

Mathematica 11.3 Integration Test Results

Test results for the 71 problems in "7.6.2 Inverse hyperbolic cosecant functions.m"

Problem 4: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{\text{ArcCsch}[a + b x]}{x} dx$$

Optimal (type 4, 162 leaves, 14 steps):

$$\text{ArcCsch}[a + b x] \text{Log}\left[1 - \frac{a e^{\text{ArcCsch}[a + b x]}}{1 - \sqrt{1 + a^2}}\right] + \text{ArcCsch}[a + b x] \text{Log}\left[1 - \frac{a e^{\text{ArcCsch}[a + b x]}}{1 + \sqrt{1 + a^2}}\right] -$$

$$\text{ArcCsch}[a + b x] \text{Log}\left[1 - e^{2 \text{ArcCsch}[a + b x]}\right] + \text{PolyLog}\left[2, \frac{a e^{\text{ArcCsch}[a + b x]}}{1 - \sqrt{1 + a^2}}\right] +$$

$$\text{PolyLog}\left[2, \frac{a e^{\text{ArcCsch}[a + b x]}}{1 + \sqrt{1 + a^2}}\right] - \frac{1}{2} \text{PolyLog}\left[2, e^{2 \text{ArcCsch}[a + b x]}\right]$$

Result (type 4, 428 leaves):

$$\frac{1}{8} \left(\pi^2 - 4 i \pi \operatorname{ArcCsch}[a + b x] - 8 \operatorname{ArcCsch}[a + b x]^2 - \right.$$

$$32 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right] \operatorname{ArcTanh}\left[\frac{(i+a) \operatorname{Cot}\left[\frac{1}{4}(\pi + 2 i \operatorname{ArcCsch}[a + b x])\right]}{\sqrt{1+a^2}}\right] -$$

$$8 \operatorname{ArcCsch}[a + b x] \operatorname{Log}\left[1 - e^{-2 \operatorname{ArcCsch}[a + b x]}\right] + 4 i \pi \operatorname{Log}\left[1 - \frac{(-1 + \sqrt{1+a^2}) e^{\operatorname{ArcCsch}[a + b x]}}{a}\right] +$$

$$8 \operatorname{ArcCsch}[a + b x] \operatorname{Log}\left[1 - \frac{(-1 + \sqrt{1+a^2}) e^{\operatorname{ArcCsch}[a + b x]}}{a}\right] -$$

$$16 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 - \frac{(-1 + \sqrt{1+a^2}) e^{\operatorname{ArcCsch}[a + b x]}}{a}\right] + 4 i \pi$$

$$\operatorname{Log}\left[1 + \frac{(1 + \sqrt{1+a^2}) e^{\operatorname{ArcCsch}[a + b x]}}{a}\right] + 8 \operatorname{ArcCsch}[a + b x] \operatorname{Log}\left[1 + \frac{(1 + \sqrt{1+a^2}) e^{\operatorname{ArcCsch}[a + b x]}}{a}\right] +$$

$$16 i \operatorname{ArcSin}\left[\frac{\sqrt{\frac{-i+a}{a}}}{\sqrt{2}}\right] \operatorname{Log}\left[1 + \frac{(1 + \sqrt{1+a^2}) e^{\operatorname{ArcCsch}[a + b x]}}{a}\right] - 4 i \pi \operatorname{Log}\left[-\frac{b x}{a + b x}\right] +$$

$$4 \operatorname{PolyLog}\left[2, e^{-2 \operatorname{ArcCsch}[a + b x]}\right] + 8 \operatorname{PolyLog}\left[2, \frac{(-1 + \sqrt{1+a^2}) e^{\operatorname{ArcCsch}[a + b x]}}{a}\right] +$$

$$\left. 8 \operatorname{PolyLog}\left[2, -\frac{(1 + \sqrt{1+a^2}) e^{\operatorname{ArcCsch}[a + b x]}}{a}\right]\right)$$

Problem 5: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{ArcCsch}[a + b x]}{x^2} dx$$

Optimal (type 3, 63 leaves, 6 steps):

$$-\frac{b \operatorname{ArcCsch}[a + b x]}{a} - \frac{\operatorname{ArcCsch}[a + b x]}{x} + \frac{2 b \operatorname{ArcTanh}\left[\frac{a + \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCsch}[a + b x]\right]}{\sqrt{1+a^2}}\right]}{a \sqrt{1+a^2}}$$

Result (type 3, 141 leaves):

$$\begin{aligned}
 & -\frac{\text{ArcCsch}[a+bx]}{x} - \frac{1}{a\sqrt{1+a^2}}b \left(\sqrt{1+a^2} \text{ArcSinh}\left[\frac{1}{a+bx}\right] + \text{Log}[x] - \right. \\
 & \left. \text{Log}\left[1+a^2+abx+a\sqrt{1+a^2}\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}} + \sqrt{1+a^2}bx\sqrt{\frac{1+a^2+2abx+b^2x^2}{(a+bx)^2}}\right] \right)
 \end{aligned}$$

Problem 7: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (e+fx)^3 (a+b \text{ArcCsch}[c+dx])^2 dx$$

Optimal (type 4, 501 leaves, 20 steps):

$$\begin{aligned}
 & \frac{b^2 f^2 (de-cf)x}{d^3} + \frac{b^2 f^3 (c+dx)^2}{12 d^4} - \frac{b f^3 (c+dx) \sqrt{1+\frac{1}{(c+dx)^2}} (a+b \text{ArcCsch}[c+dx])}{3 d^4} + \\
 & \frac{3 b f (de-cf)^2 (c+dx) \sqrt{1+\frac{1}{(c+dx)^2}} (a+b \text{ArcCsch}[c+dx])}{d^4} + \\
 & \frac{b f^2 (de-cf) (c+dx)^2 \sqrt{1+\frac{1}{(c+dx)^2}} (a+b \text{ArcCsch}[c+dx])}{d^4} + \\
 & \frac{b f^3 (c+dx)^3 \sqrt{1+\frac{1}{(c+dx)^2}} (a+b \text{ArcCsch}[c+dx])}{6 d^4} - \\
 & \frac{(de-cf)^4 (a+b \text{ArcCsch}[c+dx])^2}{4 d^4 f} + \frac{(e+fx)^4 (a+b \text{ArcCsch}[c+dx])^2}{4 f} - \\
 & \frac{2 b f^2 (de-cf) (a+b \text{ArcCsch}[c+dx]) \text{ArcTanh}[e^{\text{ArcCsch}[c+dx]}]}{d^4} + \\
 & \frac{4 b (de-cf)^3 (a+b \text{ArcCsch}[c+dx]) \text{ArcTanh}[e^{\text{ArcCsch}[c+dx]}]}{d^4} - \\
 & \frac{b^2 f^3 \text{Log}[c+dx]}{3 d^4} + \frac{3 b^2 f (de-cf)^2 \text{Log}[c+dx]}{d^4} - \\
 & \frac{b^2 f^2 (de-cf) \text{PolyLog}[2, -e^{\text{ArcCsch}[c+dx]}]}{d^4} + \frac{2 b^2 (de-cf)^3 \text{PolyLog}[2, -e^{\text{ArcCsch}[c+dx]}]}{d^4} + \\
 & \frac{b^2 f^2 (de-cf) \text{PolyLog}[2, e^{\text{ArcCsch}[c+dx]}]}{d^4} - \frac{2 b^2 (de-cf)^3 \text{PolyLog}[2, e^{\text{ArcCsch}[c+dx]}]}{d^4}
 \end{aligned}$$

