

# Mathematica 11.3 Integration Test Results

Test results for the 198 problems in "8.8 Polylogarithm function.m"

Problem 17: Unable to integrate problem.

$$\int \frac{\text{PolyLog}[3, a x]}{x^3} dx$$

Optimal (type 4, 70 leaves, 5 steps):

$$-\frac{a}{8x} + \frac{1}{8} a^2 \text{Log}[x] - \frac{1}{8} a^2 \text{Log}[1 - a x] + \frac{\text{Log}[1 - a x]}{8 x^2} - \frac{\text{PolyLog}[2, a x]}{4 x^2} - \frac{\text{PolyLog}[3, a x]}{2 x^2}$$

Result (type 9, 25 leaves):

$$\frac{\text{MeijerG}[\{\{1, 1, 1, 1\}, \{3\}\}, \{\{1, 2\}, \{0, 0, 0\}\}, -a x]}{x^2}$$

Problem 18: Unable to integrate problem.

$$\int \frac{\text{PolyLog}[3, a x]}{x^4} dx$$

Optimal (type 4, 80 leaves, 5 steps):

$$-\frac{a}{54 x^2} - \frac{a^2}{27 x} + \frac{1}{27} a^3 \text{Log}[x] - \frac{1}{27} a^3 \text{Log}[1 - a x] + \frac{\text{Log}[1 - a x]}{27 x^3} - \frac{\text{PolyLog}[2, a x]}{9 x^3} - \frac{\text{PolyLog}[3, a x]}{3 x^3}$$

Result (type 9, 25 leaves):

$$\frac{\text{MeijerG}[\{\{1, 1, 1, 1\}, \{4\}\}, \{\{1, 3\}, \{0, 0, 0\}\}, -a x]}{x^3}$$

Problem 22: Result more than twice size of optimal antiderivative.

$$\int \frac{\text{PolyLog}[2, a x^2]}{x} dx$$

Optimal (type 4, 11 leaves, 1 step):

$$\frac{1}{2} \text{PolyLog}[3, a x^2]$$

Result (type 4, 108 leaves):

$$\begin{aligned}
& -\operatorname{Log}[x]^2 \operatorname{Log}[1 - \sqrt{a} x] - \operatorname{Log}[x]^2 \operatorname{Log}[1 + \sqrt{a} x] + \operatorname{Log}[x]^2 \operatorname{Log}[1 - a x^2] - \\
& 2 \operatorname{Log}[x] \operatorname{PolyLog}[2, -\sqrt{a} x] - 2 \operatorname{Log}[x] \operatorname{PolyLog}[2, \sqrt{a} x] + \\
& \operatorname{Log}[x] \operatorname{PolyLog}[2, a x^2] + 2 \operatorname{PolyLog}[3, -\sqrt{a} x] + 2 \operatorname{PolyLog}[3, \sqrt{a} x]
\end{aligned}$$

### Problem 37: Unable to integrate problem.

$$\int \frac{\operatorname{PolyLog}[3, a x^2]}{x^5} dx$$

Optimal (type 4, 78 leaves, 6 steps):

$$-\frac{a}{16 x^2} + \frac{1}{8} a^2 \operatorname{Log}[x] - \frac{1}{16} a^2 \operatorname{Log}[1 - a x^2] + \frac{\operatorname{Log}[1 - a x^2]}{16 x^4} - \frac{\operatorname{PolyLog}[2, a x^2]}{8 x^4} - \frac{\operatorname{PolyLog}[3, a x^2]}{4 x^4}$$

Result (type 9, 30 leaves):

$$\frac{\operatorname{MeijerG}\left[\left\{\left\{1, 1, 1, 1\right\}, \{3\}\right\}, \left\{\{1, 2\}, \{0, 0, 0\}\right\}, -a x^2\right]}{2 x^4}$$

### Problem 38: Unable to integrate problem.

$$\int \frac{\operatorname{PolyLog}[3, a x^2]}{x^7} dx$$

Optimal (type 4, 88 leaves, 6 steps):

$$\begin{aligned}
& -\frac{a}{108 x^4} - \frac{a^2}{54 x^2} + \frac{1}{27} a^3 \operatorname{Log}[x] - \frac{1}{54} a^3 \operatorname{Log}[1 - a x^2] + \\
& \frac{\operatorname{Log}[1 - a x^2]}{54 x^6} - \frac{\operatorname{PolyLog}[2, a x^2]}{18 x^6} - \frac{\operatorname{PolyLog}[3, a x^2]}{6 x^6}
\end{aligned}$$

