text{ArcCsch}[c\,x]\right]\right]+$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,i\,b\,c\,\sqrt{d}\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\, i\, b\, c \sqrt{d}\, \text{PolyLog}\left[2, \frac{i \left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right]\Bigg)$$

Problem 99: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x \left(a+b\, \text{ArcCsch}[c\, x]\right)}{d+e\, x^2}\, dx$$

Optimal (type 4, 449 leaves, 26 steps):

$$\frac{\left(a+b\, \text{ArcCsch}[c\, x]\right) \text{Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c\, x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 e}+\frac{\left(a+b\, \text{ArcCsch}[c\, x]\right) \text{Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c\, x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 e}+$$
$$\frac{\left(a+b\, \text{ArcCsch}[c\, x]\right) \text{Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c\, x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 e}+\frac{\left(a+b\, \text{ArcCsch}[c\, x]\right) \text{Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c\, x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 e}-$$
$$\frac{\left(a+b\, \text{ArcCsch}[c\, x]\right) \text{Log}\left[1-e^{2\, \text{ArcCsch}[c\, x]}\right]}{e}+\frac{b\, \text{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c\, x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 e}+$$
$$\frac{b\, \text{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c\, x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 e}+\frac{b\, \text{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c\, x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 e}+$$
$$\frac{b\, \text{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c\, x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 e}-\frac{b\, \text{PolyLog}\left[2, e^{2\, \text{ArcCsch}[c\, x]}\right]}{2 e}$$

Result (type 4, 1103 leaves):

$$\frac{1}{8 e}\Bigg(b\, \pi^2-4\, i\, b\, \pi\, \text{ArcCsch}[c\, x]-8\, b\, \text{ArcCsch}[c\, x]^2+$$

$$16\, b\, \text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(c \sqrt{d}-\sqrt{e}\right) \text{Cot}\left[\frac{1}{4} \left(\pi+2\, i\, \text{ArcCsch}[c\, x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$16\, b\, \text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(c \sqrt{d}+\sqrt{e}\right) \text{Cot}\left[\frac{1}{4} \left(\pi+2\, i\, \text{ArcCsch}[c\, x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\, b\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1-e^{-2\, \text{ArcCsch}[c\, x]}\right]+2\, i\, b\, \pi\, \text{Log}\left[1-\frac{i \left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$2\,i\,b\,\pi\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$2\,i\,b\,\pi\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$2\,i\,b\,\pi\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$2\,i\,b\,\pi\,\text{Log}\Big[\sqrt{e}-\frac{i\sqrt{d}}{x}\Big]-2\,i\,b\,\pi\,\text{Log}\Big[\sqrt{e}+\frac{i\sqrt{d}}{x}\Big]+4\,a\,\text{Log}\big[d+e\,x^2\big]+$$

$$4\,b\,\text{PolyLog}\big[2,e^{-2\,\text{ArcCsch}[c\,x]}\big]+4\,b\,\text{PolyLog}\Big[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{PolyLog}\Big[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{PolyLog}\Big[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)$$

Problem 100: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{d+e\,x^2}\,dx$$

Optimal (type 4, 477 leaves, 19 steps):

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\sqrt{-d}\sqrt{e}}$$

Result (type 4, 1055 leaves):

$$\frac{1}{4\sqrt{d}\sqrt{e}}\Bigg(4\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$b\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,i\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$4\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$b\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,i\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$b\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,i\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$4\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$b\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,i\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+$$

$$b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]-2\,i\,b\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,i\,b\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$2\,i\,b\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$2\,i\,b\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)$$

Problem 101: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{x\left(d+e\,x^2\right)}\,dx$$

Optimal (type 4, 425 leaves, 19 steps):

$$\frac{(a+b\,\text{ArcCsch}[c\,x])^2}{2\,b\,d}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d}-$$
$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d}-$$
$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d}-$$
$$\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,d}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d}-\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,d}$$

Result (type 4, 1075 leaves):

$$-\frac{1}{8\,d}\Bigg(b\,\pi^2-4\,i\,b\,\pi\,\text{ArcCsch}[c\,x]-4\,b\,\text{ArcCsch}[c\,x]^2+$$
$$16\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$
$$16\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]+$$
$$2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$
$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$2\,i\,b\,\pi\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$2\,i\,b\,\pi\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$2\,i\,b\,\pi\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$8\,i\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-$$

$$2\,i\,b\,\pi\,\text{Log}\Big[\sqrt{e}-\frac{i\sqrt{d}}{x}\Big]-2\,i\,b\,\pi\,\text{Log}\Big[\sqrt{e}+\frac{i\sqrt{d}}{x}\Big]-8\,a\,\text{Log}[x]+$$

$$4\,a\,\text{Log}\big[d+e\,x^2\big]+4\,b\,\text{PolyLog}\Big[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{PolyLog}\Big[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{PolyLog}\Big[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,b\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]\Bigg)$$

Problem 102: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{x^2\left(d+e\,x^2\right)}\,dx$$

Optimal (type 4, 518 leaves, 24 steps):

$$\frac{b\,c\,\sqrt{1+\frac{1}{c^2 x^2}}}{d}-\frac{a}{d\,x}-\frac{b\,\text{ArcCsch}[c\,x]}{d\,x}+\frac{\sqrt{e}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{\sqrt{e}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}+$$
$$\frac{\sqrt{e}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{\sqrt{e}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}+\frac{b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}-$$
$$\frac{b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}+\frac{b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,(-d)^{3/2}}$$

Result (type 4, 1211 leaves):

$$-\frac{a}{d\,x}-\frac{a\,\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{d^{3/2}}+$$
$$b\left(\frac{c\,\sqrt{1+\frac{1}{c^2 x^2}}-\frac{\text{ArcCsch}[c\,x]}{x}}{d}-\frac{1}{16\,d^{3/2}}\,i\,\sqrt{e}\,\Bigg(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)+$$

$$\frac{1}{16\,d^{3/2}}\,i\sqrt{e}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)\Bigg)$$

Problem 103: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^5\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 571 leaves, 31 steps):

$$\frac{b\sqrt{1+\frac{1}{c^2 x^2}}\,x}{2\,c\,e^2}+\frac{d\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{2\,e^2\,\left(e+\frac{d}{x^2}\right)}+\frac{x^2\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{2\,e^2}-$$

$$\frac{b\,d\,\text{ArcTan}\left[\frac{\sqrt{c^2\,d-e}}{c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{2\,\sqrt{c^2\,d-e}\,e^{5/2}}-\frac{d\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{e^3}-$$

$$\frac{d\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{e^3}-\frac{d\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{e^3}-$$

$$\frac{d\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{e^3}+\frac{2\,d\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-e^{2\,\text{ArcCsch}[c\,x]}\right]}{e^3}-$$

$$\frac{b\,d\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{e^3}-\frac{b\,d\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{e^3}-$$

$$\frac{b\,d\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{e^3}-\frac{b\,d\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{e^3}+\frac{b\,d\,\text{PolyLog}\left[2,e^{2\,\text{ArcCsch}[c\,x]}\right]}{e^3}$$

Result (type 4, 1554 leaves):

$$\frac{a\,x^2}{2\,e^2}-\frac{a\,d^2}{2\,e^3\,\left(d+e\,x^2\right)}-\frac{a\,d\,\text{Log}\left[d+e\,x^2\right]}{e^3}+b\left(\frac{x\left(\sqrt{1+\frac{1}{c^2 x^2}}+c\,x\,\text{ArcCsch}[c\,x]\right)}{2\,c\,e^2}+\frac{1}{4\,e^{5/2}}\right.$$

$$\left.i\,d^{3/2}\left(-\frac{\text{ArcCsch}[c\,x]}{i\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\left(c\,\sqrt{d}+i\,\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2\,d+e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)-\frac{1}{4\,e^{5/2}}\right.$$

$$i\, d^{3/2} \left(-\frac{\text{ArcCsch}[c\, x]}{-i \sqrt{d} \sqrt{e} + e\, x} + \frac{i \left(\frac{\text{ArcSinh}\left[\frac{1}{c\, x}\right]}{\sqrt{e}} - \frac{\text{Log}\left[-\frac{2 \sqrt{d} \sqrt{e} \left(\sqrt{e} + c \left(i\, c \sqrt{d} + \sqrt{-c^2 d + e} \sqrt{1 + \frac{1}{c^2 x^2}}\right) x\right)}{\sqrt{-c^2 d + e} \left(\sqrt{d} + i \sqrt{e}\, x\right)}\right]}{\sqrt{-c^2 d + e}} \right)}{\sqrt{d}} \right) -$$

$$\frac{1}{8\, e^3} d \left(\pi^2 - 4\, i\, \pi\, \text{ArcCsch}[c\, x] - 8\, \text{ArcCsch}[c\, x]^2 + \right.$$

$$32\, \text{ArcSin}\left[\frac{\sqrt{1 + \frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(c \sqrt{d} - \sqrt{e}\right) \text{Cot}\left[\frac{1}{4} \left(\pi + 2\, i\, \text{ArcCsch}[c\, x]\right)\right]}{\sqrt{-c^2 d + e}}\right] -$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 - e^{-2\, \text{ArcCsch}[c\, x]}\right] + 4\, i\, \pi\, \text{Log}\left[1 - \frac{i \left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right] +$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 - \frac{i \left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right] +$$

$$16\, i\, \text{ArcSin}\left[\frac{\sqrt{1 + \frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1 - \frac{i \left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right] +$$

$$4\, i\, \pi\, \text{Log}\left[1 + \frac{i \left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right] +$$

$$8\, \text{ArcCsch}[c\, x]\, \text{Log}\left[1 + \frac{i \left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right] - 16\, i\, \text{ArcSin}\left[\frac{\sqrt{1 + \frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1 + \frac{i \left(\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right] - 4\, i\, \pi\, \text{Log}\left[\sqrt{e} + \frac{i \sqrt{d}}{x}\right] +$$

$$4\, \text{PolyLog}\left[2, e^{-2\, \text{ArcCsch}[c\, x]}\right] + 8\, \text{PolyLog}\left[2, \frac{i \left(-\sqrt{e} + \sqrt{-c^2 d + e}\right) e^{\text{ArcCsch}[c\, x]}}{c \sqrt{d}}\right] +$$

$$8\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)-$$

$$\frac{1}{8e^3}d\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\ \frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)\Bigg)$$

Problem 104: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^3\,(a+b\,\text{ArcCsch}[c\,x])}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 535 leaves, 29 steps):

$$-\frac{a+b\,\text{ArcCsch}[c\,x]}{2\,e\left(e+\frac{d}{x^2}\right)}+\frac{b\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\sqrt{e}\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{2\sqrt{c^2 d-e}\;e^{3/2}}+\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^2}+$$
$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^2}+$$
$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^2}-\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-e^{2\,\text{ArcCsch}[c\,x]}\right]}{e^2}+$$
$$\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^2}+$$
$$\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^2}+\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^2}-\frac{b\,\text{PolyLog}\left[2,e^{2\,\text{ArcCsch}[c\,x]}\right]}{2\,e^2}$$

Result (type 4, 1410 leaves):

$$\frac{1}{8\,e^2}\Bigg(b\,\pi^2+\frac{4\,a\,d}{d+e\,x^2}-4\,i\,b\,\pi\,\text{ArcCsch}[c\,x]+\frac{2\,b\sqrt{d}\,\text{ArcCsch}[c\,x]}{\sqrt{d}-i\sqrt{e}\,x}+$$

$$\frac{2\,b\,\sqrt{d}\,\text{ArcCsch}[c\,x]}{\sqrt{d}+\text{i}\,\sqrt{e}\,x}-8\,b\,\text{ArcCsch}[c\,x]^2-4\,b\,\text{ArcSinh}\Big[\frac{1}{c\,x}\Big]+$$

$$16\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\text{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\Big]-$$

$$16\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\text{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\Big]-$$

$$8\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+2\,\text{i}\,b\,\pi\,\text{Log}\Big[1-\frac{\text{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{\text{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$8\,\text{i}\,b\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{\text{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$2\,\text{i}\,b\,\pi\,\text{Log}\Big[1+\frac{\text{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{\text{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$8\,\text{i}\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{\text{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$2\,\text{i}\,b\,\pi\,\text{Log}\Big[1-\frac{\text{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{\text{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$8\,\text{i}\,b\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{\text{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$2\,\text{i}\,b\,\pi\,\text{Log}\Big[1+\frac{\text{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{\text{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]-$$

$$2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+\frac{2\,b\sqrt{e}\,\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}+$$

$$\frac{2\,b\sqrt{e}\,\text{Log}\left[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}+4\,a\,\text{Log}\left[d+e\,x^2\right]+$$

$$4\,b\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,b\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,b\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.4\,b\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 3, 139 leaves, 7 steps):

$$-\frac{a+b\,\text{ArcCsch}[c\,x]}{2\,e\left(d+e\,x^2\right)}+\frac{b\,c\,x\,\text{ArcTan}\left[\sqrt{-1-c^2x^2}\right]}{2\,d\,e\sqrt{-c^2x^2}}+\frac{b\,c\,x\,\text{ArcTanh}\left[\frac{\sqrt{e}\sqrt{-1-c^2x^2}}{\sqrt{c^2 d-e}}\right]}{2\,d\sqrt{c^2 d-e}\sqrt{e}\sqrt{-c^2x^2}}$$