Result (type 4, 1429 leaves):

$$a^2 e^3 x + \frac{3}{2} a^2 e^2 f x^2 + a^2 e f^2 x^3 + \frac{1}{4} a^2 f^3 x^4 +$$

$$\begin{aligned}
 & \frac{1}{6} a b \left(3 x \left(4 e^3 + 6 e^2 f x + 4 e f^2 x^2 + f^3 x^3 \right) \text{ArcCsch}[c + d x] + \frac{1}{d^4} \left(f (c + d x) \sqrt{\frac{1 + c^2 + 2 c d x + d^2 x^2}{(c + d x)^2}} \right. \right. \\
 & \quad \left. \left((-2 + 13 c^2) f^2 - 2 c d f (15 e + 2 f x) + d^2 (18 e^2 + 6 e f x + f^2 x^2) \right) - \right. \\
 & \quad \left. 3 c \left(-4 d^3 e^3 + 6 c d^2 e^2 f - 4 c^2 d e f^2 + c^3 f^3 \right) \text{ArcSinh}\left[\frac{1}{c + d x}\right] + 6 \left(2 d^3 e^3 - 6 c d^2 e^2 f + \right. \right. \\
 & \quad \left. \left. (-1 + 6 c^2) d e f^2 + c (1 - 2 c^2) f^3 \right) \text{Log}\left[\left(c + d x\right) \left(1 + \sqrt{\frac{1 + c^2 + 2 c d x + d^2 x^2}{(c + d x)^2}}\right)\right] \right) - \\
 & \frac{1}{d} b^2 e^3 \left(-\text{ArcCsch}[c + d x] \left((c + d x) \text{ArcCsch}[c + d x] - 2 \text{Log}\left[1 - e^{-\text{ArcCsch}[c + d x]}\right] + \right. \right. \\
 & \quad \left. \left. 2 \text{Log}\left[1 + e^{-\text{ArcCsch}[c + d x]}\right] \right) + 2 \text{PolyLog}\left[2, -e^{-\text{ArcCsch}[c + d x]}\right] - 2 \text{PolyLog}\left[2, e^{-\text{ArcCsch}[c + d x]}\right] \right) - \\
 & \left(3 b^2 d e^2 f x \left(\frac{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} \text{ArcCsch}[c + d x]}{d^2} + \frac{(c + d x)^2 \text{ArcCsch}[c + d x]^2}{2 d^2} - \right. \right. \\
 & \quad \frac{c \text{ArcCsch}[c + d x]^2 \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]}{2 d^2} - \frac{\text{Log}\left[\frac{1}{c + d x}\right]}{d^2} - \frac{1}{d^2} \\
 & \quad \left. 2 i c \left(i \text{ArcCsch}[c + d x] \left(\text{Log}\left[1 - e^{-\text{ArcCsch}[c + d x]}\right] - \text{Log}\left[1 + e^{-\text{ArcCsch}[c + d x]}\right] \right) + \right. \right. \\
 & \quad \left. \left. i \left(\text{PolyLog}\left[2, -e^{-\text{ArcCsch}[c + d x]}\right] - \text{PolyLog}\left[2, e^{-\text{ArcCsch}[c + d x]}\right] \right) \right) \right) + \\
 & \quad \left. \frac{c \text{ArcCsch}[c + d x]^2 \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]}{2 d^2} \right) \Bigg/ \left((c + d x) \left(-1 + \frac{c}{c + d x} \right) \right) - \\
 & \frac{1}{8 d^3} b^2 e f^2 \left(2 \left(-2 + 12 c \text{ArcCsch}[c + d x] + \text{ArcCsch}[c + d x]^2 - 6 c^2 \text{ArcCsch}[c + d x]^2 \right) \right. \\
 & \quad \text{Coth}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right] + 2 \text{ArcCsch}[c + d x] \left(-1 + 3 c \text{ArcCsch}[c + d x] \right) \\
 & \quad \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]^2 - \frac{\text{ArcCsch}[c + d x]^2 \text{Csch}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]^4}{2 (c + d x)} - 48 c \text{Log}\left[\frac{1}{c + d x}\right] + \\
 & \quad 8 \left(-1 + 6 c^2 \right) \left(\text{ArcCsch}[c + d x] \left(\text{Log}\left[1 - e^{-\text{ArcCsch}[c + d x]}\right] - \text{Log}\left[1 + e^{-\text{ArcCsch}[c + d x]}\right] \right) + \right. \\
 & \quad \left. \text{PolyLog}\left[2, -e^{-\text{ArcCsch}[c + d x]}\right] - \text{PolyLog}\left[2, e^{-\text{ArcCsch}[c + d x]}\right] \right) - \\
 & \quad 2 \text{ArcCsch}[c + d x] \left(1 + 3 c \text{ArcCsch}[c + d x] \right) \text{Sech}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]^2 - \\
 & \quad 8 (c + d x)^3 \text{ArcCsch}[c + d x]^2 \text{Sinh}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]^4 + \\
 & \quad \left. 2 \left(2 + 12 c \text{ArcCsch}[c + d x] - \text{ArcCsch}[c + d x]^2 + 6 c^2 \text{ArcCsch}[c + d x]^2 \right) \right. \\
 & \quad \left. \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right] \right) - \frac{1}{192 d (c + d x)^3 \left(-1 + \frac{c}{c + d x} \right)^3}
 \end{aligned}$$

$$\begin{aligned}
 & b^2 f^3 x^3 \left(-16 \left(2 \operatorname{ArcCsch}[c + d x] - 18 c^2 \operatorname{ArcCsch}[c + d x] + 6 c^3 \operatorname{ArcCsch}[c + d x]^2 - \right. \right. \\
 & \quad \left. \left. 3 c \left(-2 + \operatorname{ArcCsch}[c + d x]^2 \right) \right) \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right] + \right. \\
 & \quad 2 \left(2 - 24 c \operatorname{ArcCsch}[c + d x] - 3 \operatorname{ArcCsch}[c + d x]^2 + 36 c^2 \operatorname{ArcCsch}[c + d x]^2 \right) \\
 & \quad \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^2 + 3 \operatorname{ArcCsch}[c + d x]^2 \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^4 - \\
 & \quad \frac{1}{c + d x} 2 \operatorname{ArcCsch}[c + d x] \left(-1 + 6 c \operatorname{ArcCsch}[c + d x] \right) \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^4 - \\
 & \quad 64 \left(-1 + 9 c^2 \right) \operatorname{Log}\left[\frac{1}{c + d x}\right] + \\
 & \quad 192 c \left(-1 + 2 c^2 \right) \left(\operatorname{ArcCsch}[c + d x] \left(\operatorname{Log}\left[1 - e^{-\operatorname{ArcCsch}[c + d x]}\right] - \operatorname{Log}\left[1 + e^{-\operatorname{ArcCsch}[c + d x]}\right] \right) + \right. \\
 & \quad \left. \operatorname{PolyLog}\left[2, -e^{-\operatorname{ArcCsch}[c + d x]}\right] - \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcCsch}[c + d x]}\right] \right) - \\
 & \quad 2 \left(2 + 24 c \operatorname{ArcCsch}[c + d x] - 3 \operatorname{ArcCsch}[c + d x]^2 + 36 c^2 \operatorname{ArcCsch}[c + d x]^2 \right) \\
 & \quad \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^2 + 3 \operatorname{ArcCsch}[c + d x]^2 \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^4 - \\
 & \quad 32 (c + d x)^3 \operatorname{ArcCsch}[c + d x] \left(1 + 6 c \operatorname{ArcCsch}[c + d x] \right) \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^4 + \\
 & \quad 16 \left(-2 \operatorname{ArcCsch}[c + d x] + 18 c^2 \operatorname{ArcCsch}[c + d x] + 6 c^3 \operatorname{ArcCsch}[c + d x]^2 - \right. \\
 & \quad \left. 3 c \left(-2 + \operatorname{ArcCsch}[c + d x]^2 \right) \right) \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right] \left. \right)
 \end{aligned}$$