Result (type 9, 30 leaves):

$$\frac{\operatorname{MeijerG}\left[\left\{\left\{1, 1, 1, 1\right\}, \{4\}\right\}, \left\{\{1, 3\}, \{0, 0, 0\}\right\}, -a x^2\right]}{2 x^6}$$

### Problem 48: Result more than twice size of optimal antiderivative.

$$\int \frac{\operatorname{PolyLog}[2, a x^q]}{x} dx$$

Optimal (type 4, 11 leaves, 1 step):

$$\frac{\operatorname{PolyLog}[3, a x^q]}{q}$$

Result (type 4, 80 leaves):

$$-\frac{1}{6} q \operatorname{Log}[x]^2 \left( q \operatorname{Log}[x] + 3 \operatorname{Log}\left[1 - \frac{x^{-q}}{a}\right] - 3 \operatorname{Log}[1 - a x^q] \right) +$$

$$\operatorname{Log}[x] \operatorname{PolyLog}\left[2, \frac{x^{-q}}{a}\right] + \operatorname{Log}[x] \operatorname{PolyLog}[2, a x^q] + \frac{\operatorname{PolyLog}\left[3, \frac{x^{-q}}{a}\right]}{q}$$

### Problem 52: Unable to integrate problem.

$$\int x^2 \operatorname{PolyLog}[3, a x^q] dx$$

Optimal (type 5, 88 leaves, 4 steps):

$$\frac{a q^3 x^{3+q} \operatorname{Hypergeometric2F1}\left[1, \frac{3+q}{q}, 2 + \frac{3}{q}, a x^q\right]}{27 (3+q)} -$$

$$\frac{1}{27} q^2 x^3 \operatorname{Log}[1 - a x^q] - \frac{1}{9} q x^3 \operatorname{PolyLog}[2, a x^q] + \frac{1}{3} x^3 \operatorname{PolyLog}[3, a x^q]$$

Result (type 9, 41 leaves):

$$\frac{x^3 \operatorname{MeijerG}\left[\left[\{1, 1, 1, 1, \frac{-3+q}{q}\}, \{\}\right], \left[\{1\}, \{0, 0, 0, -\frac{3}{q}\}\right], -a x^q\right]}{q}$$

### Problem 53: Unable to integrate problem.

$$\int x \operatorname{PolyLog}[3, a x^q] dx$$

Optimal (type 5, 88 leaves, 4 steps):

$$\frac{a q^3 x^{2+q} \operatorname{Hypergeometric2F1}\left[1, \frac{2+q}{q}, 2 \left(1 + \frac{1}{q}\right), a x^q\right]}{8 (2+q)} -$$

$$\frac{1}{8} q^2 x^2 \operatorname{Log}[1 - a x^q] - \frac{1}{4} q x^2 \operatorname{PolyLog}[2, a x^q] + \frac{1}{2} x^2 \operatorname{PolyLog}[3, a x^q]$$

Result (type 9, 41 leaves):

$$\frac{x^2 \operatorname{MeijerG}\left[\left[\{1, 1, 1, 1, \frac{-2+q}{q}\}, \{\}\right], \left[\{1\}, \{0, 0, 0, -\frac{2}{q}\}\right], -a x^q\right]}{q}$$

### Problem 54: Unable to integrate problem.

$$\int \operatorname{PolyLog}[3, a x^q] dx$$

Optimal (type 5, 69 leaves, 4 steps):

$$\frac{a q^3 x^{1+q} \operatorname{Hypergeometric2F1}\left[1, 1 + \frac{1}{q}, 2 + \frac{1}{q}, a x^q\right]}{1+q} -$$

$$q^2 x \operatorname{Log}[1 - a x^q] - q x \operatorname{PolyLog}[2, a x^q] + x \operatorname{PolyLog}[3, a x^q]$$