Result (type 3, 271 leaves):

$$-\frac{1}{4 e}\left(\frac{2 a}{d+e x^2}+\frac{2 b \operatorname{ArcCsch}[c x]}{d+e x^2}-\frac{2 b \operatorname{ArcSinh}\left[\frac{1}{c x}\right]}{d}+\frac{b \sqrt{e} \operatorname{Log}\left[-\frac{4\left(i d e+c d \sqrt{e}\left(c \sqrt{d}+i \sqrt{-c^2 d+e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)}{b \sqrt{-c^2 d+e}\left(\sqrt{d}-i \sqrt{e} x\right)}\right]}{d \sqrt{-c^2 d+e}}+\frac{b \sqrt{e} \operatorname{Log}\left[\frac{4 i\left(d e+c d \sqrt{e}\left(i c \sqrt{d}+\sqrt{-c^2 d+e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)}{b \sqrt{-c^2 d+e}\left(\sqrt{d}+i \sqrt{e} x\right)}\right]}{d \sqrt{-c^2 d+e}}\right)$$

Problem 106: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{a+b \operatorname{ArcCsch}[c x]}{x\left(d+e x^2\right)^2} \, dx$$

Optimal (type 4, 515 leaves, 24 steps):

$$-\frac{e\left(a+b \operatorname{ArcCsch}[c x]\right)}{2 d^2\left(e+\frac{d}{x^2}\right)}+\frac{\left(a+b \operatorname{ArcCsch}[c x]\right)^2}{2 b d^2}+\frac{b \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}} x}\right]}{2 d^2 \sqrt{c^2 d-e}}-\frac{\left(a+b \operatorname{ArcCsch}[c x]\right) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 d^2}-\frac{\left(a+b \operatorname{ArcCsch}[c x]\right) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 d^2}-\frac{\left(a+b \operatorname{ArcCsch}[c x]\right) \operatorname{Log}\left[1-\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 d^2}-\frac{\left(a+b \operatorname{ArcCsch}[c x]\right) \operatorname{Log}\left[1+\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 d^2}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 d^2}-\frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 d^2}-\frac{b \operatorname{PolyLog}\left[2,-\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 d^2}-\frac{b \operatorname{PolyLog}\left[2,\frac{c \sqrt{-d} e^{\operatorname{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 d^2}$$

Result (type 4, 1382 leaves):

$$-\frac{1}{8\,d^2}\left(b\,\pi^2-\frac{4\,a\,d}{d+e\,x^2}-4\,i\,b\,\pi\,\text{ArcCsch}[c\,x]-\right.$$

$$\frac{2\,b\,\sqrt{d}\,\text{ArcCsch}[c\,x]}{\sqrt{d}-i\,\sqrt{e}\,x}-\frac{2\,b\,\sqrt{d}\,\text{ArcCsch}[c\,x]}{\sqrt{d}+i\,\sqrt{e}\,x}-4\,b\,\text{ArcCsch}[c\,x]^2+4\,b\,\text{ArcSinh}\left[\frac{1}{c\,x}\right]+$$

$$16\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]-$$

$$16\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$2\,i\,b\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$8\,i\,b\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-$$

$$2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]-2\,i\,b\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\,\sqrt{d}}{x}\right]-8\,a\,\text{Log}[x]-$$

$$\frac{2\,b\,\sqrt{e}\,\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\left(c\,\sqrt{d}+i\,\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}-$$

$$\frac{2\,b\,\sqrt{e}\,\text{Log}\left[-\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(\sqrt{e}+c\left(i\,c\,\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}+$$

$$4\,a\,\text{Log}\left[d+e\,x^2\right]+4\,b\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,b\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]\Bigg)$$

Problem 107: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x^4\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 756 leaves, 51 steps):

$$-\frac{d\left(a+b\,\text{ArcCsch}[c\,x]\right)}{4\,e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{d\left(a+b\,\text{ArcCsch}[c\,x]\right)}{4\,e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{x\left(a+b\,\text{ArcCsch}[c\,x]\right)}{e^2}+$$

$$\frac{b\,\text{ArcTanh}\left[\sqrt{1+\frac{1}{c^2\,x^2}}\right]}{c\,e^2}+\frac{b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^2\,d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,\sqrt{c^2\,d-e}\,e^2}+$$

$$\frac{b\,\sqrt{d}\,\text{ArcTanh}\left[\frac{c^2\,d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,\sqrt{c^2\,d-e}\,e^2}+\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}-$$

$$\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}+$$

$$\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}-$$

$$\frac{3\,\sqrt{-d}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}-$$

$$\frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}+\frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}-$$

$$\frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}+\frac{3\,b\,\sqrt{-d}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,e^{5/2}}$$

Result (type 4, 1593 leaves):

$$\frac{a\,x}{e^2}+\frac{a\,d\,x}{2\,e^2\left(d+e\,x^2\right)}-\frac{3\,a\,\sqrt{d}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{2\,e^{5/2}}+b\left(-\frac{1}{4\,e^2}d\right.$$

$$\left(-\frac{\text{ArcCsch}[c\,x]}{i\sqrt{d}\sqrt{e}+e\,x}-\frac{1}{\sqrt{d}}i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)\right)-$$

$$\frac{1}{4e^2}d\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)-$$

$$\frac{1}{32e^{5/2}}3\,i\sqrt{d}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]-4 i \pi \text{Log}\left[\sqrt{e}+\frac{i \sqrt{d}}{x}\right]+$$

$$4 \text{PolyLog}\left[2, e^{-2 \text{ArcCsch}[c x]}\right]+8 \text{PolyLog}\left[2, \frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]+$$

$$\left.8 \text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]\right)+$$

$$\frac{1}{32 e^{5/2}} 3 i \sqrt{d}\left(\pi^2-4 i \pi \text{ArcCsch}[c x]-8 \text{ArcCsch}[c x]^2-\right.$$

$$32 \text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{ArcTan}\left[\frac{\left(c \sqrt{d}+\sqrt{e}\right) \text{Cot}\left[\frac{1}{4}\left(\pi+2 i \text{ArcCsch}[c x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8 \text{ArcCsch}[c x] \text{Log}\left[1-e^{-2 \text{ArcCsch}[c x]}\right]+4 i \pi \text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]+$$

$$8 \text{ArcCsch}[c x] \text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]+$$

$$16 i \text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right] \text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]+$$

$$4 i \pi \text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]+$$

$$8 \text{ArcCsch}[c x] \text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]-16 i \text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c \sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]-4 i \pi \text{Log}\left[\sqrt{e}-\frac{i \sqrt{d}}{x}\right]+$$

$$4 \text{PolyLog}\left[2, e^{-2 \text{ArcCsch}[c x]}\right]+8 \text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c x]}}{c \sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\ \frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)+\frac{1}{c\,e^2}$$

$$\left(\frac{1}{2}\text{ArcCsch}[c\,x]\,\text{Coth}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]+\text{Log}\left[\text{Cosh}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]\right]-\right.$$

$$\left.\text{Log}\left[\text{Sinh}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]\right]-\frac{1}{2}\text{ArcCsch}[c\,x]\,\text{Tanh}\left[\frac{1}{2}\text{ArcCsch}[c\,x]\right]\right)\Bigg)$$

Problem 108: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{x^2\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 719 leaves, 27 steps):

$$\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,e\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,e\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{b\,\text{ArcTanh}\left[\frac{c^2\,d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,\sqrt{d}\,\sqrt{c^2\,d-e}\,e}-$$

$$\frac{b\,\text{ArcTanh}\left[\frac{c^2\,d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}}\right]}{4\,\sqrt{d}\,\sqrt{c^2\,d-e}\,e}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}-\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}+\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2\,d+e}}\right]}{4\,\sqrt{-d}\,e^{3/2}}$$

Result (type 4, 1442 leaves):

$$\frac{1}{8\,e^{3/2}}\left(-\frac{4\,a\,\sqrt{e}\,x}{d+e\,x^2}+\frac{4\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{\sqrt{d}}+b\left(\frac{2\,\text{ArcCsch}[c\,x]}{i\,\sqrt{d}-\sqrt{e}\,x}-\right.\right.$$

$$\frac{2\,\text{ArcCsch}[c\,x]}{i\,\sqrt{d}+\sqrt{e}\,x}+\frac{8\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,i\,\text{ArcCsch}[c\,x])\right]}{\sqrt{-c^2\,d+e}}\right]}{\sqrt{d}}+$$

$$\frac{8\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,(\pi+2\,i\,\text{ArcCsch}[c\,x])\right]}{\sqrt{-c^2\,d+e}}\right]}{\sqrt{d}}-$$

$$\frac{\pi\,\text{Log}\left[1-\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+\frac{2\,i\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}-$$

$$\frac{4\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\mathrm{Log}\left[1-\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+$$

$$\frac{\pi\,\mathrm{Log}\left[1+\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}-\frac{2\,i\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1+\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+$$

$$\frac{4\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\mathrm{Log}\left[1+\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+$$

$$\frac{\pi\,\mathrm{Log}\left[1-\frac{i\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}-\frac{2\,i\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1-\frac{i\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}-$$

$$\frac{4\,\mathrm{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\mathrm{Log}\left[1-\frac{i\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}-$$

$$\frac{\pi\,\mathrm{Log}\left[1+\frac{i\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+\frac{2\,i\,\mathrm{ArcCsch}[c\,x]\,\mathrm{Log}\left[1+\frac{i\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+$$

$$\frac{4\,\mathrm{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\mathrm{Log}\left[1+\frac{i\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}-\frac{\pi\,\mathrm{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]}{\sqrt{d}}+$$

$$\frac{\pi\,\mathrm{Log}\left[\sqrt{e}+\frac{i\,\sqrt{d}}{x}\right]}{\sqrt{d}}+\frac{2\,i\,\sqrt{e}\,\mathrm{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\,\left(c\,\sqrt{d}+i\,\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{d}\,\sqrt{-c^2\,d+e}}-$$

$$\frac{2\,i\,\sqrt{e}\,\mathrm{Log}\left[-\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(\sqrt{e}+c\,\left(i\,c\,\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{\sqrt{d}\,\sqrt{-c^2\,d+e}}-$$

$$\frac{2\,i\,\mathrm{PolyLog}\left[2,-\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+\frac{2\,i\,\mathrm{PolyLog}\left[2,\frac{i\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\mathrm{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]}{\sqrt{d}}+$$

$$\left.\left.\frac{2\,i\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}-\frac{2\,i\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]}{\sqrt{d}}\right)\right)$$

Problem 109: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 713 leaves, 47 steps):

$$-\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,d\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}+\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,d\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+\frac{b\,\text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2x^2}}}\right]}{4\,d^{3/2}\sqrt{c^2 d-e}}+$$

$$\frac{b\,\text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2x^2}}}\right]}{4\,d^{3/2}\sqrt{c^2 d-e}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}+$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}-\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}+$$

$$\frac{\left(a+b\,\text{ArcCsch}[c\,x]\right)\text{Log}\left[1+\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}-$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}-\frac{b\,\text{PolyLog}\left[2,\frac{c\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{3/2}\sqrt{e}}$$

Result (type 4, 1520 leaves):

$$\frac{a\,x}{2\,d\left(d+e\,x^2\right)}+\frac{a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{2\,d^{3/2}\sqrt{e}}+$$

$$b\left(-\frac{1}{4\,d}\left(-\frac{\text{ArcCsch}[c\,x]}{\mathbb{i}\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{\mathbb{i}\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(\mathbb{i}\,\sqrt{e}+c\,\left(c\,\sqrt{d}+\mathbb{i}\,\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(\mathbb{i}\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)-\right.$$

$$\frac{1}{4\,d}\left(-\frac{\text{ArcCsch}[c\,x]}{-\mathbb{i}\,\sqrt{d}\,\sqrt{e}+e\,x}+\frac{\mathbb{i}\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(\sqrt{e}+c\,\left(\mathbb{i}\,c\,\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(\sqrt{d}+\mathbb{i}\,\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{32\,d^{3/2}\,\sqrt{e}}\,\mathbb{i}\left(\pi^2-4\,\mathbb{i}\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\mathbb{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,\mathbb{i}\,\pi\,\text{Log}\left[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$16\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,\mathbb{i}\,\pi\,\text{Log}\left[1+\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,\mathbb{i}\,\pi\,\text{Log}\left[\sqrt{e}+\frac{\mathbb{i}\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\,-\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)-$$

$$\frac{1}{32\,d^{3/2}\sqrt{e}}\,\mathbb{i}\left(\pi^2-4\,\mathbb{i}\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\mathbb{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,\mathbb{i}\,\pi\,\text{Log}\left[1+\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,\mathbb{i}\,\pi\,\text{Log}\left[1-\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,\mathbb{i}\,\pi\,\text{Log}\left[\sqrt{e}-\frac{\mathbb{i}\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,-\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\ \frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right) e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)\Bigg)$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{x^2\left(d+e\,x^2\right)^2}\,dx$$