Problem 8: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int (e + f x)^2 (a + b \operatorname{ArcCsch}[c + d x])^2 dx$$

Optimal (type 4, 351 leaves, 17 steps):

$$\begin{aligned}
 & \frac{b^2 f^2 x}{3 d^2} + \frac{2 b f (d e - c f) (c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} (a + b \operatorname{ArcCsch}[c + d x])}{d^3} + \\
 & \frac{b f^2 (c + d x)^2 \sqrt{1 + \frac{1}{(c + d x)^2}} (a + b \operatorname{ArcCsch}[c + d x])}{3 d^3} - \frac{(d e - c f)^3 (a + b \operatorname{ArcCsch}[c + d x])^2}{3 d^3 f} + \\
 & \frac{(e + f x)^3 (a + b \operatorname{ArcCsch}[c + d x])^2}{3 f} - \frac{2 b f^2 (a + b \operatorname{ArcCsch}[c + d x]) \operatorname{ArcTanh}\left[e^{\operatorname{ArcCsch}[c + d x]}\right]}{3 d^3} + \\
 & \frac{4 b (d e - c f)^2 (a + b \operatorname{ArcCsch}[c + d x]) \operatorname{ArcTanh}\left[e^{\operatorname{ArcCsch}[c + d x]}\right]}{d^3} + \frac{2 b^2 f (d e - c f) \operatorname{Log}[c + d x]}{d^3} - \\
 & \frac{b^2 f^2 \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcCsch}[c + d x]}\right]}{3 d^3} + \frac{2 b^2 (d e - c f)^2 \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcCsch}[c + d x]}\right]}{d^3} + \\
 & \frac{b^2 f^2 \operatorname{PolyLog}\left[2, e^{\operatorname{ArcCsch}[c + d x]}\right]}{3 d^3} - \frac{2 b^2 (d e - c f)^2 \operatorname{PolyLog}\left[2, e^{\operatorname{ArcCsch}[c + d x]}\right]}{d^3}
 \end{aligned}$$

Result (type 4, 864 leaves):

$$a^2 e^2 x + a^2 e f x^2 + \frac{1}{3} a^2 f^2 x^3 +$$

$$\frac{1}{3} a b \left(2 x (3 e^2 + 3 e f x + f^2 x^2) \operatorname{ArcCsch}[c + d x] + \frac{1}{d^3} \left(-f (c + d x) \sqrt{\frac{1 + c^2 + 2 c d x + d^2 x^2}{(c + d x)^2}} \right. \right. \\ \left. \left. (5 c f - d (6 e + f x)) + 2 c (3 d^2 e^2 - 3 c d e f + c^2 f^2) \operatorname{ArcSinh}\left[\frac{1}{c + d x}\right] + \right. \right. \\ \left. \left. (6 d^2 e^2 - 12 c d e f + (-1 + 6 c^2) f^2) \operatorname{Log}\left[(c + d x) \left(1 + \sqrt{\frac{1 + c^2 + 2 c d x + d^2 x^2}{(c + d x)^2}}\right)\right] \right) \right) -$$

$$\frac{1}{d} b^2 e^2 (-\operatorname{ArcCsch}[c + d x] ((c + d x) \operatorname{ArcCsch}[c + d x] - 2 \operatorname{Log}[1 - e^{-\operatorname{ArcCsch}[c + d x]}] + \\ 2 \operatorname{Log}[1 + e^{-\operatorname{ArcCsch}[c + d x]}]) + 2 \operatorname{PolyLog}[2, -e^{-\operatorname{ArcCsch}[c + d x]}] - 2 \operatorname{PolyLog}[2, e^{-\operatorname{ArcCsch}[c + d x]}]) -$$

$$\left(2 b^2 d e f x \left(\frac{(c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} \operatorname{ArcCsch}[c + d x]}{d^2} + \frac{(c + d x)^2 \operatorname{ArcCsch}[c + d x]^2}{2 d^2} - \right. \right.$$

$$\left. \frac{c \operatorname{ArcCsch}[c + d x]^2 \operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]}{2 d^2} - \frac{\operatorname{Log}\left[\frac{1}{c + d x}\right]}{d^2} - \frac{1}{d^2} \right.$$

$$2 i c (i \operatorname{ArcCsch}[c + d x] (\operatorname{Log}[1 - e^{-\operatorname{ArcCsch}[c + d x]}] - \operatorname{Log}[1 + e^{-\operatorname{ArcCsch}[c + d x]}]) + \\ i (\operatorname{PolyLog}[2, -e^{-\operatorname{ArcCsch}[c + d x]}] - \operatorname{PolyLog}[2, e^{-\operatorname{ArcCsch}[c + d x]}])) +$$

$$\left. \frac{c \operatorname{ArcCsch}[c + d x]^2 \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]}{2 d^2} \right) / \left((c + d x) \left(-1 + \frac{c}{c + d x}\right) \right) -$$

$$\frac{1}{24 d^3} b^2 f^2 \left(2 (-2 + 12 c \operatorname{ArcCsch}[c + d x] + \operatorname{ArcCsch}[c + d x]^2 - 6 c^2 \operatorname{ArcCsch}[c + d x]^2) \right.$$

$$\operatorname{Coth}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right] + 2 \operatorname{ArcCsch}[c + d x] (-1 + 3 c \operatorname{ArcCsch}[c + d x])$$

$$\operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^2 - \frac{\operatorname{ArcCsch}[c + d x]^2 \operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^4}{2 (c + d x)} - 48 c \operatorname{Log}\left[\frac{1}{c + d x}\right] +$$

$$8 (-1 + 6 c^2) (\operatorname{ArcCsch}[c + d x] (\operatorname{Log}[1 - e^{-\operatorname{ArcCsch}[c + d x]}] - \operatorname{Log}[1 + e^{-\operatorname{ArcCsch}[c + d x]}]) + \\ \operatorname{PolyLog}[2, -e^{-\operatorname{ArcCsch}[c + d x]}] - \operatorname{PolyLog}[2, e^{-\operatorname{ArcCsch}[c + d x]}]) -$$

$$2 \operatorname{ArcCsch}[c + d x] (1 + 3 c \operatorname{ArcCsch}[c + d x]) \operatorname{Sech}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^2 -$$

$$8 (c + d x)^3 \operatorname{ArcCsch}[c + d x]^2 \operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]^4 +$$

$$2 (2 + 12 c \operatorname{ArcCsch}[c + d x] - \operatorname{ArcCsch}[c + d x]^2 + 6 c^2 \operatorname{ArcCsch}[c + d x]^2)$$

$$\left. \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]\right)$$

Problem 9: Result more than twice size of optimal antiderivative.

$$\int (e + f x) (a + b \operatorname{ArcCsch}[c + d x])^2 dx$$

Optimal (type 4, 194 leaves, 11 steps):

$$\frac{b f (c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} (a + b \operatorname{ArcCsch}[c + d x])}{d^2} - \frac{(d e - c f)^2 (a + b \operatorname{ArcCsch}[c + d x])^2}{2 d^2 f} + \frac{(e + f x)^2 (a + b \operatorname{ArcCsch}[c + d x])^2}{2 f} + \frac{4 b (d e - c f) (a + b \operatorname{ArcCsch}[c + d x]) \operatorname{ArcTanh}\left[e^{\operatorname{ArcCsch}[c + d x]}\right]}{d^2} + \frac{b^2 f \operatorname{Log}[c + d x]}{d^2} + \frac{2 b^2 (d e - c f) \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcCsch}[c + d x]}\right]}{d^2} - \frac{2 b^2 (d e - c f) \operatorname{PolyLog}\left[2, e^{\operatorname{ArcCsch}[c + d x]}\right]}{d^2}$$