Result (type 9, 39 leaves):

$$\frac{x \operatorname{MeijerG}\left[\left\{\left\{1, 1, 1, 1, \frac{-1+q}{q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, -\frac{1}{q}\}\right\}, -a x^q\right]}{q}$$

Problem 56: Unable to integrate problem.

$$\int \frac{\operatorname{PolyLog}[3, a x^q]}{x^2} dx$$

Optimal (type 5, 84 leaves, 4 steps):

$$\frac{a q^3 x^{-1+q} \operatorname{Hypergeometric2F1}\left[1, -\frac{1-q}{q}, 2 - \frac{1}{q}, a x^q\right]}{1 - q} + \frac{q^2 \operatorname{Log}[1 - a x^q]}{x} - \frac{q \operatorname{PolyLog}[2, a x^q]}{x} - \frac{\operatorname{PolyLog}[3, a x^q]}{x}$$

Result (type 9, 37 leaves):

$$\frac{\operatorname{MeijerG}\left[\left\{\left\{1, 1, 1, 1, 1 + \frac{1}{q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, \frac{1}{q}\}\right\}, -a x^q\right]}{q x}$$

Problem 57: Unable to integrate problem.

$$\int \frac{\operatorname{PolyLog}[3, a x^q]}{x^3} dx$$

Optimal (type 5, 95 leaves, 4 steps):

$$\frac{a q^3 x^{-2+q} \operatorname{Hypergeometric2F1}\left[1, -\frac{2-q}{q}, 2 \left(1 - \frac{1}{q}\right), a x^q\right]}{8 (2 - q)} + \frac{q^2 \operatorname{Log}[1 - a x^q]}{8 x^2} - \frac{q \operatorname{PolyLog}[2, a x^q]}{4 x^2} - \frac{\operatorname{PolyLog}[3, a x^q]}{2 x^2}$$

Result (type 9, 41 leaves):

$$\frac{\operatorname{MeijerG}\left[\left\{\left\{1, 1, 1, 1, \frac{2+q}{q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, \frac{2}{q}\}\right\}, -a x^q\right]}{q x^2}$$

Problem 58: Unable to integrate problem.

$$\int \frac{\operatorname{PolyLog}[3, a x^q]}{x^4} dx$$

Optimal (type 5, 93 leaves, 4 steps):

$$-\frac{a q^3 x^{-3+q} \text{Hypergeometric2F1}\left[1, -\frac{3-q}{q}, 2-\frac{3}{q}, a x^q\right]}{27(3-q)} + \frac{q^2 \text{Log}[1-a x^q]}{27 x^3} - \frac{q \text{PolyLog}[2, a x^q]}{9 x^3} - \frac{\text{PolyLog}[3, a x^q]}{3 x^3}$$

Result (type 9, 41 leaves):

$$-\frac{\text{MeijerG}\left[\left\{\left\{1, 1, 1, 1, \frac{3+q}{q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, \frac{3}{q}\}\right\}, -a x^q\right]}{q x^3}$$

**Problem 74: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{PolyLog}[2, a x^2]}{\sqrt{d x}} dx$$

Optimal (type 4, 115 leaves, 8 steps):

$$-\frac{32 \sqrt{d x}}{d} + \frac{16 \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{a^{1/4} \sqrt{d}} + \frac{16 \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{a^{1/4} \sqrt{d}} + \frac{8 \sqrt{d x} \text{Log}[1-a x^2]}{d} + \frac{2 \sqrt{d x} \text{PolyLog}[2, a x^2]}{d}$$

Result (type 5, 57 leaves):

$$\frac{1}{2 \sqrt{d x} \text{Gamma}\left[\frac{9}{4}\right]} 5 x \text{Gamma}\left[\frac{5}{4}\right] \left(-16 + 16 \text{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, a x^2\right] + 4 \text{Log}[1-a x^2] + \text{PolyLog}[2, a x^2]\right)$$

**Problem 75: Result unnecessarily involves higher level functions.**

$$\int \frac{\text{PolyLog}[2, a x^2]}{(d x)^{3/2}} dx$$

Optimal (type 4, 103 leaves, 7 steps):

$$-\frac{16 a^{1/4} \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{16 a^{1/4} \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{8 \text{Log}[1-a x^2]}{d \sqrt{d x}} - \frac{2 \text{PolyLog}[2, a x^2]}{d \sqrt{d x}}$$