Optimal (type 4, 758 leaves, 50 steps) :

$$\frac{b\,c\,\sqrt{1+\frac{1}{c^2 x^2}}}{d^2}-\frac{a}{d^2 x}-\frac{b\,\text{ArcCsch}[c\,x]}{d^2 x}+\frac{e\,(a+b\,\text{ArcCsch}[c\,x])}{4\,d^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-$$

$$\frac{e\,(a+b\,\text{ArcCsch}[c\,x])}{4\,d^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{b\,e\,\text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{4\,d^{5/2}\,\sqrt{c^2 d-e}}-$$

$$\frac{b\,e\,\text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{4\,d^{5/2}\,\sqrt{c^2 d-e}}-\frac{3\,\sqrt{e}\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{5/2}}+$$

$$\frac{3\,\sqrt{e}\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{5/2}}-$$

$$\frac{3\,\sqrt{e}\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{5/2}}+$$

$$\frac{3\,\sqrt{e}\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{5/2}}+$$

$$\frac{3\,b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{5/2}}-\frac{3\,b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{5/2}}+$$

$$\frac{3\,b\,\sqrt{e}\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{5/2}}-\frac{3\,b\,\sqrt{e}\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{4\,(-d)^{5/2}}$$

Result (type 4, 1487 leaves):

$$\frac{1}{8\,d^{5/2}}\left(-\frac{8\,a\,\sqrt{d}}{x}-\frac{4\,a\,\sqrt{d}\,e\,x}{d+e\,x^2}-12\,a\,\sqrt{e}\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]+\right.$$

$$b\left(8\,c\,\sqrt{d}\,\sqrt{1+\frac{1}{c^2\,x^2}}-\frac{8\,\sqrt{d}\,\text{ArcCsch}[c\,x]}{x}-\frac{2\,\sqrt{d}\,e\,\text{ArcCsch}[c\,x]}{-\mathbb{i}\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{2\,\sqrt{d}\,e\,\text{ArcCsch}[c\,x]}{\mathbb{i}\,\sqrt{d}\,\sqrt{e}+e\,x}-\right.$$

$$24\,\mathbb{i}\,\sqrt{e}\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(c\,\sqrt{d}-\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\mathbb{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\Big]-$$

$$24\,\mathbb{i}\,\sqrt{e}\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(c\,\sqrt{d}+\sqrt{e}\right)\,\text{Cot}\left[\frac{1}{4}\,\left(\pi+2\,\mathbb{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\Big]+$$

$$3\,\sqrt{e}\,\pi\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$6\,\mathbb{i}\,\sqrt{e}\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$12\,\sqrt{e}\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$3\,\sqrt{e}\,\pi\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$6\,\mathbb{i}\,\sqrt{e}\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$12\,\sqrt{e}\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1+\frac{\mathbb{i}\,\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]-$$

$$3\,\sqrt{e}\,\pi\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$6\,\mathbb{i}\,\sqrt{e}\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$12\,\sqrt{e}\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{\mathbb{i}\,\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\Big]+$$

$$3\sqrt{e}\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$6\,i\sqrt{e}\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-12\sqrt{e}\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+3\sqrt{e}\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]-$$

$$3\sqrt{e}\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+\frac{2\,i\,e\,\text{Log}\left[\frac{2\sqrt{d}\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}-$$

$$\frac{2\,i\,e\,\text{Log}\left[-\frac{2\sqrt{d}\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}+$$

$$6\,i\sqrt{e}\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$6\,i\sqrt{e}\,\text{PolyLog}\left[2,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-$$

$$6\,i\sqrt{e}\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.\left.6\,i\sqrt{e}\,\text{PolyLog}\left[2,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)\right)$$

Problem 111: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x^5\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 676 leaves, 33 steps):

$$\frac{b\,c\,d\,\sqrt{1+\frac{1}{c^2 x^2}}}{8\,(c^2 d-e)\,e^2\,\left(e+\frac{d}{x^2}\right)x}-\frac{a+b\,\text{ArcCsch}[c\,x]}{4\,e\,\left(e+\frac{d}{x^2}\right)^2}-\frac{a+b\,\text{ArcCsch}[c\,x]}{2\,e^2\,\left(e+\frac{d}{x^2}\right)}+$$

$$\frac{b\,(c^2 d-2\,e)\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{8\,(c^2 d-e)^{3/2}\,e^{5/2}}+\frac{b\,\text{ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}\right]}{2\,\sqrt{c^2 d-e}\,e^{5/2}}+$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^3}+$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^3}-$$

$$\frac{(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-e^{2\,\text{ArcCsch}[c\,x]}\right]}{e^3}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^3}+$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2\,e^3}+\frac{b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^3}+$$

$$\frac{b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2\,e^3}-\frac{b\,\text{PolyLog}\left[2,e^{2\,\text{ArcCsch}[c\,x]}\right]}{2\,e^3}$$

Result (type 4, 2023 leaves):

$$-\frac{a\,d^2}{4\,e^3\,(d+e\,x^2)^2}+\frac{a\,d}{e^3\,(d+e\,x^2)}+\frac{a\,\text{Log}\left[d+e\,x^2\right]}{2\,e^3}+$$

$$b\left(-\frac{1}{16\,e^{5/2}}d\left(\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}{\sqrt{d}\,(c^2 d-e)\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\right.\right.$$

$$\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{1}{d\,(c^2 d-e)^{3/2}}i\,(2\,c^2 d-e)\,\text{Log}\left[\left(4\,d\,\sqrt{c^2 d-e}\,\sqrt{e}\right.\right.$$

$$\left.\left.\left(\sqrt{e}+i\,c\left(c\sqrt{d}-\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)\right/\left(\left(2 c^2 d-e\right)\left(\sqrt{d}+i\sqrt{e}\,x\right)\right)\right]\right)-$$

$$\frac{1}{16 e^{5/2}}d\left(-\frac{i\,c\sqrt{e}\,\sqrt{1+\frac{1}{c^2 x^2}}\,x}{\sqrt{d}\left(c^2 d-e\right)\left(i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\sqrt{e}}+\right.$$

$$\frac{1}{d\left(c^2 d-e\right)^{3/2}}i\left(2 c^2 d-e\right)\text{Log}\left[\left(4\,i\,d\sqrt{c^2 d-e}\,\sqrt{e}\right.\right.$$

$$\left.\left.\left.\left(i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)\right/\left(\left(2 c^2 d-e\right)\left(\sqrt{d}-i\sqrt{e}\,x\right)\right)\right]\right)-$$

$$\frac{1}{16 e^{5/2}}7\,i\sqrt{d}\left(-\frac{\text{ArcCsch}[c\,x]}{i\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{16 e^{5/2}}7\,i\sqrt{d}\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{16\,e^3}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+\right.$$

$$32\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{ArcTan}\Big[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\big[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\big]}{\sqrt{-c^2\,d+e}}\Big]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\big[1-e^{-2\,\text{ArcCsch}[c\,x]}\big]+4\,i\,\pi\,\text{Log}\Big[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$16\,i\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\,\text{Log}\Big[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$4\,i\,\pi\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-16\,i\,\text{ArcSin}\Big[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]$$

$$\text{Log}\Big[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]-4\,i\,\pi\,\text{Log}\Big[\sqrt{e}+\frac{i\sqrt{d}}{x}\Big]+$$

$$4\,\text{PolyLog}\big[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\big]+8\,\text{PolyLog}\Big[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]+$$

$$\left.8\,\text{PolyLog}\Big[2,\,-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\Big]\right)+$$

$$\frac{1}{16\,e^3}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\Big[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\Big]\right.$$

$$\text{ArcTan}\Big[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\big[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\big]}{\sqrt{-c^2\,d+e}}\Big]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\,\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\,\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]+$$

$$\left.\left.8\,\text{PolyLog}\left[2,\,\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]\right)\right)$$

Problem 112: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{x^3\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 3, 167 leaves, 6 steps):

$$-\frac{b\,c\,x\,\sqrt{-1-c^2\,x^2}}{8\,\left(c^2\,d-e\right)\,e\,\sqrt{-c^2\,x^2}\,\left(d+e\,x^2\right)}+\frac{x^4\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{4\,d\,\left(d+e\,x^2\right)^2}+\frac{b\,c\,\left(c^2\,d-2\,e\right)\,x\,\text{ArcTanh}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2\,x^2}}{\sqrt{c^2\,d-e}}\right]}{8\,d\,\left(c^2\,d-e\right)^{3/2}\,e^{3/2}\,\sqrt{-c^2\,x^2}}$$

Result (type 3, 375 leaves):

$$-\frac{1}{16 e^2}\left(-\frac{4 a d}{\left(d+e x^2\right)^2}+\frac{8 a}{d+e x^2}-\frac{2 b c e \sqrt{1+\frac{1}{c^2 x^2}}\, x}{\left(-c^2 d+e\right)\left(d+e x^2\right)}+\frac{4 b\left(d+2 e x^2\right) \text{ArcCsch}[c x]}{\left(d+e x^2\right)^2}-\frac{4 b \text{ArcSinh}\left[\frac{1}{c x}\right]}{d}+\frac{1}{d\left(-c^2 d+e\right)^{3/2}} b \sqrt{e}\left(-c^2 d+2 e\right) \text{Log}\left[\left(16 d e^{3/2} \sqrt{-c^2 d+e}\left(\sqrt{e}+c\left(-\mathrm{i} c \sqrt{d}+\sqrt{-c^2 d+e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)\right)\Big/\left(b\left(-c^2 d+2 e\right)\left(\mathrm{i} \sqrt{d}+\sqrt{e}\, x\right)\right)\right]+\frac{1}{d\left(-c^2 d+e\right)^{3/2}} b \sqrt{e}\left(-c^2 d+2 e\right) \text{Log}\left[-\left(\left(16\, \mathrm{i}\, d e^{3/2} \sqrt{-c^2 d+e}\left(\sqrt{e}+c\left(\mathrm{i} c \sqrt{d}+\sqrt{-c^2 d+e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)\right)\Big/\left(b\left(c^2 d-2 e\right)\left(\sqrt{d}+\mathrm{i} \sqrt{e}\, x\right)\right)\right)\right]\right)$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x\,(a+b \text{ArcCsch}[c x])}{\left(d+e x^2\right)^3}\, dx$$

Optimal (type 3, 205 leaves, 8 steps):

$$\frac{b c x \sqrt{-1-c^2 x^2}}{8 d\left(c^2 d-e\right) \sqrt{-c^2 x^2}\left(d+e x^2\right)}-\frac{a+b \text{ArcCsch}[c x]}{4 e\left(d+e x^2\right)^2}+\frac{b c x \text{ArcTan}\left[\sqrt{-1-c^2 x^2}\right]}{4 d^2 e \sqrt{-c^2 x^2}}+\frac{b c\left(3 c^2 d-2 e\right) x \text{ArcTanh}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{\sqrt{c^2 d-e}}\right]}{8 d^2\left(c^2 d-e\right)^{3/2} \sqrt{e} \sqrt{-c^2 x^2}}$$

Result (type 3, 368 leaves):

$$\frac{1}{16}\left(-\frac{4\,a}{e\,\left(d+e\,x^2\right)^2}+\frac{2\,b\,c\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x}{d\,\left(c^2\,d-e\right)\,\left(d+e\,x^2\right)}-\frac{4\,b\,\text{ArcCsch}[c\,x]}{e\,\left(d+e\,x^2\right)^2}+\frac{4\,b\,\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d^2\,e}+\right.$$

$$\left(b\,\left(3\,c^2\,d-2\,e\right)\,\text{Log}\left[\left(16\,d^2\,\sqrt{e}\,\sqrt{-c^2\,d+e}\,\left(\sqrt{e}+c\,\left(-\text{i}\,c\,\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)\right)\right/\right.$$

$$\left.\left.\left(b\,\left(-3\,c^2\,d+2\,e\right)\,\left(\text{i}\,\sqrt{d}+\sqrt{e}\,x\right)\right)\right]\right)\Big/\left(d^2\,\sqrt{e}\,\left(-c^2\,d+e\right)^{3/2}\right)+$$

$$\left(b\,\left(3\,c^2\,d-2\,e\right)\,\text{Log}\left[-\left(\left(16\,\text{i}\,d^2\,\sqrt{e}\,\sqrt{-c^2\,d+e}\,\left(\sqrt{e}+c\,\left(\text{i}\,c\,\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)\right)\right/\right.\right.$$

$$\left.\left.\left.\left.\left(b\,\left(3\,c^2\,d-2\,e\right)\,\left(\sqrt{d}+\text{i}\,\sqrt{e}\,x\right)\right)\right)\right]\right)\Big/\left(d^2\,\sqrt{e}\,\left(-c^2\,d+e\right)^{3/2}\right)\right)$$