Result (type 4, 427 leaves):

$$\frac{1}{2 d^2} \left(2 a^2 (d e - c f) (c + d x) + a^2 f (c + d x)^2 + 2 a b f (c + d x) \left(\sqrt{1 + \frac{1}{(c + d x)^2}} + (c + d x) \operatorname{ArcCsch}[c + d x] \right) + 2 b^2 f \left((c + d x) \sqrt{1 + \frac{1}{(c + d x)^2}} \operatorname{ArcCsch}[c + d x] + \frac{1}{2} (c + d x)^2 \operatorname{ArcCsch}[c + d x]^2 - \operatorname{Log}\left[\frac{1}{c + d x}\right] \right) + 4 a b d e \left((c + d x) \operatorname{ArcCsch}[c + d x] + \operatorname{Log}\left[\frac{\operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]}{2 (c + d x)}\right] - \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]\right] \right) - 4 a b c f \left((c + d x) \operatorname{ArcCsch}[c + d x] + \operatorname{Log}\left[\frac{\operatorname{Csch}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]}{2 (c + d x)}\right] - \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]\right] \right) + 2 b^2 d e \left(\operatorname{ArcCsch}[c + d x] \left((c + d x) \operatorname{ArcCsch}[c + d x] - 2 \operatorname{Log}\left[1 - e^{-\operatorname{ArcCsch}[c + d x]}\right] \right) + 2 \operatorname{Log}\left[1 + e^{-\operatorname{ArcCsch}[c + d x]}\right] - 2 \operatorname{PolyLog}\left[2, -e^{-\operatorname{ArcCsch}[c + d x]}\right] + 2 \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcCsch}[c + d x]}\right] \right) - 2 b^2 c f \left(\operatorname{ArcCsch}[c + d x] \left((c + d x) \operatorname{ArcCsch}[c + d x] - 2 \operatorname{Log}\left[1 - e^{-\operatorname{ArcCsch}[c + d x]}\right] \right) + 2 \operatorname{Log}\left[1 + e^{-\operatorname{ArcCsch}[c + d x]}\right] - 2 \operatorname{PolyLog}\left[2, -e^{-\operatorname{ArcCsch}[c + d x]}\right] + 2 \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcCsch}[c + d x]}\right] \right) \right)$$

Problem 10: Result more than twice size of optimal antiderivative.

$$\int (a + b \operatorname{ArcCsch}[c + d x])^2 dx$$

Optimal (type 4, 85 leaves, 8 steps):

$$\frac{(c + d x) (a + b \operatorname{ArcCsch}[c + d x])^2}{d} + \frac{4 b (a + b \operatorname{ArcCsch}[c + d x]) \operatorname{ArcTanh}\left[e^{\operatorname{ArcCsch}[c + d x]}\right]}{d} + \frac{2 b^2 \operatorname{PolyLog}\left[2, -e^{\operatorname{ArcCsch}[c + d x]}\right]}{d} - \frac{2 b^2 \operatorname{PolyLog}\left[2, e^{\operatorname{ArcCsch}[c + d x]}\right]}{d}$$

Result (type 4, 176 leaves):

$$\frac{1}{d} \left(a^2 c + a^2 d x + 2 a b (c + d x) \operatorname{ArcCsch}[c + d x] + b^2 c \operatorname{ArcCsch}[c + d x]^2 + b^2 d x \operatorname{ArcCsch}[c + d x]^2 - 2 b^2 \operatorname{ArcCsch}[c + d x] \operatorname{Log}\left[1 - e^{-\operatorname{ArcCsch}[c + d x]}\right] + 2 b^2 \operatorname{ArcCsch}[c + d x] \operatorname{Log}\left[1 + e^{-\operatorname{ArcCsch}[c + d x]}\right] + 2 a b \operatorname{Log}\left[\operatorname{Cosh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]\right] - 2 a b \operatorname{Log}\left[\operatorname{Sinh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]\right] - 2 b^2 \operatorname{PolyLog}\left[2, -e^{-\operatorname{ArcCsch}[c + d x]}\right] + 2 b^2 \operatorname{PolyLog}\left[2, e^{-\operatorname{ArcCsch}[c + d x]}\right] \right)$$

Problem 11: Unable to integrate problem.

$$\int \frac{(a + b \operatorname{ArcCsch}[c + d x])^2}{e + f x} dx$$

Optimal (type 4, 475 leaves, 17 steps):

$$\begin{aligned}
 & - \frac{(a + b \operatorname{ArcCsch}[c + d x])^2 \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCsch}[c + d x]}\right]}{f} + \\
 & \frac{(a + b \operatorname{ArcCsch}[c + d x])^2 \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCsch}[c + d x]}(d e - c f)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{f} + \\
 & \frac{(a + b \operatorname{ArcCsch}[c + d x])^2 \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCsch}[c + d x]}(d e - c f)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{f} - \\
 & \frac{b(a + b \operatorname{ArcCsch}[c + d x]) \operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCsch}[c + d x]}\right]}{f} + \\
 & \frac{2 b(a + b \operatorname{ArcCsch}[c + d x]) \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCsch}[c + d x]}(d e - c f)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{f} + \\
 & \frac{2 b(a + b \operatorname{ArcCsch}[c + d x]) \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCsch}[c + d x]}(d e - c f)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{f} + \frac{b^2 \operatorname{PolyLog}\left[3, e^{2 \operatorname{ArcCsch}[c + d x]}\right]}{2 f} - \\
 & \frac{2 b^2 \operatorname{PolyLog}\left[3, -\frac{e^{\operatorname{ArcCsch}[c + d x]}(d e - c f)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{f} - \frac{2 b^2 \operatorname{PolyLog}\left[3, -\frac{e^{\operatorname{ArcCsch}[c + d x]}(d e - c f)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{f}
 \end{aligned}$$

Result (type 8, 22 leaves):

$$\int \frac{(a + b \operatorname{ArcCsch}[c + d x])^2}{e + f x} dx$$

Problem 12: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCsch}[c + d x])^2}{(e + f x)^2} dx$$

Optimal (type 4, 448 leaves, 12 steps):

$$\frac{d (a + b \operatorname{ArcCsch}[c + d x])^2}{f (d e - c f)} - \frac{(a + b \operatorname{ArcCsch}[c + d x])^2}{f (e + f x)} -$$

$$\frac{2 b d (a + b \operatorname{ArcCsch}[c + d x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCsch}[c + d x]} (d e - c f)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(d e - c f) \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}} +$$

$$\frac{2 b d (a + b \operatorname{ArcCsch}[c + d x]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCsch}[c + d x]} (d e - c f)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(d e - c f) \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}} -$$

$$\frac{2 b^2 d \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCsch}[c + d x]} (d e - c f)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(d e - c f) \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}} + \frac{2 b^2 d \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCsch}[c + d x]} (d e - c f)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(d e - c f) \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}$$