Result (type 5, 62 leaves):

$$\left(x \text{Gamma}\left[\frac{3}{4}\right] \left(16 a x^2 \text{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, a x^2\right] + 12 \text{Log}[1-a x^2] - 3 \text{PolyLog}[2, a x^2]\right)\right) / \left(2 (d x)^{3/2} \text{Gamma}\left[\frac{7}{4}\right]\right)$$

### Problem 76: Result unnecessarily involves higher level functions.

$$\int \frac{\text{PolyLog}[2, a x^2]}{(d x)^{5/2}} dx$$

Optimal (type 4, 111 leaves, 7 steps):

$$\frac{16 a^{3/4} \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{9 d^{5/2}} + \frac{16 a^{3/4} \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{9 d^{5/2}} + \frac{8 \text{Log}[1 - a x^2]}{9 d (d x)^{3/2}} - \frac{2 \text{PolyLog}[2, a x^2]}{3 d (d x)^{3/2}}$$

Result (type 5, 62 leaves):

$$\left( x \text{Gamma}\left[\frac{1}{4}\right] \left( 16 a x^2 \text{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, a x^2\right] + 4 \text{Log}[1 - a x^2] - 3 \text{PolyLog}[2, a x^2] \right) \right) / \left( 18 (d x)^{5/2} \text{Gamma}\left[\frac{5}{4}\right] \right)$$

### Problem 77: Result unnecessarily involves higher level functions.

$$\int \frac{\text{PolyLog}[2, a x^2]}{(d x)^{7/2}} dx$$

Optimal (type 4, 126 leaves, 8 steps):

$$-\frac{32 a}{25 d^3 \sqrt{d x}} - \frac{16 a^{5/4} \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{25 d^{7/2}} + \frac{16 a^{5/4} \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{25 d^{7/2}} + \frac{8 \text{Log}[1 - a x^2]}{25 d (d x)^{5/2}} - \frac{2 \text{PolyLog}[2, a x^2]}{5 d (d x)^{5/2}}$$

Result (type 5, 70 leaves):

$$-\left( \left( x \text{Gamma}\left[-\frac{1}{4}\right] \left( -48 a x^2 + 16 a^2 x^4 \text{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, a x^2\right] + 12 \text{Log}[1 - a x^2] - 15 \text{PolyLog}[2, a x^2] \right) \right) \right) / \left( 150 (d x)^{7/2} \text{Gamma}\left[\frac{3}{4}\right] \right)$$

### Problem 78: Result unnecessarily involves higher level functions.

$$\int (d x)^{5/2} \text{PolyLog}[3, a x^2] dx$$

Optimal (type 4, 161 leaves, 10 steps):

$$\frac{128 d (d x)^{3/2}}{1029 a} + \frac{128 (d x)^{7/2}}{2401 d} + \frac{64 d^{5/2} \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{343 a^{7/4}} - \frac{64 d^{5/2} \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{343 a^{7/4}} - \frac{32 (d x)^{7/2} \text{Log}[1 - a x^2]}{343 d} - \frac{8 (d x)^{7/2} \text{PolyLog}[2, a x^2]}{49 d} + \frac{2 (d x)^{7/2} \text{PolyLog}[3, a x^2]}{7 d}$$

Result (type 5, 89 leaves):

$$-\left(\left(11 d (d x)^{3/2} \text{Gamma}\left[\frac{11}{4}\right] \left(-448 - 192 a x^2 + 448 \text{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, a x^2\right] + 336 a x^2 \text{Log}[1 - a x^2] + 588 a x^2 \text{PolyLog}[2, a x^2] - 1029 a x^2 \text{PolyLog}[3, a x^2]\right)\right) / \left(14406 a \text{Gamma}\left[\frac{15}{4}\right]\right)\right)$$