Problem 114: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{x\,\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 657 leaves, 28 steps):

$$-\frac{b c e \sqrt{1+\frac{1}{c^2 x^2}}}{8 d^2 \left(c^2 d-e\right) \left(e+\frac{d}{x^2}\right) x}+\frac{e^2 (a+b \text{ArcCsch}[c x])}{4 d^3 \left(e+\frac{d}{x^2}\right)^2}-\frac{e (a+b \text{ArcCsch}[c x])}{d^3 \left(e+\frac{d}{x^2}\right)}+$$

$$\frac{(a+b \text{ArcCsch}[c x])^2}{2 b d^3}-\frac{b \left(c^2 d-2 e\right) \sqrt{e} \text{ ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}} x}\right]}{8 d^3 \left(c^2 d-e\right)^{3/2}}+$$

$$\frac{b \sqrt{e} \text{ ArcTan}\left[\frac{\sqrt{c^2 d-e}}{c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}} x}\right]}{d^3 \sqrt{c^2 d-e}}-\frac{(a+b \text{ArcCsch}[c x]) \text{ Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 d^3}-$$

$$\frac{(a+b \text{ArcCsch}[c x]) \text{ Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 d^3}-\frac{(a+b \text{ArcCsch}[c x]) \text{ Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 d^3}-$$

$$\frac{(a+b \text{ArcCsch}[c x]) \text{ Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 d^3}-\frac{b \text{ PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 d^3}-$$

$$\frac{b \text{ PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{2 d^3}-\frac{b \text{ PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 d^3}-\frac{b \text{ PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{2 d^3}$$

Result (type 4, 2077 leaves):

$$\frac{a}{4 d \left(d+e x^2\right)^2}+\frac{a}{2 d^2 \left(d+e x^2\right)}+\frac{a \text{ Log}[x]}{d^3}-\frac{a \text{ Log}\left[d+e x^2\right]}{2 d^3}+$$

$$b \left(\frac{1}{16 d^2}\sqrt{e} \left(\frac{i c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}} x}{\sqrt{d} \left(c^2 d-e\right) \left(-i \sqrt{d}+\sqrt{e} x\right)}-\frac{\text{ArcCsch}[c x]}{\sqrt{e} \left(-i \sqrt{d}+\sqrt{e} x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c x}\right]}{d \sqrt{e}}+\right.\right.$$

$$\left.\frac{i \left(2 c^2 d-e\right) \text{ Log}\left[\frac{4 d \sqrt{c^2 d-e} \sqrt{e} \left(\sqrt{e}+i c \left(c \sqrt{d}-\sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}\right) x\right)}{\left(2 c^2 d-e\right) \left(\sqrt{d}+i \sqrt{e} x\right)}\right]}{d \left(c^2 d-e\right)^{3/2}}\right)+\frac{1}{16 d^2}$$

$$\sqrt{e}\left(-\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}{\sqrt{d}\,(c^2d-e)\,(i\sqrt{d}+\sqrt{e}\,x)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\,(i\sqrt{d}+\sqrt{e}\,x)^2}-\frac{\text{ArcSinh}[\frac{1}{c\,x}]}{d\,\sqrt{e}}+\frac{1}{d\,(c^2d-e)^{3/2}}\right.$$

$$i\,(2c^2d-e)\,\text{Log}\Big[\Big(4\,i\,d\,\sqrt{c^2d-e}\,\sqrt{e}\,\Big(i\sqrt{e}+c\,\Big(c\sqrt{d}+\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}\Big)\,x\Big)\Big)\Big/$$

$$\left.\Big((2c^2d-e)\,(\sqrt{d}-i\sqrt{e}\,x)\Big)\Big]\right)-\frac{1}{16\,d^{5/2}}$$

$$5\,i\,\sqrt{e}\left(-\frac{\text{ArcCsch}[c\,x]}{i\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}[\frac{1}{c\,x}]}{\sqrt{e}}-\frac{\text{Log}\Big[\frac{2\sqrt{d}\,\sqrt{e}\,\big(i\sqrt{e}+c\,\big(c\sqrt{d}+i\sqrt{-c^2d+e}\,\sqrt{1+\frac{1}{c^2x^2}}\big)\,x\big)}{\sqrt{-c^2d+e}\,(i\sqrt{d}+\sqrt{e}\,x)}\Big]}{\sqrt{-c^2d+e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{16\,d^{5/2}}5\,i\,\sqrt{e}\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}[\frac{1}{c\,x}]}{\sqrt{e}}-\frac{\text{Log}\Big[-\frac{2\sqrt{d}\,\sqrt{e}\,\big(\sqrt{e}+c\,\big(i\,c\sqrt{d}+\sqrt{-c^2d+e}\,\sqrt{1+\frac{1}{c^2x^2}}\big)\,x\big)}{\sqrt{-c^2d+e}\,(\sqrt{d}+i\sqrt{e}\,x)}\Big]}{\sqrt{-c^2d+e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{2\,d^3}\left(-\text{ArcCsch}[c\,x]\,\left(\text{ArcCsch}[c\,x]+2\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]\right)+\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]\right)-$$

$$\frac{1}{16\,d^3}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.8\,\text{PolyLog}\left[2,\,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)-$$

$$\frac{1}{16\,d^3}\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\right.$$

$$\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left.\left.8\,\text{PolyLog}\left[2,\,\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\right)\right)$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 1106 leaves, 35 steps) :

$$-\frac{b\,c\,\sqrt{-d}\,\sqrt{1+\frac{1}{c^2 x^2}}}{16\,(c^2 d-e)\,e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{b\,c\,\sqrt{-d}\,\sqrt{1+\frac{1}{c^2 x^2}}}{16\,(c^2 d-e)\,e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}+$$

$$\frac{\sqrt{-d}\,(a+b\,\text{ArcCsch}[c\,x])}{16\,e^{3/2}\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)^2}+\frac{3\,(a+b\,\text{ArcCsch}[c\,x])}{16\,e^2\left(\sqrt{-d}\,\sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{-d}\,(a+b\,\text{ArcCsch}[c\,x])}{16\,e^{3/2}\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)^2}-$$

$$\frac{3\,(a+b\,\text{ArcCsch}[c\,x])}{16\,e^2\left(\sqrt{-d}\,\sqrt{e}+\frac{d}{x}\right)}-\frac{3\,b\,\text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,\sqrt{d}\,\sqrt{c^2 d-e}\,e^2}+$$

$$\frac{b\,\text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,\sqrt{d}\,(c^2 d-e)^{3/2}\,e}-\frac{3\,b\,\text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,\sqrt{d}\,\sqrt{c^2 d-e}\,e^2}+$$

$$\frac{b\,\text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d}\,\sqrt{e}}{x}}{c\,\sqrt{d}\,\sqrt{c^2 d-e}\,\sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16\,\sqrt{d}\,(c^2 d-e)^{3/2}\,e}+\frac{3\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-$$

$$\frac{3\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+\frac{3\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-$$

$$\frac{3\,(a+b\,\text{ArcCsch}[c\,x])\,\text{Log}\left[1+\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-\frac{3\,b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+$$

$$\frac{3\,b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}-\frac{3\,b\,\text{PolyLog}\left[2,-\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}+\frac{3\,b\,\text{PolyLog}\left[2,\frac{c\,\sqrt{-d}\,e^{\text{ArcCsch}[c\,x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16\,\sqrt{-d}\,e^{5/2}}$$

Result (type 4, 2045 leaves):

$$\frac{a\,d\,x}{4\,e^2\,(d+e\,x^2)^2}-\frac{5\,a\,x}{8\,e^2\,(d+e\,x^2)}+\frac{3\,a\,\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right]}{8\,\sqrt{d}\,e^{5/2}}+$$

$$b\left(\frac{1}{16 e^2}\,i\sqrt{d}\left(\frac{i c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}{\sqrt{d}\left(c^2d-e\right)\left(-i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c x]}{\sqrt{e}\left(-i\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c x}\right]}{d\sqrt{e}}+\right.\right.$$

$$\left.\frac{i\left(2c^2d-e\right)\text{Log}\left[\frac{4d\sqrt{c^2d-e}\,\sqrt{e}\left(\sqrt{e}+i c\left(c\sqrt{d}-\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\left(2c^2d-e\right)\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{d\left(c^2d-e\right)^{3/2}}\right)-\frac{1}{16 e^2}$$

$$i\sqrt{d}\left(-\frac{i c\sqrt{e}\sqrt{1+\frac{1}{c^2x^2}}\,x}{\sqrt{d}\left(c^2d-e\right)\left(i\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c x]}{\sqrt{e}\left(i\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c x}\right]}{d\sqrt{e}}+\frac{1}{d\left(c^2d-e\right)^{3/2}}\right.$$

$$i\left(2c^2d-e\right)\text{Log}\left[\left(4 i d\sqrt{c^2d-e}\,\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)\right)\Big/\right.$$

$$\left.\left.\left(\left(2c^2d-e\right)\left(\sqrt{d}-i\sqrt{e}\,x\right)\right)\right]\right)+\frac{1}{16 e^2}$$

$$5\left(-\frac{\text{ArcCsch}[c x]}{i\sqrt{d}\,\sqrt{e}+e x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\left(i\sqrt{e}+c\left(c\sqrt{d}+i\sqrt{-c^2d+e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d+e}\left(i\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2d+e}}\right)}{\sqrt{d}}\right)+\frac{1}{16 e^2}$$

$$5\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{128\sqrt{d}\,e^{5/2}}3\,i\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)-$$

$$\frac{1}{128\sqrt{d}\,e^{5/2}}3\,i\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\ \frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)\Bigg)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 1106 leaves, 63 steps):

$$-\frac{b c \sqrt{1+\frac{1}{c^2 x^2}}}{16 \sqrt{-d} \left(c^2 d-e\right) \sqrt{e} \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}-\frac{b c \sqrt{1+\frac{1}{c^2 x^2}}}{16 \sqrt{-d} \left(c^2 d-e\right) \sqrt{e} \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}+$$

$$\frac{a+b \text{ArcCsch}[c x]}{16 \sqrt{-d} \sqrt{e} \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)^2}+\frac{a+b \text{ArcCsch}[c x]}{16 d e \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}-\frac{a+b \text{ArcCsch}[c x]}{16 \sqrt{-d} \sqrt{e} \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)^2}-$$

$$\frac{a+b \text{ArcCsch}[c x]}{16 d e \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}-\frac{b \text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{3/2} \left(c^2 d-e\right)^{3/2}}-$$

$$\frac{b \text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{3/2} \sqrt{c^2 d-e}\, e}-\frac{b \text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{3/2} \left(c^2 d-e\right)^{3/2}}-$$

$$\frac{b \text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{3/2} \sqrt{c^2 d-e}\, e}-\frac{\left(a+b \text{ArcCsch}[c x]\right) \text{Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{3/2} e^{3/2}}+$$

$$\frac{\left(a+b \text{ArcCsch}[c x]\right) \text{Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{3/2} e^{3/2}}-\frac{\left(a+b \text{ArcCsch}[c x]\right) \text{Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{3/2} e^{3/2}}+$$

$$\frac{\left(a+b \text{ArcCsch}[c x]\right) \text{Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{3/2} e^{3/2}}+\frac{b \text{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{3/2} e^{3/2}}-$$

$$\frac{b \text{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{3/2} e^{3/2}}+\frac{b \text{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{3/2} e^{3/2}}-\frac{b \text{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{3/2} e^{3/2}}$$