Result (type 4, 2061 leaves):

$$-\frac{a^2}{f (e + f x)} -$$

$$\left(2 a b (c + d x)^2 \left(f + \frac{d e - c f}{c + d x} \right)^2 \left(\frac{\operatorname{ArcCsch}[c + d x]}{f + \frac{d e}{c + d x} - \frac{c f}{c + d x}} - \frac{2 \operatorname{ArcTan}\left[\frac{d e - c f - f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]}{\sqrt{-d^2 e^2 + 2 c d e f - (1 + c^2) f^2}}\right]}{\sqrt{-d^2 e^2 + 2 c d e f - (1 + c^2) f^2}} \right) \right) /$$

$$(d (-d e + c f) (e + f x)^2) -$$

$$\frac{1}{d (e + f x)^2} b^2 (c + d x)^2 \left(f + \frac{d e - c f}{c + d x} \right)^2 \left(\frac{\operatorname{ArcCsch}[c + d x]^2}{(-d e + c f) \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right)} + \right.$$

$$\frac{1}{d e - c f} 2 \left(-\frac{i \pi \operatorname{ArcTanh}\left[\frac{-d e + c f + f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + d x]\right]}{\sqrt{f^2 + (d e - c f)^2}}\right]}{\sqrt{f^2 + (d e - c f)^2}} - \frac{1}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}} \right) 2 \left(\frac{\pi}{2} - \right.$$

$$i \operatorname{ArcCsch}[c + d x] \left. \operatorname{ArcTanh}\left[\frac{(f - i (d e - c f)) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch}[c + d x]\right)\right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}\right] \right) -$$

$$2 \operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] \operatorname{ArcTanh}\left[\frac{(-f - i (d e - c f)) \operatorname{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch}[c + d x]\right)\right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}\right] +$$

$$\left(\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] - 2 i \right.$$

$$\left. \left(\operatorname{ArcTanh}\left[\left((f - i (d e - c f)) \operatorname{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch}[c + d x]\right)\right]\right)\right] / \right.$$

$$\left. \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right) - \operatorname{ArcTanh}\left[\left((-f - i (d e - c f)) \right.$$

$$\begin{aligned}
 & \tan\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsch}[c + dx]\right)\right] - \operatorname{PolyLog}\left[2, \left(i\left(f + i\sqrt{-d^2e^2 + 2cdef - f^2 - c^2f^2}\right)\left(f - i(de - cf) - \sqrt{-d^2e^2 + 2cdef - f^2 - c^2f^2}\right) \tan\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsch}[c + dx]\right)\right]\right)\right] / \\
 & \left(\left(de - cf\right)\left(f - i(de - cf) + \sqrt{-d^2e^2 + 2cdef - f^2 - c^2f^2}\right) \tan\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsch}[c + dx]\right)\right]\right)
 \end{aligned}$$

Problem 13: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{(a + b \operatorname{ArcCsch}[c + dx])^2}{(e + fx)^3} dx$$

Optimal (type 4, 1024 leaves, 23 steps):

$$\begin{aligned}
 & - \frac{b d^2 f \sqrt{1 + \frac{1}{(c+dx)^2}} (a + b \operatorname{ArcCsch}[c + dx])}{(de - cf) (d^2 e^2 - 2 c d e f + (1 + c^2) f^2) \left(f + \frac{de - cf}{c + dx}\right)} + \frac{d^2 (a + b \operatorname{ArcCsch}[c + dx])^2}{2 f (de - cf)^2} - \\
 & \frac{(a + b \operatorname{ArcCsch}[c + dx])^2}{2 f (e + f x)^2} + \frac{b d^2 f^2 (a + b \operatorname{ArcCsch}[c + dx]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCsch}[c + dx]} (de - cf)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(de - cf)^2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)^{3/2}} - \\
 & \frac{2 b d^2 (a + b \operatorname{ArcCsch}[c + dx]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCsch}[c + dx]} (de - cf)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(de - cf)^2 \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}} - \\
 & \frac{b d^2 f^2 (a + b \operatorname{ArcCsch}[c + dx]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCsch}[c + dx]} (de - cf)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(de - cf)^2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)^{3/2}} + \\
 & \frac{2 b d^2 (a + b \operatorname{ArcCsch}[c + dx]) \operatorname{Log}\left[1 + \frac{e^{\operatorname{ArcCsch}[c + dx]} (de - cf)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(de - cf)^2 \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}} + \\
 & \frac{b^2 d^2 f \operatorname{Log}\left[f + \frac{de - cf}{c + dx}\right]}{(de - cf)^2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)} + \frac{b^2 d^2 f^2 \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCsch}[c + dx]} (de - cf)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(de - cf)^2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)^{3/2}} - \\
 & \frac{2 b^2 d^2 \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCsch}[c + dx]} (de - cf)}{f - \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(de - cf)^2 \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}} - \\
 & \frac{b^2 d^2 f^2 \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCsch}[c + dx]} (de - cf)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(de - cf)^2 (d^2 e^2 - 2 c d e f + (1 + c^2) f^2)^{3/2}} + \frac{2 b^2 d^2 \operatorname{PolyLog}\left[2, -\frac{e^{\operatorname{ArcCsch}[c + dx]} (de - cf)}{f + \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}\right]}{(de - cf)^2 \sqrt{d^2 e^2 - 2 c d e f + (1 + c^2) f^2}}
 \end{aligned}$$

Result (type 4, 8348 leaves):

$$\begin{aligned}
 & - \frac{a^2}{2 f (e + f x)^2} - \\
 & \left(a b (de - cf + f (c + dx))^3 \left(\frac{f (de - cf) \sqrt{1 + \frac{1}{(c+dx)^2}} - 2 \operatorname{ArcCsch}[c + dx]}{f + \frac{de - cf}{c + dx}} + \frac{f \operatorname{ArcCsch}[c + dx]}{\left(f + \frac{de - cf}{c + dx}\right)^2} \right) \right. \\
 & \left. \left(2 (2 d^2 e^2 - 4 c d e f + (1 + 2 c^2) f^2) \operatorname{ArcTan}\left[\frac{de - cf - f \operatorname{Tanh}\left[\frac{1}{2} \operatorname{ArcCsch}[c + dx]\right]}{\sqrt{-d^2 e^2 + 2 c d e f - (1 + c^2) f^2}} \right] \right) \right) /
 \end{aligned}$$

$$\begin{aligned}
 & 2 \operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] \operatorname{ArcTanh}\left[\frac{(-f - i(d e - c f)) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}\right] + \\
 & \left(\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] - 2 i \right. \\
 & \quad \left. \left(\operatorname{ArcTanh}\left[\left((f - i(d e - c f)) \operatorname{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \right. \right. \\
 & \quad \left. \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) - \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right]\right) \right) \\
 & \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)} \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}{\sqrt{2} \sqrt{-i(d e - c f)} \sqrt{f + \frac{d e - c f}{c + d x}}}\right] + \left(\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] + \right. \\
 & \quad \left. 2 i \left(\operatorname{ArcTanh}\left[\left((f - i(d e - c f)) \operatorname{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \right. \right. \\
 & \quad \left. \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) - \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right]\right) \right) \\
 & \operatorname{Log}\left[\frac{e^{\frac{1}{2} i\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)} \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}{\sqrt{2} \sqrt{-i(d e - c f)} \sqrt{f + \frac{d e - c f}{c + d x}}}\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] + 2 i \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right]\right) \\
 & \operatorname{Log}\left[1 - \left(i \left(f - i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) \left(f - i(d e - c f) - \right. \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right) / \right. \\
 & \quad \left. \left((d e - c f) \left(f - i(d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right) + \right. \\
 & \left. \left(-\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] + 2 i \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right]\right) \right) \\
 & \operatorname{Log}\left[1 - \left(i \left(f + i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) \left(f - i(d e - c f) - \right. \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right) / \right. \\
 & \quad \left. \left((d e - c f) \left(f + i(d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right) + \right. \\
 & \left. \left(-\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] + 2 i \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right]\right) \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right) + \\
 & i \left(\operatorname{PolyLog} \left[2, \left(i \left(f - i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right] \right) / \\
 & \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right) - \operatorname{PolyLog} \left[\right. \\
 & 2, \left(i \left(f + i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right] \right) / \\
 & \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right) \right] \right) -
 \end{aligned}$$

$$\frac{1}{(d e - c f)^2 (d^2 e^2 - 2 c d e f + f^2 + c^2 f^2) (d e - c f + f (c + d x))^3}$$