**Problem 79: Result unnecessarily involves higher level functions.**

$$\int (d x)^{3/2} \text{PolyLog}[3, a x^2] dx$$

Optimal (type 4, 161 leaves, 10 steps):

$$\frac{128 d \sqrt{d x}}{125 a} + \frac{128 (d x)^{5/2}}{625 d} - \frac{64 d^{3/2} \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{125 a^{5/4}} - \frac{64 d^{3/2} \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{125 a^{5/4}} - \frac{32 (d x)^{5/2} \text{Log}[1 - a x^2]}{125 d} - \frac{8 (d x)^{5/2} \text{PolyLog}[2, a x^2]}{25 d} + \frac{2 (d x)^{5/2} \text{PolyLog}[3, a x^2]}{5 d}$$

Result (type 5, 89 leaves):

$$-\frac{1}{1250 a \text{Gamma}\left[\frac{13}{4}\right]} 9 d \sqrt{d x} \text{Gamma}\left[\frac{9}{4}\right] \left(-320 - 64 a x^2 + 320 \text{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, a x^2\right] + 80 a x^2 \text{Log}[1 - a x^2] + 100 a x^2 \text{PolyLog}[2, a x^2] - 125 a x^2 \text{PolyLog}[3, a x^2]\right)$$

**Problem 80: Result unnecessarily involves higher level functions.**

$$\int \sqrt{d x} \text{PolyLog}[3, a x^2] dx$$

Optimal (type 4, 146 leaves, 9 steps):

$$\frac{128 (d x)^{3/2}}{81 d} + \frac{64 \sqrt{d} \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{27 a^{3/4}} - \frac{64 \sqrt{d} \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{27 a^{3/4}} - \frac{32 (d x)^{3/2} \text{Log}[1 - a x^2]}{27 d} - \frac{8 (d x)^{3/2} \text{PolyLog}[2, a x^2]}{9 d} + \frac{2 (d x)^{3/2} \text{PolyLog}[3, a x^2]}{3 d}$$

Result (type 5, 68 leaves):

$$-\frac{1}{162 \text{Gamma}\left[\frac{11}{4}\right]} 7 x \sqrt{d x} \text{Gamma}\left[\frac{7}{4}\right] \left(-64 + 64 \text{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, a x^2\right] + 48 \text{Log}[1 - a x^2] + 36 \text{PolyLog}[2, a x^2] - 27 \text{PolyLog}[3, a x^2]\right)$$

### Problem 81: Result unnecessarily involves higher level functions.

$$\int \frac{\text{PolyLog}[3, a x^2]}{\sqrt{d x}} dx$$

Optimal (type 4, 134 leaves, 9 steps):

$$\frac{128 \sqrt{d x}}{d} - \frac{64 \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{a^{1/4} \sqrt{d}} - \frac{64 \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{a^{1/4} \sqrt{d}} - \frac{32 \sqrt{d x} \text{Log}[1 - a x^2]}{d} - \frac{8 \sqrt{d x} \text{PolyLog}[2, a x^2]}{d} + \frac{2 \sqrt{d x} \text{PolyLog}[3, a x^2]}{d}$$

Result (type 5, 68 leaves):

$$-\frac{1}{2 \sqrt{d x} \text{Gamma}\left[\frac{9}{4}\right]} 5 x \text{Gamma}\left[\frac{5}{4}\right] \left(-64 + 64 \text{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, a x^2\right] + 16 \text{Log}[1 - a x^2] + 4 \text{PolyLog}[2, a x^2] - \text{PolyLog}[3, a x^2]\right)$$

### Problem 82: Result unnecessarily involves higher level functions.

$$\int \frac{\text{PolyLog}[3, a x^2]}{(d x)^{3/2}} dx$$

Optimal (type 4, 122 leaves, 8 steps):

$$-\frac{64 a^{1/4} \text{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{64 a^{1/4} \text{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{d^{3/2}} + \frac{32 \text{Log}[1 - a x^2]}{d \sqrt{d x}} - \frac{8 \text{PolyLog}[2, a x^2]}{d \sqrt{d x}} - \frac{2 \text{PolyLog}[3, a x^2]}{d \sqrt{d x}}$$