Result (type 4, 2053 leaves) :

$$-\frac{a x}{4 e \left(d+e x^2\right)^2}+\frac{a x}{8 d e \left(d+e x^2\right)}+\frac{a \text{ArcTan}\left[\frac{\sqrt{e} x}{\sqrt{d}}\right]}{8 d^{3/2} e^{3/2}}+$$

$$b\left(-\frac{1}{16\sqrt{d}\,e}\,\mathrm{i}\left(\frac{\mathrm{i}\,c\sqrt{e}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}{\sqrt{d}\,\left(c^2d-e\right)\left(-\mathrm{i}\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\,\left(-\mathrm{i}\sqrt{d}+\sqrt{e}\,x\right)^2}-\right.\right.$$

$$\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\sqrt{e}}+\frac{1}{d\left(c^2d-e\right)^{3/2}}\mathrm{i}\left(2c^2d-e\right)\text{Log}\left[\left(4d\sqrt{c^2d-e}\,\sqrt{e}\right.\right.$$

$$\left.\left.\left.\left(\sqrt{e}+\mathrm{i}\,c\left(c\sqrt{d}-\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)\right)\Big/\left(\left(2c^2d-e\right)\left(\sqrt{d}+\mathrm{i}\sqrt{e}\,x\right)\right)\right]\right)+$$

$$\frac{1}{16\sqrt{d}\,e}\,\mathrm{i}\left(-\frac{\mathrm{i}\,c\sqrt{e}\,\sqrt{1+\frac{1}{c^2x^2}}\,x}{\sqrt{d}\,\left(c^2d-e\right)\left(\mathrm{i}\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\,\left(\mathrm{i}\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\sqrt{e}}+\right.$$

$$\frac{1}{d\left(c^2d-e\right)^{3/2}}\mathrm{i}\left(2c^2d-e\right)\text{Log}\left[\left(4\,\mathrm{i}\,d\sqrt{c^2d-e}\,\sqrt{e}\right.\right.$$

$$\left.\left.\left.\left(\mathrm{i}\sqrt{e}+c\left(c\sqrt{d}+\sqrt{c^2d-e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)\right)\Big/\left(\left(2c^2d-e\right)\left(\sqrt{d}-\mathrm{i}\sqrt{e}\,x\right)\right)\right]\right)-$$

$$\frac{1}{16\,d\,e}\left(-\frac{\text{ArcCsch}[c\,x]}{\mathrm{i}\sqrt{d}\,\sqrt{e}+e\,x}-\frac{\mathrm{i}\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\sqrt{d}\,\sqrt{e}\,\left(\mathrm{i}\sqrt{e}+c\left(c\sqrt{d}+\mathrm{i}\sqrt{-c^2d+e}\,\sqrt{1+\frac{1}{c^2x^2}}\right)x\right)}{\sqrt{-c^2d+e}\,\left(\mathrm{i}\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2d+e}}\right)}{\sqrt{d}}\right)-$$

$$\frac{1}{16\,d\,e}\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)x\right)}{\sqrt{-c^2\,d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{128\,d^{3/2}\,e^{3/2}}\,i\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2\,d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2\,d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,-\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\mathbb{e}^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)-$$

$$\frac{1}{128\,d^{3/2}\,e^{3/2}}\,\mathbb{i}\,\Bigg(\pi^2-4\,\mathbb{i}\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,\mathbb{i}\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\mathbb{e}^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,\mathbb{i}\,\pi\,\text{Log}\left[1+\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\mathbb{e}^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\mathbb{e}^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\mathbb{e}^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,\mathbb{i}\,\pi\,\text{Log}\left[1-\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\mathbb{e}^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\mathbb{e}^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,\mathbb{i}\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{\mathbb{i}\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)\mathbb{e}^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,\mathbb{i}\,\pi\,\text{Log}\left[\sqrt{e}-\frac{\mathbb{i}\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\mathbb{e}^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,-\frac{\mathbb{i}\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)\mathbb{e}^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,\ \frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)\Bigg)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{a+b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x^2\right)^3}\,dx$$

Optimal (type 4, 1096 leaves, 81 steps):

$$-\frac{b c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}}}{16 (-d)^{3/2} \left(c^2 d-e\right) \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}-\frac{b c \sqrt{e} \sqrt{1+\frac{1}{c^2 x^2}}}{16 (-d)^{3/2} \left(c^2 d-e\right) \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}+$$

$$\frac{\sqrt{e} \left(a+b \text{ArcCsch}[c x]\right)}{16 (-d)^{3/2} \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)^2}-\frac{5 \left(a+b \text{ArcCsch}[c x]\right)}{16 d^2 \left(\sqrt{-d} \sqrt{e}-\frac{d}{x}\right)}-\frac{\sqrt{e} \left(a+b \text{ArcCsch}[c x]\right)}{16 (-d)^{3/2} \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)^2}+$$

$$\frac{5 \left(a+b \text{ArcCsch}[c x]\right)}{16 d^2 \left(\sqrt{-d} \sqrt{e}+\frac{d}{x}\right)}+\frac{5 b \text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{5/2} \sqrt{c^2 d-e}}+$$

$$\frac{b e \text{ArcTanh}\left[\frac{c^2 d-\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{5/2} \left(c^2 d-e\right)^{3/2}}+\frac{5 b \text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{5/2} \sqrt{c^2 d-e}}+$$

$$\frac{b e \text{ArcTanh}\left[\frac{c^2 d+\frac{\sqrt{-d} \sqrt{e}}{x}}{c \sqrt{d} \sqrt{c^2 d-e} \sqrt{1+\frac{1}{c^2 x^2}}}\right]}{16 d^{5/2} \left(c^2 d-e\right)^{3/2}}+\frac{3 \left(a+b \text{ArcCsch}[c x]\right) \text{Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}-$$

$$\frac{3 \left(a+b \text{ArcCsch}[c x]\right) \text{Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}+\frac{3 \left(a+b \text{ArcCsch}[c x]\right) \text{Log}\left[1-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}-$$

$$\frac{3 \left(a+b \text{ArcCsch}[c x]\right) \text{Log}\left[1+\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}-\frac{3 b \text{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}+$$

$$\frac{3 b \text{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}-\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}-\frac{3 b \text{PolyLog}\left[2,-\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}+\frac{3 b \text{PolyLog}\left[2,\frac{c \sqrt{-d}\, e^{\text{ArcCsch}[c x]}}{\sqrt{e}+\sqrt{-c^2 d+e}}\right]}{16 (-d)^{5/2} \sqrt{e}}$$

Result (type 4, 2038 leaves):

$$\frac{a x}{4 d \left(d+e x^2\right)^2}+\frac{3 a x}{8 d^2 \left(d+e x^2\right)}+\frac{3 a \text{ArcTan}\left[\frac{\sqrt{e}\, x}{\sqrt{d}}\right]}{8 d^{5/2} \sqrt{e}}+$$

$$b\left(\frac{1}{16\,d^{3/2}}\,i\left(\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\,\left(c^2\,d-e\right)\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\,\left(-i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\right.\right.$$

$$\left.\frac{i\,\left(2\,c^2\,d-e\right)\,\text{Log}\left[\frac{4\,d\,\sqrt{c^2\,d-e}\,\sqrt{e}\,\left(\sqrt{e}+i\,c\,\left(c\,\sqrt{d}-\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\left(2\,c^2\,d-e\right)\,\left(\sqrt{d}+i\,\sqrt{e}\,x\right)}\right]}{d\,\left(c^2\,d-e\right)^{3/2}}\right)-\frac{1}{16\,d^{3/2}}$$

$$i\left(-\frac{i\,c\,\sqrt{e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x}{\sqrt{d}\,\left(c^2\,d-e\right)\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}-\frac{\text{ArcCsch}[c\,x]}{\sqrt{e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)^2}-\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{d\,\sqrt{e}}+\frac{1}{d\,\left(c^2\,d-e\right)^{3/2}}\right.$$

$$i\,\left(2\,c^2\,d-e\right)\,\text{Log}\left[\left(4\,i\,d\,\sqrt{c^2\,d-e}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\,\left(c\,\sqrt{d}+\sqrt{c^2\,d-e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)\right)\Big/\right.$$

$$\left.\left.\left(\left(2\,c^2\,d-e\right)\,\left(\sqrt{d}-i\,\sqrt{e}\,x\right)\right)\right]\right)-\frac{1}{16\,d^2}$$

$$3\left(-\frac{\text{ArcCsch}[c\,x]}{i\,\sqrt{d}\,\sqrt{e}+e\,x}-\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[\frac{2\,\sqrt{d}\,\sqrt{e}\,\left(i\,\sqrt{e}+c\,\left(c\,\sqrt{d}+i\,\sqrt{-c^2\,d+e}\,\sqrt{1+\frac{1}{c^2\,x^2}}\right)\,x\right)}{\sqrt{-c^2\,d+e}\,\left(i\,\sqrt{d}+\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2\,d+e}}\right)}{\sqrt{d}}\right)-\frac{1}{16\,d^2}$$

$$3\left(-\frac{\text{ArcCsch}[c\,x]}{-i\sqrt{d}\,\sqrt{e}+e\,x}+\frac{i\left(\frac{\text{ArcSinh}\left[\frac{1}{c\,x}\right]}{\sqrt{e}}-\frac{\text{Log}\left[-\frac{2\sqrt{d}\,\sqrt{e}\left(\sqrt{e}+c\left(i\,c\sqrt{d}+\sqrt{-c^2 d+e}\,\sqrt{1+\frac{1}{c^2 x^2}}\right)x\right)}{\sqrt{-c^2 d+e}\left(\sqrt{d}+i\sqrt{e}\,x\right)}\right]}{\sqrt{-c^2 d+e}}\right)}{\sqrt{d}}\right)+$$

$$\frac{1}{128\,d^{5/2}\sqrt{e}}\,3\,i\left(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2+\right.$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}-\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1+\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1+\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}+\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,\,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,\,\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{PolyLog}\left[2,-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]\Bigg)-$$

$$\frac{1}{128\,d^{5/2}\sqrt{e}}3\,i\Bigg(\pi^2-4\,i\,\pi\,\text{ArcCsch}[c\,x]-8\,\text{ArcCsch}[c\,x]^2-$$

$$32\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{ArcTan}\left[\frac{\left(c\sqrt{d}+\sqrt{e}\right)\text{Cot}\left[\frac{1}{4}\left(\pi+2\,i\,\text{ArcCsch}[c\,x]\right)\right]}{\sqrt{-c^2 d+e}}\right]-$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-e^{-2\,\text{ArcCsch}[c\,x]}\right]+4\,i\,\pi\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]\text{Log}\left[1+\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$4\,i\,\pi\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$8\,\text{ArcCsch}[c\,x]\,\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-16\,i\,\text{ArcSin}\left[\frac{\sqrt{1-\frac{\sqrt{e}}{c\sqrt{d}}}}{\sqrt{2}}\right]$$

$$\text{Log}\left[1-\frac{i\left(\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]-4\,i\,\pi\,\text{Log}\left[\sqrt{e}-\frac{i\sqrt{d}}{x}\right]+$$

$$4\,\text{PolyLog}\left[2,e^{-2\,\text{ArcCsch}[c\,x]}\right]+8\,\text{PolyLog}\left[2,-\frac{i\left(-\sqrt{e}+\sqrt{-c^2 d+e}\right)e^{\text{ArcCsch}[c\,x]}}{c\sqrt{d}}\right]+$$

$$\left. \left. 8\,\text{PolyLog}\left[2,\ \frac{i\left(\sqrt{e}+\sqrt{-c^2\,d+e}\right)\,e^{\text{ArcCsch}[c\,x]}}{c\,\sqrt{d}}\right]\right)\right)$$

Problem 118: Result unnecessarily involves higher level functions.

$$\int x^5\,\sqrt{d+e\,x^2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 3, 413 leaves, 12 steps):

$$-\frac{b\left(23\,c^4\,d^2-12\,c^2\,d\,e-75\,e^2\right)\,x\,\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{1680\,c^5\,e^2\,\sqrt{-c^2\,x^2}}-\frac{b\left(29\,c^2\,d+25\,e\right)\,x\,\sqrt{-1-c^2\,x^2}\,\left(d+e\,x^2\right)^{3/2}}{840\,c^3\,e^2\,\sqrt{-c^2\,x^2}}+\frac{b\,x\,\sqrt{-1-c^2\,x^2}\,\left(d+e\,x^2\right)^{5/2}}{42\,c\,e^2\,\sqrt{-c^2\,x^2}}+\frac{d^2\,\left(d+e\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^3}-\frac{2\,d\,\left(d+e\,x^2\right)^{5/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5\,e^3}+\frac{\left(d+e\,x^2\right)^{7/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{7\,e^3}+\frac{b\left(105\,c^6\,d^3+35\,c^4\,d^2\,e+63\,c^2\,d\,e^2-75\,e^3\right)\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{1680\,c^6\,e^{5/2}\,\sqrt{-c^2\,x^2}}+\frac{8\,b\,c\,d^{7/2}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\right]}{105\,e^3\,\sqrt{-c^2\,x^2}}$$

Result (type 6, 713 leaves):

$$\left(b\, d \sqrt{1+\frac{1}{c^2 x^2}}\; x^3 \left(-\left(105 c^6 d^3+35 c^4 d^2 e+63 c^2 d e^2-75 e^3\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.$$
$$\left(c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+$$
$$4\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]$$
$$\left(\left(35 c^6 d^2 e^2 x^2+63 c^4 d e^3 x^2-75 c^2 e^4 x^2+c^8 d^3 \left(-128 d+105 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\right.\right.$$
$$\left.\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+32 c^8 d^3 x^2 \left(e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$
$$\left.\left.\left.\left.c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/$$
$$\left(840 c^5 e^2 \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+\right.\right.$$
$$\left.c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$
$$\left(-4 d\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+\right.$$
$$\left.\left.x^2 \left(e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$
$$\frac{1}{1680 c^5 e^3}\sqrt{d+e x^2} \left(16 a c^5 \left(8 d^3-4 d^2 e x^2+3 d e^2 x^4+15 e^3 x^6\right)+\right.$$
$$b\, e \sqrt{1+\frac{1}{c^2 x^2}}\; x \left(75 e^2-2 c^2 e \left(19 d+25 e x^2\right)+c^4 \left(-41 d^2+22 d e x^2+40 e^2 x^4\right)\right)+$$
$$\left.16 b c^5 \left(8 d^3-4 d^2 e x^2+3 d e^2 x^4+15 e^3 x^6\right) \text{ArcCsch}[c x]\right)$$