4
c
d
e
f

$$(c + d x)^3 \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right)^3$$

$$\left(\frac{i \pi \operatorname{ArcTanh} \left[\frac{-d e + c f + f \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcCsCh}[c + d x] \right]}{\sqrt{f^2 + (d e - c f)^2}} \right]}{\sqrt{f^2 + (d e - c f)^2}} - \right.$$

$$\left. \frac{1}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}} \left(2 \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right) \right)$$

$$\operatorname{ArcTanh} \left[\frac{(f - i (d e - c f)) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}} \right] -$$

$$\begin{aligned}
 & 2 \operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] \operatorname{ArcTanh}\left[\frac{(-f - i(d e - c f)) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}\right] + \\
 & \left(\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] - 2 i \right. \\
 & \quad \left. \left(\operatorname{ArcTanh}\left[\left((f - i(d e - c f)) \operatorname{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \right. \right. \\
 & \quad \left. \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) - \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right) \left. \right) \\
 & \operatorname{Log}\left[\frac{e^{-\frac{1}{2} i\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)} \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}{\sqrt{2} \sqrt{-i(d e - c f)} \sqrt{f + \frac{d e - c f}{c + d x}}}\right] + \left(\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] + \right. \\
 & \quad 2 i \left(\operatorname{ArcTanh}\left[\left((f - i(d e - c f)) \operatorname{Cot}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \right. \\
 & \quad \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) - \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right) \left. \right) \\
 & \operatorname{Log}\left[\frac{e^{\frac{1}{2} i\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)} \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}{\sqrt{2} \sqrt{-i(d e - c f)} \sqrt{f + \frac{d e - c f}{c + d x}}}\right] - \\
 & \left(\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] + 2 i \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right) \\
 & \operatorname{Log}\left[1 - \left(i\left(f - i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\left(f - i(d e - c f) - \right. \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \\
 & \quad \left(\left(d e - c f\right)\left(f - i(d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right) + \\
 & \left(-\operatorname{ArcCos}\left[-\frac{i f}{d e - c f}\right] + 2 i \operatorname{ArcTanh}\left[\left((-f - i(d e - c f)) \operatorname{Tan}\left[\right. \right. \right. \right. \\
 & \quad \left. \left. \frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\right) \\
 & \operatorname{Log}\left[1 - \left(i\left(f + i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\left(f - i(d e - c f) - \right. \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsSch}[c + d x]\right)\right]\right)\right] /
 \end{aligned}$$

$$\begin{aligned}
 & \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right) + \\
 & i \left(\operatorname{PolyLog} \left[2, \left(i \left(f - i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right] \right) / \\
 & \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right) - \operatorname{PolyLog} \left[\right. \\
 & 2, \left(i \left(f + i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right] \right) / \\
 & \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right] \right) \right) \right] + \\
 & \frac{1}{(d e - c f)^2 (d^2 e^2 - 2 c d e f + f^2 + c^2 f^2) (d e - c f + f (c + d x))^3} \\
 & \frac{(c + d x)^3}{\left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right)^3} \\
 & \left(\frac{i \pi \operatorname{ArcTanh} \left[\frac{-d e + c f + f \operatorname{Tanh} \left[\frac{1}{2} \operatorname{ArcCsCh}[c + d x] \right]}{\sqrt{f^2 + (d e - c f)^2}} \right]}{\sqrt{f^2 + (d e - c f)^2}} - \right. \\
 & \left. \frac{1}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}} \right) 2 \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \\
 & \operatorname{ArcTanh} \left[\frac{(f - i (d e - c f)) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}} \right] - \\
 & 2 \operatorname{ArcCos} \left[-\frac{i f}{d e - c f} \right] \operatorname{ArcTanh} \left[\frac{(-f - i (d e - c f)) \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x] \right) \right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}} \right] +
 \end{aligned}$$

$$\begin{aligned}
 & \left(\operatorname{ArcCos} \left[-\frac{i f}{d e - c f} \right] - 2 i \right. \\
 & \quad \left(\operatorname{ArcTanh} \left[\left((f - i (d e - c f)) \cot \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) / \right. \right. \\
 & \quad \quad \left. \left. \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] - \operatorname{ArcTanh} \left[\left((-f - i (d e - c f)) \right. \right. \right. \\
 & \quad \quad \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] \right) \right) \\
 & \operatorname{Log} \left[\frac{e^{-\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right)} \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}{\sqrt{2} \sqrt{-i (d e - c f)} \sqrt{f + \frac{d e - c f}{c + d x}}} \right] + \left(\operatorname{ArcCos} \left[-\frac{i f}{d e - c f} \right] + \right. \\
 & \quad 2 i \left(\operatorname{ArcTanh} \left[\left((f - i (d e - c f)) \cot \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) / \right. \right. \\
 & \quad \quad \left. \left. \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] - \operatorname{ArcTanh} \left[\left((-f - i (d e - c f)) \right. \right. \right. \\
 & \quad \quad \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] \right) \right) \\
 & \operatorname{Log} \left[\frac{e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right)} \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}{\sqrt{2} \sqrt{-i (d e - c f)} \sqrt{f + \frac{d e - c f}{c + d x}}} \right] - \\
 & \left(\operatorname{ArcCos} \left[-\frac{i f}{d e - c f} \right] + 2 i \operatorname{ArcTanh} \left[\left((-f - i (d e - c f)) \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] \right) \\
 & \operatorname{Log} \left[1 - \left(i \left(f - i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right) \right) / \\
 & \quad \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right) \right] + \\
 & \left(-\operatorname{ArcCos} \left[-\frac{i f}{d e - c f} \right] + 2 i \operatorname{ArcTanh} \left[\left((-f - i (d e - c f)) \operatorname{Tan} \left[\right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] \right) \\
 & \operatorname{Log} \left[1 - \left(i \left(f + i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right) \right) / \\
 & \quad \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right) \right] +
 \end{aligned}$$

$$\begin{aligned}
 & i \left(\text{PolyLog}\left[2, \left(i \left(f - i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \right. \right. \\
 & \quad \left. \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \text{ Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcCsch}[c + d x] \right)\right]\right)\right]\right) / \\
 & \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcCsch}[c + d x] \right)\right]\right)\right) - \text{PolyLog}\left[\right. \\
 & 2, \left(i \left(f + i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \text{ Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcCsch}[c + d x] \right)\right]\right)\right)\right] / \\
 & \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcCsch}[c + d x] \right)\right]\right)\right) \right] \Bigg) + \\
 & \frac{1}{(d e - c f)^2 (d^2 e^2 - 2 c d e f + f^2 + c^2 f^2) (d e - c f + f (c + d x))^3} \\
 & \frac{c^2}{f^2} \\
 & (c + d x)^3 \\
 & \left(f + \frac{d e}{c + d x} - \frac{c f}{c + d x} \right)^3 \\
 & \left(\frac{i \pi \text{ArcTanh}\left[\frac{-d e + c f + f \text{Tanh}\left[\frac{1}{2} \text{ArcCsch}[c + d x]\right]}{\sqrt{f^2 + (d e - c f)^2}}\right]}{\sqrt{f^2 + (d e - c f)^2}} \right) - \\
 & \frac{1}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}} \left(2 \left(\frac{\pi}{2} - i \text{ArcCsch}[c + d x] \right) \right. \\
 & \quad \left. \text{ArcTanh}\left[\frac{(f - i (d e - c f)) \text{Cot}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcCsch}[c + d x] \right)\right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}\right] \right) - \\
 & 2 \text{ArcCos}\left[-\frac{i f}{d e - c f}\right] \text{ArcTanh}\left[\frac{(-f - i (d e - c f)) \text{Tan}\left[\frac{1}{2} \left(\frac{\pi}{2} - i \text{ArcCsch}[c + d x] \right)\right]}{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}\right] + \\
 & \left(\text{ArcCos}\left[-\frac{i f}{d e - c f}\right] - 2 i \right)
 \end{aligned}$$