Result (type 5, 71 leaves):

$$\left(x \text{Gamma}\left[\frac{3}{4}\right] \left(64 a x^2 \text{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, a x^2\right] + 48 \text{Log}[1 - a x^2] - 12 \text{PolyLog}[2, a x^2] - 3 \text{PolyLog}[3, a x^2]\right)\right) / \left(2 (d x)^{3/2} \text{Gamma}\left[\frac{7}{4}\right]\right)$$

### Problem 83: Result unnecessarily involves higher level functions.

$$\int \frac{\text{PolyLog}[3, a x^2]}{(d x)^{5/2}} dx$$

Optimal (type 4, 132 leaves, 8 steps):



$$\frac{64 a^{3/4} \operatorname{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{27 d^{5/2}} + \frac{64 a^{3/4} \operatorname{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{27 d^{5/2}} +$$

$$\frac{32 \operatorname{Log}\left[1 - a x^2\right]}{27 d (d x)^{3/2}} - \frac{8 \operatorname{PolyLog}\left[2, a x^2\right]}{9 d (d x)^{3/2}} - \frac{2 \operatorname{PolyLog}\left[3, a x^2\right]}{3 d (d x)^{3/2}}$$

Result (type 5, 71 leaves):

$$\left( x \operatorname{Gamma}\left[\frac{1}{4}\right] \left( 64 a x^2 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, a x^2\right] + 16 \operatorname{Log}\left[1 - a x^2\right] - \right. \right.$$

$$\left. \left. 12 \operatorname{PolyLog}\left[2, a x^2\right] - 9 \operatorname{PolyLog}\left[3, a x^2\right] \right) \right) / \left( 54 (d x)^{5/2} \operatorname{Gamma}\left[\frac{5}{4}\right] \right)$$

**Problem 84: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{PolyLog}\left[3, a x^2\right]}{(d x)^{7/2}} dx$$

Optimal (type 4, 147 leaves, 9 steps):

$$-\frac{128 a}{125 d^3 \sqrt{d x}} - \frac{64 a^{5/4} \operatorname{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{125 d^{7/2}} + \frac{64 a^{5/4} \operatorname{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{125 d^{7/2}} +$$

$$\frac{32 \operatorname{Log}\left[1 - a x^2\right]}{125 d (d x)^{5/2}} - \frac{8 \operatorname{PolyLog}\left[2, a x^2\right]}{25 d (d x)^{5/2}} - \frac{2 \operatorname{PolyLog}\left[3, a x^2\right]}{5 d (d x)^{5/2}}$$

Result (type 5, 79 leaves):

$$-\left( \left( x \operatorname{Gamma}\left[-\frac{1}{4}\right] \left( -192 a x^2 + 64 a^2 x^4 \operatorname{Hypergeometric2F1}\left[\frac{3}{4}, 1, \frac{7}{4}, a x^2\right] + 48 \operatorname{Log}\left[1 - a x^2\right] - \right. \right. \right.$$

$$\left. \left. 60 \operatorname{PolyLog}\left[2, a x^2\right] - 75 \operatorname{PolyLog}\left[3, a x^2\right] \right) \right) / \left( 750 (d x)^{7/2} \operatorname{Gamma}\left[\frac{3}{4}\right] \right)$$

**Problem 85: Result unnecessarily involves higher level functions.**

$$\int \frac{\operatorname{PolyLog}\left[3, a x^2\right]}{(d x)^{9/2}} dx$$

Optimal (type 4, 147 leaves, 9 steps):

$$-\frac{128 a}{1029 d^3 (d x)^{3/2}} + \frac{64 a^{7/4} \operatorname{ArcTan}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{343 d^{9/2}} + \frac{64 a^{7/4} \operatorname{ArcTanh}\left[\frac{a^{1/4} \sqrt{d x}}{\sqrt{d}}\right]}{343 d^{9/2}} +$$

$$\frac{32 \operatorname{Log}\left[1 - a x^2\right]}{343 d (d x)^{7/2}} - \frac{8 \operatorname{PolyLog}\left[2, a x^2\right]}{49 d (d x)^{7/2}} - \frac{2 \operatorname{PolyLog}\left[3, a x^2\right]}{7 d (d x)^{7/2}}$$