Problem 119: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x^3 \sqrt{d+e x^2}\, \left(a+b\, \text{ArcCsch}[c x]\right) dx$$

Optimal (type 3, 302 leaves, 11 steps):

$$\frac{b\left(c^2 d-9 e\right) x \sqrt{-1-c^2 x^2}\,\sqrt{d+e x^2}}{120 c^3 e \sqrt{-c^2 x^2}}+\frac{b x \sqrt{-1-c^2 x^2}\,\left(d+e x^2\right)^{3/2}}{20 c e \sqrt{-c^2 x^2}}-\frac{d\left(d+e x^2\right)^{3/2}\left(a+b\,\text{ArcCsch}[c x]\right)}{3 e^2}+\frac{\left(d+e x^2\right)^{5/2}\left(a+b\,\text{ArcCsch}[c x]\right)}{5 e^2}-\frac{b\left(15 c^4 d^2+10 c^2 d e-9 e^2\right) x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{120 c^4 e^{3/2} \sqrt{-c^2 x^2}}-\frac{2 b c d^{5/2} x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\,\sqrt{-1-c^2 x^2}}\right]}{15 e^2 \sqrt{-c^2 x^2}}$$

Result (type 6, 635 leaves):

$$-\left(\left(\left(b d \sqrt{1+\frac{1}{c^2 x^2}}\, x^3 \left(-\left(15 c^4 d^2+10 c^2 d e-9 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(\left(10 c^4 d e^2 x^2-9 c^2 e^3 x^2+c^6 d^2 \left(-16 d+15 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+4 c^6 d^2 x^2 \left(e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/ \left(60 c^3 e \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right) \left(-4 d\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)+\frac{1}{120 c^3 e^2}\sqrt{d+e x^2} \left(8 a c^3 \left(-2 d^2+d e x^2+3 e^2 x^4\right)+b e \sqrt{1+\frac{1}{c^2 x^2}}\, x \left(-9 e+c^2 \left(7 d+6 e x^2\right)\right)+8 b c^3 \left(-2 d^2+d e x^2+3 e^2 x^4\right) \text{ArcCsch}[c x]\right)$$

Problem 120: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x \sqrt{d+e x^2}\, (a+b \,\text{ArcCsch}[c x])\, dx$$

Optimal (type 3, 203 leaves, 9 steps):

$$\frac{b x \sqrt{-1-c^2 x^2}\, \sqrt{d+e x^2}}{6 c \sqrt{-c^2 x^2}} + \frac{\left(d+e x^2\right)^{3/2} (a+b \,\text{ArcCsch}[c x])}{3 e} + \frac{b \left(3 c^2 d-e\right) x \,\text{ArcTan}\left[\frac{\sqrt{e}\, \sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{6 c^2 \sqrt{e}\, \sqrt{-c^2 x^2}} + \frac{b c d^{3/2} x \,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\, \sqrt{-1-c^2 x^2}}\right]}{3 e \sqrt{-c^2 x^2}}$$

Result (type 6, 556 leaves):

$$\left(b d \sqrt{1+\frac{1}{c^2 x^2}}\, x^3 \left(-\left(3 c^2 d-e\right) \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -c^2 x^2, -\frac{e x^2}{d}\right] \left(c^2 d \,\text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + e \,\text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right]\right) + 2 \,\text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] \left(-2 \left(c^2 e^2 x^2 + c^4 d \left(2 d - 3 e x^2\right)\right) \text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -c^2 x^2, -\frac{e x^2}{d}\right] + c^4 d x^2 \left(e \,\text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \,\text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -c^2 x^2, -\frac{e x^2}{d}\right]\right)\right)\right)\right) \Bigg/ \left(3 c \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2 \,\text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -\frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + c^2 d \,\text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -\frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right] + e \,\text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -\frac{1}{c^2 x^2}, -\frac{d}{e x^2}\right]\right) \left(-4 d \,\text{AppellF1}\left[1, \frac{1}{2}, \frac{1}{2}, 2, -c^2 x^2, -\frac{e x^2}{d}\right] + x^2 \left(e \,\text{AppellF1}\left[2, \frac{1}{2}, \frac{3}{2}, 3, -c^2 x^2, -\frac{e x^2}{d}\right] + c^2 d \,\text{AppellF1}\left[2, \frac{3}{2}, \frac{1}{2}, 3, -c^2 x^2, -\frac{e x^2}{d}\right]\right)\right)\right) + \frac{1}{6 c e} \sqrt{d+e x^2} \left(b e \sqrt{1+\frac{1}{c^2 x^2}}\, x + 2 a c \left(d+e x^2\right) + 2 b c \left(d+e x^2\right) \text{ArcCsch}[c x]\right)$$

Problem 126: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2}\, (a+b \,\text{ArcCsch}[c x])}{x^4}\, dx$$

Optimal (type 4, 389 leaves, 8 steps):

$$-\frac{2 b c^3 \left(c^2 d-2 e\right) x^2 \sqrt{d+e x^2}}{9 d \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2}}-\frac{2 b c \left(c^2 d-2 e\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{9 d \sqrt{-c^2 x^2}}+\frac{b c \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{9 x^2 \sqrt{-c^2 x^2}}-\frac{\left(d+e x^2\right)^{3/2} \left(a+b \,\text{ArcCsch}[c x]\right)}{3 d x^3}+\frac{2 b c^2 \left(c^2 d-2 e\right) x \sqrt{d+e x^2}\, \text{EllipticE}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{9 d \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}-\frac{b \left(c^2 d-3 e\right) e x \sqrt{d+e x^2}\, \text{EllipticF}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{9 d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}$$

Result (type 8, 25 leaves):

$$\int \frac{\sqrt{d+e x^2} \left(a+b \,\text{ArcCsch}[c x]\right)}{x^4} \, dx$$

Problem 127: Unable to integrate problem.

$$\int \frac{\sqrt{d+e x^2} \left(a+b \,\text{ArcCsch}[c x]\right)}{x^6} \, dx$$

Optimal (type 4, 527 leaves, 9 steps):

$$\frac{b c^3 \left(24 c^4 d^2-19 c^2 d e-31 e^2\right) x^2 \sqrt{d+e x^2}}{225 d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2}}+\frac{b c \left(24 c^4 d^2-19 c^2 d e-31 e^2\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{225 d^2 \sqrt{-c^2 x^2}}-\frac{b c \left(12 c^2 d+e\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{225 d x^2 \sqrt{-c^2 x^2}}+\frac{b c \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{3/2}}{25 d x^4 \sqrt{-c^2 x^2}}-\frac{\left(d+e x^2\right)^{3/2} \left(a+b \,\text{ArcCsch}[c x]\right)}{5 d x^5}+\frac{2 e \left(d+e x^2\right)^{3/2} \left(a+b \,\text{ArcCsch}[c x]\right)}{15 d^2 x^3}-\left(b c^2 \left(24 c^4 d^2-19 c^2 d e-31 e^2\right) x \sqrt{d+e x^2}\, \text{EllipticE}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]\right)\Big/\left(225 d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}\right)+\left(2 b e \left(6 c^4 d^2-4 c^2 d e-15 e^2\right) x \sqrt{d+e x^2}\, \text{EllipticF}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]\right)\Big/\left(225 d^3 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}\right)$$

Result (type 8, 25 leaves):

$$\int \frac{\sqrt{d+e x^2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{x^6}\,dx$$

Problem 128: Result unnecessarily involves higher level functions.

$$\int x^3\,\left(d+e x^2\right)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 3, 384 leaves, 12 steps):

$$-\frac{b\left(3 c^4 d^2+38 c^2 d e-25 e^2\right) x \sqrt{-1-c^2 x^2}\,\sqrt{d+e x^2}}{560 c^5 e \sqrt{-c^2 x^2}}+\frac{b\left(13 c^2 d-25 e\right) x \sqrt{-1-c^2 x^2}\,\left(d+e x^2\right)^{3/2}}{840 c^3 e \sqrt{-c^2 x^2}}+\frac{b x \sqrt{-1-c^2 x^2}\,\left(d+e x^2\right)^{5/2}}{42 c e \sqrt{-c^2 x^2}}-\frac{d\left(d+e x^2\right)^{5/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5 e^2}+\frac{\left(d+e x^2\right)^{7/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{7 e^2}-\frac{b\left(35 c^6 d^3+35 c^4 d^2 e-63 c^2 d e^2+25 e^3\right) x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2 x^2}}{c\sqrt{d+e x^2}}\right]}{560 c^6 e^{3/2}\sqrt{-c^2 x^2}}-\frac{2 b c d^{7/2} x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\,\sqrt{-1-c^2 x^2}}\right]}{35 e^2\sqrt{-c^2 x^2}}$$

Result (type 6, 687 leaves):

$$-\left(\left(\left(b\,d\,\sqrt{1+\frac{1}{c^2 x^2}}\;x^3\right.\right.\right.$$

$$\left(-\left(35\,c^6\,d^3+35\,c^4\,d^2\,e-63\,c^2\,d\,e^2+25\,e^3\right)\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right.$$

$$\left(c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)+$$

$$4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]$$

$$\left(\left(35\,c^6\,d^2\,e^2\,x^2-63\,c^4\,d\,e^3\,x^2+25\,c^2\,e^4\,x^2+c^8\,d^3\,\left(-32\,d+35\,e\,x^2\right)\right)\right.$$

$$\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+8\,c^8\,d^3\,x^2\,\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},\right.\right.$$

$$\left.\left.\left.\left.\left.3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\right)\Big/$$

$$\left(280\,c^5\,e\,\left(1+c^2\,x^2\right)\,\sqrt{d+e\,x^2}\,\left(-4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+\right.\right.$$

$$\left.c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)$$

$$\left(-4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+x^2\,\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+\right.\right.$$

$$\left.\left.\left.\left.c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)+$$

$$\frac{1}{1680\,c^5\,e^2}\sqrt{d+e\,x^2}\,\left(-48\,a\,c^5\,\left(2\,d-5\,e\,x^2\right)\,\left(d+e\,x^2\right)^2+b\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\right.$$

$$\left(75\,e^2-2\,c^2\,e\,\left(82\,d+25\,e\,x^2\right)+c^4\,\left(57\,d^2+106\,d\,e\,x^2+40\,e^2\,x^4\right)\right)-$$

$$\left.48\,b\,c^5\,\left(2\,d-5\,e\,x^2\right)\,\left(d+e\,x^2\right)^2\,\text{ArcCsch}[c\,x]\right)$$

Problem 129: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int x\,\left(d+e\,x^2\right)^{3/2}\,\left(a+b\,\text{ArcCsch}[c\,x]\right)\,dx$$

Optimal (type 3, 270 leaves, 10 steps):

$$\frac{b\left(7 c^2 d-3 e\right) x \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{40 c^3 \sqrt{-c^2 x^2}}+\frac{b x \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{3/2}}{20 c \sqrt{-c^2 x^2}}+\frac{\left(d+e x^2\right)^{5/2} (a+b \text{ArcCsch}[c x])}{5 e}+\frac{b \left(15 c^4 d^2-10 c^2 d e+3 e^2\right) x \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{40 c^4 \sqrt{e} \sqrt{-c^2 x^2}}+\frac{b c d^{5/2} x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1-c^2 x^2}}\right]}{5 e \sqrt{-c^2 x^2}}$$

Result (type 6, 610 leaves):

$$\left(b d \sqrt{1+\frac{1}{c^2 x^2}} x^3 \left(-\left(15 c^4 d^2-10 c^2 d e+3 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(\left(-10 c^4 d e^2 x^2+3 c^2 e^3 x^2+c^6 d^2 \left(-8 d+15 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+2 c^6 d^2 x^2 \left(e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/ \left(20 c^3 \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right) \left(-4 d \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+\frac{1}{40 c^3 e}\sqrt{d+e x^2} \left(8 a c^3 \left(d+e x^2\right)^2+b e \sqrt{1+\frac{1}{c^2 x^2}} x \left(-3 e+c^2 \left(9 d+2 e x^2\right)\right)+8 b c^3 \left(d+e x^2\right)^2 \text{ArcCsch}[c x]\right)$$

Problem 136: Unable to integrate problem.