$$\begin{aligned}
 & \left(\operatorname{ArcTanh} \left[\left((f - i (d e - c f)) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] / \right. \\
 & \quad \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) - \operatorname{ArcTanh} \left[\left((-f - i (d e - c f)) \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] \Big) \\
 & \operatorname{Log} \left[\frac{e^{-\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right)} \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}{\sqrt{2} \sqrt{-i (d e - c f)} \sqrt{f + \frac{d e - c f}{c + d x}}} \right] + \left(\operatorname{ArcCos} \left[-\frac{i f}{d e - c f} \right] + \right. \\
 & \quad 2 i \left(\operatorname{ArcTanh} \left[\left((f - i (d e - c f)) \operatorname{Cot} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] / \right. \\
 & \quad \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) - \operatorname{ArcTanh} \left[\left((-f - i (d e - c f)) \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] \Big) \\
 & \operatorname{Log} \left[\frac{e^{\frac{1}{2} i \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right)} \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}}{\sqrt{2} \sqrt{-i (d e - c f)} \sqrt{f + \frac{d e - c f}{c + d x}}} \right] - \\
 & \left(\operatorname{ArcCos} \left[-\frac{i f}{d e - c f} \right] + 2 i \operatorname{ArcTanh} \left[\left((-f - i (d e - c f)) \right. \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] \Big) \\
 & \operatorname{Log} \left[1 - \left(i \left(f - i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] / \right. \\
 & \quad \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] \Big) + \\
 & \left(-\operatorname{ArcCos} \left[-\frac{i f}{d e - c f} \right] + 2 i \operatorname{ArcTanh} \left[\left((-f - i (d e - c f)) \operatorname{Tan} \left[\right. \right. \right. \right. \\
 & \quad \left. \left. \left. \frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] / \left(\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \right] \Big) \\
 & \operatorname{Log} \left[1 - \left(i \left(f + i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \right. \\
 & \quad \left. \left. \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] / \right. \\
 & \quad \left((d e - c f) \left(f - i (d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right. \right. \\
 & \quad \left. \left. \operatorname{Tan} \left[\frac{1}{2} \left(\frac{\pi}{2} - i \operatorname{ArcCsch} [c + d x] \right) \right] \right) \right] \Big) + \\
 & i \left(\operatorname{PolyLog} \left[2, \left(i \left(f - i \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \right) \left(f - i (d e - c f) - \right. \right. \right. \right. \right.
 \end{aligned}$$

$$\frac{\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2} \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x]\right)\right]}{\left((d e - c f)\left(f - i(d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x]\right)\right]\right) - \operatorname{PolyLog}\left[2, \left(i\left(f + i\sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right)\left(f - i(d e - c f) - \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x]\right)\right]\right)\right]}{\left((d e - c f)\left(f - i(d e - c f) + \sqrt{-d^2 e^2 + 2 c d e f - f^2 - c^2 f^2}\right) \operatorname{Tan}\left[\frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{ArcCsCh}[c + d x]\right)\right]\right)}$$

Problem 23: Result unnecessarily involves higher level functions.

$$\int \frac{\operatorname{ArcCsCh}[a x^n]}{x} dx$$

Optimal (type 4, 61 leaves, 7 steps):

$$\frac{\operatorname{ArcCsCh}[a x^n]^2}{2 n} - \frac{\operatorname{ArcCsCh}[a x^n] \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCsCh}[a x^n]}\right]}{n} - \frac{\operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCsCh}[a x^n]}\right]}{2 n}$$

Result (type 5, 64 leaves):

$$-\frac{x^{-n} \operatorname{HypergeometricPFQ}\left[\left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right\}, \left\{\frac{3}{2}, \frac{3}{2}\right\}, -\frac{x^{-2 n}}{a^2}\right]}{a n} + \left(\operatorname{ArcCsCh}[a x^n] - \operatorname{ArcSinh}\left[\frac{x^{-n}}{a}\right]\right) \operatorname{Log}[x]$$

Problem 25: Result more than twice size of optimal antiderivative.

$$\int \operatorname{ArcCsCh}\left[c e^{a+b x}\right] dx$$

Optimal (type 4, 77 leaves, 7 steps):

$$\frac{\operatorname{ArcCsCh}\left[c e^{a+b x}\right]^2}{2 b} - \frac{\operatorname{ArcCsCh}\left[c e^{a+b x}\right] \operatorname{Log}\left[1 - e^{2 \operatorname{ArcCsCh}\left[c e^{a+b x}\right]}\right]}{b} - \frac{\operatorname{PolyLog}\left[2, e^{2 \operatorname{ArcCsCh}\left[c e^{a+b x}\right]}\right]}{2 b}$$

Result (type 4, 236 leaves):

$$\begin{aligned}
 & x \operatorname{ArcCsch}\left[c e^{a+bx}\right] + \left(e^{-a-bx} \sqrt{1+c^2 e^{2(a+bx)}} \right. \\
 & \left. \left(\operatorname{Log}\left[-c^2 e^{2(a+bx)}\right]^2 + \operatorname{ArcTanh}\left[\sqrt{1+c^2 e^{2(a+bx)}}\right] \left(-8bx + 4 \operatorname{Log}\left[-c^2 e^{2(a+bx)}\right]\right) - \right. \right. \\
 & \left. \left. 4 \operatorname{Log}\left[-c^2 e^{2(a+bx)}\right] \operatorname{Log}\left[\frac{1}{2}\left(1+\sqrt{1+c^2 e^{2(a+bx)}}\right)\right] + 2 \operatorname{Log}\left[\frac{1}{2}\left(1+\sqrt{1+c^2 e^{2(a+bx)}}\right)\right]^2 - \right. \right. \\
 & \left. \left. 4 \operatorname{PolyLog}\left[2, \frac{1}{2}\left(1-\sqrt{1+c^2 e^{2(a+bx)}}\right)\right]\right) \right) / \left(8bc \sqrt{1+\frac{e^{-2(a+bx)}}{c^2}} \right)
 \end{aligned}$$

Problem 38: Result unnecessarily involves higher level functions.

$$\int e^{\operatorname{ArcCsch}[ax^2]} x^4 dx$$

Optimal (type 4, 202 leaves, 8 steps):

$$-\frac{2\sqrt{1+\frac{1}{a^2x^4}}}{5a^2\left(a+\frac{1}{x^2}\right)x} + \frac{2\sqrt{1+\frac{1}{a^2x^4}}x}{5a^2} + \frac{x^3}{3a} + \frac{1}{5}\sqrt{1+\frac{1}{a^2x^4}}x^5 +$$

$$\frac{2\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\operatorname{EllipticE}\left[2\operatorname{ArcCot}\left[\sqrt{a}x\right], \frac{1}{2}\right]}{5a^{7/2}\sqrt{1+\frac{1}{a^2x^4}}}$$

$$\frac{\sqrt{\frac{a^2+\frac{1}{x^4}}{\left(a+\frac{1}{x^2}\right)^2}}\left(a+\frac{1}{x^2}\right)\operatorname{EllipticF}\left[2\operatorname{ArcCot}\left[\sqrt{a}x\right], \frac{1}{2}\right]}{5a^{7/2}\sqrt{1+\frac{1}{a^2x^4}}}$$