Result (type 5, 84 leaves):

$$-\left(\left(\sqrt{d x} \operatorname{Gamma}\left[-\frac{3}{4}\right]\left(-64 a x^2+192 a^2 x^4 \operatorname{Hypergeometric2F1}\left[\frac{1}{4}, 1, \frac{5}{4}, a x^2\right]+48 \operatorname{Log}\left[1-a x^2\right]-84 \operatorname{PolyLog}\left[2, a x^2\right]-147 \operatorname{PolyLog}\left[3, a x^2\right]\right)\right) / \left(686 d^5 x^4 \operatorname{Gamma}\left[\frac{1}{4}\right]\right)\right)$$

**Problem 88: Unable to integrate problem.**

$$\int \frac{\operatorname{PolyLog}\left[2, a x^q\right]}{\sqrt{d x}} d x$$

Optimal (type 5, 93 leaves, 4 steps):

$$\frac{8 a q^2 x^q \sqrt{d x} \operatorname{Hypergeometric2F1}\left[1, \frac{1+q}{q}, \frac{1}{2}\left(4+\frac{1}{q}\right), a x^q\right]}{d(1+2 q)} + \frac{4 q \sqrt{d x} \operatorname{Log}\left[1-a x^q\right]}{d} + \frac{2 \sqrt{d x} \operatorname{PolyLog}\left[2, a x^q\right]}{d}$$

Result (type 9, 48 leaves):

$$\frac{x \operatorname{MeijerG}\left[\left[\left\{\left\{1, 1, 1, 1-\frac{1}{2 q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, -\frac{1}{2 q}\}\right\}, -a x^q\right]\right]}{q \sqrt{d x}}$$

**Problem 89: Unable to integrate problem.**

$$\int \frac{\operatorname{PolyLog}\left[2, a x^q\right]}{(d x)^{3 / 2}} d x$$

Optimal (type 5, 97 leaves, 4 steps):

$$-\frac{8 a q^2 x^q \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}\left(2-\frac{1}{q}\right), \frac{1}{2}\left(4-\frac{1}{q}\right), a x^q\right]}{d(1-2 q) \sqrt{d x}} + \frac{4 q \operatorname{Log}\left[1-a x^q\right]}{d \sqrt{d x}} - \frac{2 \operatorname{PolyLog}\left[2, a x^q\right]}{d \sqrt{d x}}$$

Result (type 9, 48 leaves):

$$\frac{x \operatorname{MeijerG}\left[\left[\left\{\left\{1, 1, 1, 1+\frac{1}{2 q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, \frac{1}{2 q}\}\right\}, -a x^q\right]\right]}{q(d x)^{3 / 2}}$$

**Problem 90: Unable to integrate problem.**

$$\int \frac{\operatorname{PolyLog}\left[2, a x^q\right]}{(d x)^{5 / 2}} d x$$

Optimal (type 5, 105 leaves, 4 steps):

$$\frac{8 a q^2 x^{-1+q} \operatorname{Hypergeometric2F1}\left[1, \frac{1}{2}\left(2 - \frac{3}{q}\right), \frac{1}{2}\left(4 - \frac{3}{q}\right), a x^q\right]}{9 d^2 (3 - 2 q) \sqrt{d x}} + \frac{4 q \operatorname{Log}[1 - a x^q]}{9 d (d x)^{3/2}} - \frac{2 \operatorname{PolyLog}[2, a x^q]}{3 d (d x)^{3/2}}$$

Result (type 9, 48 leaves):

$$\frac{x \operatorname{MeijerG}\left[\left\{\left\{1, 1, 1, 1 + \frac{3}{2q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, \frac{3}{2q}\}\right\}, -a x^q\right]}{q (d x)^{5/2}}$$

### Problem 91: Unable to integrate problem.

$$\int (d x)^{3/2} \operatorname{PolyLog}[3, a x^q] dx$$

Optimal (type 5, 125 leaves, 5 steps):

$$\frac{16 a d q^3 x^{2+q} \sqrt{d x} \operatorname{Hypergeometric2F1}\left[1, \frac{5+q}{q}, \frac{1}{2}\left(4 + \frac{5}{q}\right), a x^q\right]}{125 (5 + 2 q)} - \frac{8 q^2 (d x)^{5/2} \operatorname{Log}[1 - a x^q]}{125 d} - \frac{4 q (d x)^{5/2} \operatorname{PolyLog}[2, a x^q]}{25 d} + \frac{2 (d x)^{5/2} \operatorname{PolyLog}[3, a x^q]}{5 d}$$