$$\int \frac{\left(d+e x^2\right)^{3/2} (a+b \text{ArcCsch}[c x])}{x^6} \, dx$$

Optimal (type 4, 492 leaves, 9 steps):

$$\frac{b\,c^3\left(8\,c^4\,d^2-23\,c^2\,d\,e+23\,e^2\right)x^2\sqrt{d+e\,x^2}}{75\,d\,\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}}+\frac{b\,c\left(8\,c^4\,d^2-23\,c^2\,d\,e+23\,e^2\right)\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{75\,d\,\sqrt{-c^2\,x^2}}-\frac{4\,b\,c\left(c^2\,d-2\,e\right)\sqrt{-1-c^2\,x^2}\,\sqrt{d+e\,x^2}}{75\,x^2\,\sqrt{-c^2\,x^2}}+\frac{b\,c\,\sqrt{-1-c^2\,x^2}\,\left(d+e\,x^2\right)^{3/2}}{25\,x^4\,\sqrt{-c^2\,x^2}}-\frac{\left(d+e\,x^2\right)^{5/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{5\,d\,x^5}-\left(b\,c^2\left(8\,c^4\,d^2-23\,c^2\,d\,e+23\,e^2\right)x\,\sqrt{d+e\,x^2}\,\text{EllipticE}\left[\text{ArcTan}[c\,x],1-\frac{e}{c^2\,d}\right]\right)\Big/\left(75\,d\,\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}\,\sqrt{\frac{d+e\,x^2}{d\left(1+c^2\,x^2\right)}}\right)+\left(b\,e\left(4\,c^4\,d^2-11\,c^2\,d\,e+15\,e^2\right)x\,\sqrt{d+e\,x^2}\,\text{EllipticF}\left[\text{ArcTan}[c\,x],1-\frac{e}{c^2\,d}\right]\right)\Big/\left(75\,d^2\,\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}\,\sqrt{\frac{d+e\,x^2}{d\left(1+c^2\,x^2\right)}}\right)$$

Result (type 8, 25 leaves):

$$\int\frac{\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{x^6}\,dx$$

Problem 137: Unable to integrate problem.

$$\int\frac{\left(d+e\,x^2\right)^{3/2}\left(a+b\,\text{ArcCsch}[c\,x]\right)}{x^8}\,dx$$

Optimal (type 4, 643 leaves, 10 steps):

$$-\frac{b c^3 \left(240 c^6 d^3-528 c^4 d^2 e+193 c^2 d e^2+247 e^3\right) x^2 \sqrt{d+e x^2}}{3675 d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2}}-\frac{1}{3675 d^2 \sqrt{-c^2 x^2}}$$

$$b c \left(240 c^6 d^3-528 c^4 d^2 e+193 c^2 d e^2+247 e^3\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}+$$

$$\frac{b c \left(120 c^4 d^2-159 c^2 d e-37 e^2\right) \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{3675 d x^2 \sqrt{-c^2 x^2}}-$$

$$\frac{b c \left(30 c^2 d-11 e\right) \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{3/2}}{1225 d x^4 \sqrt{-c^2 x^2}}+\frac{b c \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{5/2}}{49 d x^6 \sqrt{-c^2 x^2}}-$$

$$\frac{\left(d+e x^2\right)^{5/2} \left(a+b \text{ArcCsch}[c x]\right)}{7 d x^7}+\frac{2 e \left(d+e x^2\right)^{5/2} \left(a+b \text{ArcCsch}[c x]\right)}{35 d^2 x^5}+$$

$$\left(b c^2 \left(240 c^6 d^3-528 c^4 d^2 e+193 c^2 d e^2+247 e^3\right) x \sqrt{d+e x^2} \text{ EllipticE}\left[\text{ArcTan}[c x], 1-\frac{e}{c^2 d}\right]\right) \Bigg/$$

$$\left(3675 d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}\right)-$$

$$\left(b e \left(120 c^6 d^3-249 c^4 d^2 e+71 c^2 d e^2+210 e^3\right) x \sqrt{d+e x^2} \text{ EllipticF}\left[\text{ArcTan}[c x], 1-\frac{e}{c^2 d}\right]\right) \Bigg/$$

$$\left(3675 d^3 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}\right)$$

Result (type 8, 25 leaves):

$$\int \frac{\left(d+e x^2\right)^{3/2} \left(a+b \text{ArcCsch}[c x]\right)}{x^8} \, dx$$

Problem 138: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 \left(a+b \text{ArcCsch}[c x]\right)}{\sqrt{d+e x^2}} \, dx$$

Optimal (type 3, 329 leaves, 11 steps):

$$-\frac{b \left(19 c^2 d+9 e\right) x \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{120 c^3 e^2 \sqrt{-c^2 x^2}}+$$

$$\frac{b x \sqrt{-1-c^2 x^2} \left(d+e x^2\right)^{3/2}}{20 c e^2 \sqrt{-c^2 x^2}}+\frac{d^2 \sqrt{d+e x^2} \left(a+b \text{ArcCsch}[c x]\right)}{e^3}-$$

$$\frac{2 d \left(d+e x^2\right)^{3/2} \left(a+b \text{ArcCsch}[c x]\right)}{3 e^3}+\frac{\left(d+e x^2\right)^{5/2} \left(a+b \text{ArcCsch}[c x]\right)}{5 e^3}+$$

$$\frac{b \left(45 c^4 d^2+10 c^2 d e+9 e^2\right) x \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{120 c^4 e^{5/2} \sqrt{-c^2 x^2}}+\frac{8 b c d^{5/2} x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1-c^2 x^2}}\right]}{15 e^3 \sqrt{-c^2 x^2}}$$

Result (type 6, 637 leaves):

$$\left(b\, d\, \sqrt{1+\frac{1}{c^2 x^2}}\, x^3 \left(-\left(45 c^4 d^2+10 c^2 d e+9 e^2\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+ 4\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(\left(10 c^4 d e^2 x^2+9 c^2 e^3 x^2+c^6 d^2 \left(-64 d+45 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+ 16 c^6 d^2 x^2 \left(e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+ c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/$$

$$\left(60 c^3 e^2 \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+ c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right) \left(-4 d\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+ x^2 \left(e\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)+$$

$$\frac{1}{120 c^3 e^3}\sqrt{d+e x^2} \left(8 a c^3 \left(8 d^2-4 d e x^2+3 e^2 x^4\right)+ b e \sqrt{1+\frac{1}{c^2 x^2}}\, x \left(-9 e+c^2 \left(-13 d+6 e x^2\right)\right)+ 8 b c^3 \left(8 d^2-4 d e x^2+3 e^2 x^4\right) \text{ArcCsch}[c x]\right)$$

Problem 139: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^3 \left(a+b\, \text{ArcCsch}[c x]\right)}{\sqrt{d+e x^2}}\, dx$$

Optimal (type 3, 229 leaves, 10 steps) :

$$\frac{b x \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{6 c e \sqrt{-c^2 x^2}}-\frac{d \sqrt{d+e x^2} \left(a+b \text{ArcCsch}[c x]\right)}{e^2}+\frac{\left(d+e x^2\right)^{3/2} \left(a+b \text{ArcCsch}[c x]\right)}{3 e^2}-\frac{b \left(3 c^2 d+e\right) x \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{6 c^2 e^{3/2} \sqrt{-c^2 x^2}}-\frac{2 b c d^{3/2} x \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1-c^2 x^2}}\right]}{3 e^2 \sqrt{-c^2 x^2}}$$

Result (type 6, 560 leaves):

$$-\left(\left(b d \sqrt{1+\frac{1}{c^2 x^2}} x^3 \left(-\left(3 c^2 d+e\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right] \left(c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+4 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right] \left(\left(c^2 e^2 x^2+c^4 d \left(-4 d+3 e x^2\right)\right) \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+c^4 d x^2 \left(e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right) \Big/ \left(3 c e \left(1+c^2 x^2\right) \sqrt{d+e x^2} \left(-4 c^2 e x^2 \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right) \left(-4 d \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2 \left(e \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)+\frac{1}{6 c e^2}\sqrt{d+e x^2} \left(-4 a c d+b e \sqrt{1+\frac{1}{c^2 x^2}} x+2 a c e x^2+2 b c \left(-2 d+e x^2\right) \text{ArcCsch}[c x]\right)$$

Problem 140: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x \left(a+b \text{ArcCsch}[c x]\right)}{\sqrt{d+e x^2}} \, dx$$

Optimal (type 3, 135 leaves, 9 steps):

$$\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsch}[c x])}{e}+\frac{b x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2 x^2}}{c\,\sqrt{d+e x^2}}\right]}{\sqrt{e}\,\sqrt{-c^2 x^2}}+\frac{b c\,\sqrt{d}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d}\,\sqrt{-1-c^2 x^2}}\right]}{e\,\sqrt{-c^2 x^2}}$$

Result (type 6, 271 leaves):

$$\left(3 b\,(c^2 d-e)\,\sqrt{1+\frac{1}{c^2 x^2}}\,\sqrt{d+e x^2}\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e\,(1+c^2 x^2)}{-c^2 d+e},1+c^2 x^2\right]\right)\Big/$$
$$\left(c e x\left(3\,(c^2 d-e)\,\text{AppellF1}\left[\frac{1}{2},-\frac{1}{2},1,\frac{3}{2},\frac{e\,(1+c^2 x^2)}{-c^2 d+e},1+c^2 x^2\right]+\right.\right.$$
$$(1+c^2 x^2)\left(2\,(c^2 d-e)\,\text{AppellF1}\left[\frac{3}{2},-\frac{1}{2},2,\frac{5}{2},\frac{e\,(1+c^2 x^2)}{-c^2 d+e},1+c^2 x^2\right]+\right.$$
$$\left.\left.\left.e\,\text{AppellF1}\left[\frac{3}{2},\frac{1}{2},1,\frac{5}{2},\frac{e\,(1+c^2 x^2)}{-c^2 d+e},1+c^2 x^2\right]\right)\right)\right)+\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsch}[c x])}{e}$$

Problem 146: Unable to integrate problem.

$$\int \frac{a+b\,\text{ArcCsch}[c x]}{x^4\,\sqrt{d+e x^2}}\,dx$$

Optimal (type 4, 425 leaves, 8 steps):

$$-\frac{b c^3\,(2 c^2 d+5 e)\,x^2\,\sqrt{d+e x^2}}{9 d^2\,\sqrt{-c^2 x^2}\,\sqrt{-1-c^2 x^2}}-\frac{b c\,(2 c^2 d+5 e)\,\sqrt{-1-c^2 x^2}\,\sqrt{d+e x^2}}{9 d^2\,\sqrt{-c^2 x^2}}+$$
$$\frac{b c\,\sqrt{-1-c^2 x^2}\,\sqrt{d+e x^2}}{9 d x^2\,\sqrt{-c^2 x^2}}-\frac{\sqrt{d+e x^2}\,(a+b\,\text{ArcCsch}[c x])}{3 d x^3}+\frac{2 e\,\sqrt{d+e x^2}\,(a+b\,\text{ArcCsch}[c x])}{3 d^2 x}+$$
$$\frac{b c^2\,(2 c^2 d+5 e)\,x\,\sqrt{d+e x^2}\,\text{EllipticE}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{9 d^2\,\sqrt{-c^2 x^2}\,\sqrt{-1-c^2 x^2}\,\sqrt{\frac{d+e x^2}{d\,(1+c^2 x^2)}}}-$$
$$\frac{b e\,(c^2 d+6 e)\,x\,\sqrt{d+e x^2}\,\text{EllipticF}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{9 d^3\,\sqrt{-c^2 x^2}\,\sqrt{-1-c^2 x^2}\,\sqrt{\frac{d+e x^2}{d\,(1+c^2 x^2)}}}$$

Result (type 8, 25 leaves):

$$\int \frac{a+b\,\text{ArcCsch}[c x]}{x^4\,\sqrt{d+e x^2}}\,dx$$

Problem 147: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int \frac{x^5 \left(a + b\, \text{ArcCsch}[c\, x]\right)}{\left(d + e\, x^2\right)^{3/2}}\, dx$$

Optimal (type 3, 256 leaves, 10 steps):

$$\frac{b\, x \sqrt{-1 - c^2 x^2} \sqrt{d + e\, x^2}}{6\, c\, e^2 \sqrt{-c^2 x^2}} - \frac{d^2 \left(a + b\, \text{ArcCsch}[c\, x]\right)}{e^3 \sqrt{d + e\, x^2}} - \frac{2\, d \sqrt{d + e\, x^2} \left(a + b\, \text{ArcCsch}[c\, x]\right)}{e^3} + \frac{\left(d + e\, x^2\right)^{3/2} \left(a + b\, \text{ArcCsch}[c\, x]\right)}{3\, e^3} - \frac{b \left(9\, c^2 d + e\right) x\, \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+e\, x^2}}\right]}{6\, c^2 e^{5/2} \sqrt{-c^2 x^2}} - \frac{8\, b\, c\, d^{3/2}\, x\, \text{ArcTan}\left[\frac{\sqrt{d+e\, x^2}}{\sqrt{d} \sqrt{-1-c^2 x^2}}\right]}{3\, e^3 \sqrt{-c^2 x^2}}$$

Result (type 6, 592 leaves):

$$-\left(\left(b\,d\,\sqrt{1+\frac{1}{c^2 x^2}}\,x^3\left(-\left(9 c^2 d+e\right)\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]\right.\right.\right.$$

$$\left(c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)+$$

$$4\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\left(\left(c^2 e^2 x^2+c^4 d\left(-16 d+9 e x^2\right)\right)\right.$$

$$\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+4 c^4 d x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,\right.\right.$$

$$\left.\left.\left.\left.\left.-c^2 x^2,-\frac{e x^2}{d}\right]+c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)\Big/$$

$$\left(3 c e^2\left(1+c^2 x^2\right)\sqrt{d+e x^2}\left(-4 c^2 e x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+\right.\right.$$

$$\left.c^2 d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)$$

$$\left(-4 d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2 x^2,-\frac{e x^2}{d}\right]+x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]+\right.\right.$$

$$\left.\left.\left.\left.c^2 d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2 x^2,-\frac{e x^2}{d}\right]\right)\right)\right)\right)+$$

$$\frac{1}{6 c e^3\sqrt{d+e x^2}}\left(b\,e\sqrt{1+\frac{1}{c^2 x^2}}\,x\left(d+e x^2\right)-\right.$$