Result (type 5, 126 leaves):

$$\begin{aligned}
 & -\frac{1}{15a\left(ax^2\right)^{3/2}}2\sqrt{2}e^{-\operatorname{ArcCsch}[ax^2]}\left(\frac{e^{\operatorname{ArcCsch}[ax^2]}}{-1+e^{2\operatorname{ArcCsch}[ax^2]}}\right)^{5/2}x^3\left(-1-2e^{2\operatorname{ArcCsch}[ax^2]}\right) - \\
 & 3e^{4\operatorname{ArcCsch}[ax^2]}+\left(1-e^{2\operatorname{ArcCsch}[ax^2]}\right)^{5/2}\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, e^{2\operatorname{ArcCsch}[ax^2]}\right]
 \end{aligned}$$

Problem 40: Result unnecessarily involves higher level functions.

$$\int e^{\operatorname{ArcCsch}[ax^2]} x^2 dx$$

Optimal (type 4, 86 leaves, 5 steps):

$$\frac{x}{a} + \frac{1}{3} \sqrt{1 + \frac{1}{a^2 x^4}} x^3 - \frac{\sqrt{\frac{a^2 + \frac{1}{x^4}}{\left(a + \frac{1}{x^2}\right)^2}} \left(a + \frac{1}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{3 a^{5/2} \sqrt{1 + \frac{1}{a^2 x^4}}}$$

Result (type 5, 113 leaves):

$$-\frac{1}{3 a \sqrt{a x^2}} 2 \sqrt{2} e^{-\text{ArcCsch}[a x^2]} \left(\frac{e^{\text{ArcCsch}[a x^2]}}{-1 + e^{2 \text{ArcCsch}[a x^2]}}\right)^{3/2} x \left(1 - 2 e^{2 \text{ArcCsch}[a x^2]} - \left(1 - e^{2 \text{ArcCsch}[a x^2]}\right)^{3/2} \text{Hypergeometric2F1}\left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, e^{2 \text{ArcCsch}[a x^2]}\right]\right)$$

Problem 42: Result unnecessarily involves higher level functions.

$$\int e^{\text{ArcCsch}[a x^2]} dx$$

Optimal (type 4, 165 leaves, 7 steps):

$$-\frac{1}{a x} - \frac{2 \sqrt{1 + \frac{1}{a^2 x^4}}}{\left(a + \frac{1}{x^2}\right) x} + \sqrt{1 + \frac{1}{a^2 x^4}} x + \frac{2 \sqrt{\frac{a^2 + \frac{1}{x^4}}{\left(a + \frac{1}{x^2}\right)^2}} \left(a + \frac{1}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{a^{3/2} \sqrt{1 + \frac{1}{a^2 x^4}}} - \frac{\sqrt{\frac{a^2 + \frac{1}{x^4}}{\left(a + \frac{1}{x^2}\right)^2}} \left(a + \frac{1}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{a^{3/2} \sqrt{1 + \frac{1}{a^2 x^4}}}$$

Result (type 5, 96 leaves):

$$\frac{1}{3 \sqrt{a x^2}} \sqrt{2} e^{\text{ArcCsch}[a x^2]} \sqrt{\frac{e^{\text{ArcCsch}[a x^2]}}{-1 + e^{2 \text{ArcCsch}[a x^2]}}} x \left(3 - 2 \sqrt{1 - e^{2 \text{ArcCsch}[a x^2]}} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, e^{2 \text{ArcCsch}[a x^2]}\right]\right)$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{e^{\text{ArcCsch}[a x^2]}}{x^2} dx$$

Optimal (type 4, 91 leaves, 5 steps):

$$-\frac{1}{3 a x^3} - \frac{\sqrt{1 + \frac{1}{a^2 x^4}}}{3 x} - \frac{\sqrt{\frac{a^2 + \frac{1}{x^4}}{\left(a + \frac{1}{x^2}\right)^2}} \left(a + \frac{1}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{3 \sqrt{a} \sqrt{1 + \frac{1}{a^2 x^4}}}$$

Result (type 4, 95 leaves):

$$-\frac{1}{3 x^3} \left(\frac{1}{a} + \sqrt{1 + \frac{1}{a^2 x^4}} x^2 + \frac{1}{\sqrt{1 + a^2 x^4}} \right. \\ \left. 2 (-1)^{1/4} \sqrt{1 + \frac{1}{a^2 x^4}} x^2 (a x^2)^{3/2} \text{EllipticF}\left[i \text{ArcSinh}\left[(-1)^{1/4} \sqrt{a x^2}\right], -1\right] \right)$$

Problem 46: Result unnecessarily involves higher level functions.

$$\int \frac{e^{\text{ArcCsch}[a x^2]}}{x^4} dx$$

Optimal (type 4, 181 leaves, 7 steps):

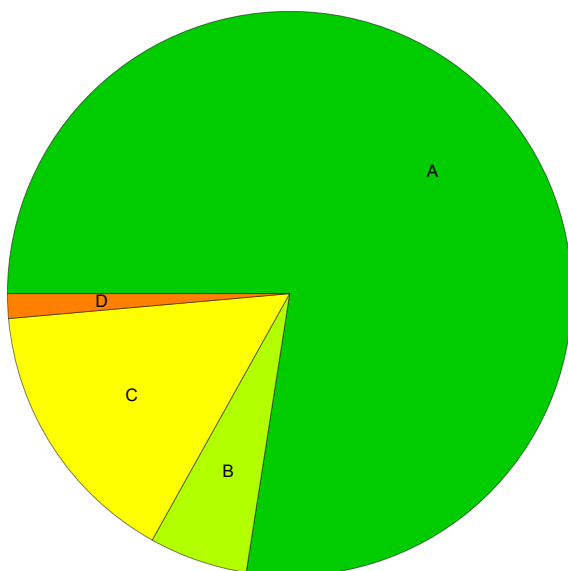
$$-\frac{1}{5 a x^5} - \frac{\sqrt{1 + \frac{1}{a^2 x^4}}}{5 x^3} - \frac{2 a^2 \sqrt{1 + \frac{1}{a^2 x^4}}}{5 \left(a + \frac{1}{x^2}\right) x} + \frac{2 \sqrt{a} \sqrt{\frac{a^2 + \frac{1}{x^4}}{\left(a + \frac{1}{x^2}\right)^2}} \left(a + \frac{1}{x^2}\right) \text{EllipticE}\left[2 \text{ArcCot}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{5 \sqrt{1 + \frac{1}{a^2 x^4}}} \\ \frac{\sqrt{a} \sqrt{\frac{a^2 + \frac{1}{x^4}}{\left(a + \frac{1}{x^2}\right)^2}} \left(a + \frac{1}{x^2}\right) \text{EllipticF}\left[2 \text{ArcCot}\left[\sqrt{a} x\right], \frac{1}{2}\right]}{5 \sqrt{1 + \frac{1}{a^2 x^4}}}$$

Result (type 5, 119 leaves):

$$-\frac{1}{10 x^3} e^{-\text{ArcCsch}[a x^2]} \sqrt{\frac{e^{\text{ArcCsch}[a x^2]}}{-2 + 2 e^{2 \text{ArcCsch}[a x^2]}}} (a x^2)^{3/2} \left(-3 + 2 e^{2 \text{ArcCsch}[a x^2]} + \right. \\ \left. e^{4 \text{ArcCsch}[a x^2]} + 8 \sqrt{1 - e^{2 \text{ArcCsch}[a x^2]}} \text{Hypergeometric2F1}\left[-\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, e^{2 \text{ArcCsch}[a x^2]}\right] \right)$$

Summary of Integration Test Results

71 integration problems



A - 55 optimal antiderivatives

B - 4 more than twice size of optimal antiderivatives

C - 11 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 0 integration timeouts