2
a
c
$\left(8 d^2+4 d e x^2-e^2 x^4\right)-2$
b
c
$\left(8 d^2+4 d e x^2-e^2 x^4\right)$

$\left.\text{ArcCsch}[c\,x]\right)$

Problem 148: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{x^3\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e x^2\right)^{3/2}}\,dx$$

Optimal (type 3, 160 leaves, 9 steps):

$$\frac{d\,(a+b\,\text{ArcCsch}[c\,x])}{e^2\sqrt{d+e\,x^2}}+\frac{\sqrt{d+e\,x^2}\,(a+b\,\text{ArcCsch}[c\,x])}{e^2}+\frac{b\,x\,\text{ArcTan}\left[\frac{\sqrt{e}\,\sqrt{-1-c^2\,x^2}}{c\,\sqrt{d+e\,x^2}}\right]}{e^{3/2}\sqrt{-c^2\,x^2}}+\frac{2\,b\,c\,\sqrt{d}\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\right]}{e^2\sqrt{-c^2\,x^2}}$$

Result (type 6, 334 leaves) :

$$\left(2\,b\,c\,d\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x^3\left(-\left(\left(2\,c^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\right)+\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\Big/\left(4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]-x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/\left(e\,(1+c^2\,x^2)\,\sqrt{d+e\,x^2}\right)+\frac{(2\,d+e\,x^2)\,(a+b\,\text{ArcCsch}[c\,x])}{e^2\sqrt{d+e\,x^2}}$$

Problem 149: Result unnecessarily involves higher level functions and more than twice size of optimal antiderivative.

$$\int\frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{(d+e\,x^2)^{3/2}}\,dx$$

Optimal (type 3, 82 leaves, 4 steps) :

$$-\frac{a+b\,\text{ArcCsch}[c\,x]}{e\,\sqrt{d+e\,x^2}}-\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\right]}{\sqrt{d}\,e\,\sqrt{-c^2\,x^2}}$$

Result (type 6, 192 leaves) :

$$-\left(\left(2 b c^3 \sqrt{1+\frac{1}{c^2 x^2}}\, x^3\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)\Big/\left(\left(1+c^2 x^2\right)\sqrt{d+e x^2}\left(-4 c^2 e x^2\, \text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+c^2 d\, \text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]+e\, \text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2 x^2},-\frac{d}{e x^2}\right]\right)\right)\right)-\frac{a+b\, \text{ArcCsch}[c x]}{e \sqrt{d+e x^2}}$$

Problem 155: Unable to integrate problem.

$$\int \frac{a+b\, \text{ArcCsch}[c x]}{x^2 \left(d+e x^2\right)^{3/2}}\, dx$$

Optimal (type 4, 321 leaves, 7 steps):

$$\frac{b c^3 x^2 \sqrt{d+e x^2}}{d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2}}+\frac{b c \sqrt{-1-c^2 x^2} \sqrt{d+e x^2}}{d^2 \sqrt{-c^2 x^2}}-\frac{a+b\, \text{ArcCsch}[c x]}{d x \sqrt{d+e x^2}}-\frac{2 e x \left(a+b\, \text{ArcCsch}[c x]\right)}{d^2 \sqrt{d+e x^2}}-\frac{b c^2 x \sqrt{d+e x^2}\, \text{EllipticE}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{d^2 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}+\frac{2 b e x \sqrt{d+e x^2}\, \text{EllipticF}\left[\text{ArcTan}[c x],1-\frac{e}{c^2 d}\right]}{d^3 \sqrt{-c^2 x^2} \sqrt{-1-c^2 x^2} \sqrt{\frac{d+e x^2}{d \left(1+c^2 x^2\right)}}}$$

Result (type 8, 25 leaves):

$$\int \frac{a+b\, \text{ArcCsch}[c x]}{x^2 \left(d+e x^2\right)^{3/2}}\, dx$$

Problem 156: Result unnecessarily involves higher level functions.

$$\int \frac{x^5 \left(a+b\, \text{ArcCsch}[c x]\right)}{\left(d+e x^2\right)^{5/2}}\, dx$$

Optimal (type 3, 251 leaves, 10 steps):

$$\frac{b c d x \sqrt{-1-c^2 x^2}}{3 \left(c^2 d-e\right) e^2 \sqrt{-c^2 x^2} \sqrt{d+e x^2}}-\frac{d^2 \left(a+b\, \text{ArcCsch}[c x]\right)}{3 e^3 \left(d+e x^2\right)^{3/2}}+\frac{2 d \left(a+b\, \text{ArcCsch}[c x]\right)}{e^3 \sqrt{d+e x^2}}+\frac{\sqrt{d+e x^2} \left(a+b\, \text{ArcCsch}[c x]\right)}{e^3}+\frac{b x\, \text{ArcTan}\left[\frac{\sqrt{e} \sqrt{-1-c^2 x^2}}{c \sqrt{d+e x^2}}\right]}{e^{5/2} \sqrt{-c^2 x^2}}+\frac{8 b c \sqrt{d}\, x\, \text{ArcTan}\left[\frac{\sqrt{d+e x^2}}{\sqrt{d} \sqrt{-1-c^2 x^2}}\right]}{3 e^3 \sqrt{-c^2 x^2}}$$

Result (type 6, 428 leaves):

$$\left(2\,b\,c\,d\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x^3\left(-\left(\left(8\,c^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]-c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]-e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\right)-\left(3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\Big/\left(-4\,d\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+x^2\left(e\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-c^2\,x^2,-\frac{e\,x^2}{d}\right]\right)\right)\right)\right)\Big/$$

$$\left(3\,e^2\left(1+c^2\,x^2\right)\sqrt{d+e\,x^2}\right)+\left(b\,c\,d\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\;x\left(d+e\,x^2\right)+a\left(c^2\,d-e\right)\left(8\,d^2+12\,d\,e\,x^2+3\,e^2\,x^4\right)+b\left(c^2\,d-e\right)\left(8\,d^2+12\,d\,e\,x^2+3\,e^2\,x^4\right)\text{ArcCsch}[c\,x]\right)\Big/\left(3\left(c^2\,d-e\right)e^3\left(d+e\,x^2\right)^{3/2}\right)$$

Problem 157: Result unnecessarily involves higher level functions.

$$\int\frac{x^3\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 169 leaves, 7 steps):

$$-\frac{b\,c\,x\,\sqrt{-1-c^2\,x^2}}{3\left(c^2\,d-e\right)e\,\sqrt{-c^2\,x^2}\,\sqrt{d+e\,x^2}}+\frac{d\left(a+b\,\text{ArcCsch}[c\,x]\right)}{3\,e^2\left(d+e\,x^2\right)^{3/2}}-\frac{a+b\,\text{ArcCsch}[c\,x]}{e^2\,\sqrt{d+e\,x^2}}-\frac{2\,b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\right]}{3\,\sqrt{d}\,e^2\,\sqrt{-c^2\,x^2}}$$

Result (type 6, 273 leaves):

$$-\left(\left(4\,b\,c^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(3\,e\,\left(1+c^2\,x^2\right)\,\sqrt{d+e\,x^2}\,\left(-4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\right)\right)+\left(b\,c\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\left(d+e\,x^2\right)+a\,\left(c^2\,d-e\right)\,\left(2\,d+3\,e\,x^2\right)+b\,\left(c^2\,d-e\right)\,\left(2\,d+3\,e\,x^2\right)\,\text{ArcCsch}[c\,x]\right)\Big/\left(3\,e^2\,\left(-c^2\,d+e\right)\,\left(d+e\,x^2\right)^{3/2}\right)$$

Problem 158: Result unnecessarily involves higher level functions.

$$\int\frac{x\,\left(a+b\,\text{ArcCsch}[c\,x]\right)}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 3, 144 leaves, 5 steps):

$$\frac{b\,c\,x\,\sqrt{-1-c^2\,x^2}}{3\,d\,\left(c^2\,d-e\right)\,\sqrt{-c^2\,x^2}\,\sqrt{d+e\,x^2}}-\frac{a+b\,\text{ArcCsch}[c\,x]}{3\,e\,\left(d+e\,x^2\right)^{3/2}}-\frac{b\,c\,x\,\text{ArcTan}\left[\frac{\sqrt{d+e\,x^2}}{\sqrt{d}\,\sqrt{-1-c^2\,x^2}}\right]}{3\,d^{3/2}\,e\,\sqrt{-c^2\,x^2}}$$

Result (type 6, 257 leaves):

$$-\left(\left(2\,b\,c^3\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x^3\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\Big/\left(3\,d\,\left(1+c^2\,x^2\right)\,\sqrt{d+e\,x^2}\,\left(-4\,c^2\,e\,x^2\,\text{AppellF1}\left[1,\frac{1}{2},\frac{1}{2},2,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+c^2\,d\,\text{AppellF1}\left[2,\frac{1}{2},\frac{3}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]+e\,\text{AppellF1}\left[2,\frac{3}{2},\frac{1}{2},3,-\frac{1}{c^2\,x^2},-\frac{d}{e\,x^2}\right]\right)\right)\right)+\left(a\,d\,\left(-c^2\,d+e\right)+b\,c\,e\,\sqrt{1+\frac{1}{c^2\,x^2}}\,x\,\left(d+e\,x^2\right)+b\,d\,\left(-c^2\,d+e\right)\,\text{ArcCsch}[c\,x]\right)\Big/\left(3\,d\,\left(c^2\,d-e\right)\,e\,\left(d+e\,x^2\right)^{3/2}\right)$$

Problem 164: Unable to integrate problem.

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{\left(d+e\,x^2\right)^{5/2}}\,dx$$

Optimal (type 4, 278 leaves, 5 steps):

$$\frac{x\,(a+b\,\text{ArcCsch}[c\,x])}{3\,d\,(d+e\,x^2)^{3/2}}+\frac{2\,x\,(a+b\,\text{ArcCsch}[c\,x])}{3\,d^2\,\sqrt{d+e\,x^2}}-$$

$$\frac{b\,c\,\sqrt{e}\,x\,\sqrt{-1-c^2\,x^2}\,\text{EllipticE}\left[\text{ArcTan}\left[\frac{\sqrt{e}\,x}{\sqrt{d}}\right],1-\frac{c^2\,d}{e}\right]}{3\,d^{3/2}\,(c^2\,d-e)\,\sqrt{-c^2\,x^2}\,\sqrt{\frac{d\,(1+c^2\,x^2)}{d+e\,x^2}}\,\sqrt{d+e\,x^2}}-$$

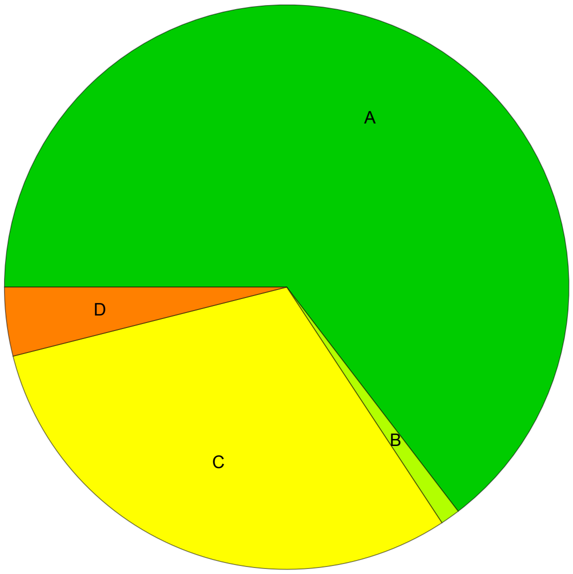
$$\frac{b\,(3\,c^2\,d-2\,e)\,x\,\sqrt{d+e\,x^2}\,\text{EllipticF}\left[\text{ArcTan}[c\,x],1-\frac{e}{c^2\,d}\right]}{3\,d^3\,(c^2\,d-e)\,\sqrt{-c^2\,x^2}\,\sqrt{-1-c^2\,x^2}\,\sqrt{\frac{d+e\,x^2}{d\,(1+c^2\,x^2)}}}$$

Result (type 8, 22 leaves):

$$\int\frac{a+b\,\text{ArcCsch}[c\,x]}{(d+e\,x^2)^{5/2}}\,dx$$

Summary of Integration Test Results

178 integration problems



A - 115 optimal antiderivatives

B - 2 more than twice size of optimal antiderivatives

C - 54 unnecessarily complex antiderivatives

D - 7 unable to integrate problems

E - 0 integration timeouts