Result (type 9, 50 leaves):

$$-\frac{1}{q} x (d x)^{3/2} \operatorname{MeijerG}\left[\left\{\left\{1, 1, 1, 1, 1 - \frac{5}{2q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, -\frac{5}{2q}\}\right\}, -a x^q\right]$$

### Problem 92: Unable to integrate problem.

$$\int \sqrt{d x} \operatorname{PolyLog}[3, a x^q] dx$$

Optimal (type 5, 124 leaves, 5 steps):

$$\frac{16 a q^3 x^{1+q} \sqrt{d x} \operatorname{Hypergeometric2F1}\left[1, \frac{3+q}{q}, \frac{1}{2}\left(4 + \frac{3}{q}\right), a x^q\right]}{27 (3 + 2 q)} - \frac{8 q^2 (d x)^{3/2} \operatorname{Log}[1 - a x^q]}{27 d} - \frac{4 q (d x)^{3/2} \operatorname{PolyLog}[2, a x^q]}{9 d} + \frac{2 (d x)^{3/2} \operatorname{PolyLog}[3, a x^q]}{3 d}$$

Result (type 9, 50 leaves):

$$-\frac{1}{q} x \sqrt{d x} \operatorname{MeijerG}\left[\left\{\left\{1, 1, 1, 1, 1 - \frac{3}{2q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, -\frac{3}{2q}\}\right\}, -a x^q\right]$$

### Problem 93: Unable to integrate problem.

$$\int \frac{\text{PolyLog}[3, a x^q]}{\sqrt{d x}} dx$$

Optimal (type 5, 115 leaves, 5 steps):

$$\frac{16 a q^3 x^q \sqrt{d x} \text{Hypergeometric2F1}\left[1, \frac{1+q}{q}, \frac{1}{2}\left(4 + \frac{1}{q}\right), a x^q\right]}{d(1+2q)} - \frac{8 q^2 \sqrt{d x} \text{Log}[1 - a x^q]}{d} - \frac{4 q \sqrt{d x} \text{PolyLog}[2, a x^q]}{d} + \frac{2 \sqrt{d x} \text{PolyLog}[3, a x^q]}{d}$$

Result (type 9, 50 leaves):

$$-\frac{1}{q \sqrt{d x}} x \text{MeijerG}\left[\left\{\left\{1, 1, 1, 1, 1 - \frac{1}{2q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, -\frac{1}{2q}\}\right\}, -a x^q\right]$$

### Problem 94: Unable to integrate problem.

$$\int \frac{\text{PolyLog}[3, a x^q]}{(d x)^{3/2}} dx$$

Optimal (type 5, 119 leaves, 5 steps):

$$\frac{16 a q^3 x^q \text{Hypergeometric2F1}\left[1, \frac{1}{2}\left(2 - \frac{1}{q}\right), \frac{1}{2}\left(4 - \frac{1}{q}\right), a x^q\right]}{d(1-2q) \sqrt{d x}} + \frac{8 q^2 \text{Log}[1 - a x^q]}{d \sqrt{d x}} - \frac{4 q \text{PolyLog}[2, a x^q]}{d \sqrt{d x}} - \frac{2 \text{PolyLog}[3, a x^q]}{d \sqrt{d x}}$$

Result (type 9, 50 leaves):

$$-\frac{x \text{MeijerG}\left[\left\{\left\{1, 1, 1, 1, 1 + \frac{1}{2q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, \frac{1}{2q}\}\right\}, -a x^q\right]}{q (d x)^{3/2}}$$

### Problem 95: Unable to integrate problem.

$$\int \frac{\text{PolyLog}[3, a x^q]}{(d x)^{5/2}} dx$$

Optimal (type 5, 129 leaves, 5 steps):

$$\frac{16 a q^3 x^{-1+q} \text{Hypergeometric2F1}\left[1, \frac{1}{2}\left(2 - \frac{3}{q}\right), \frac{1}{2}\left(4 - \frac{3}{q}\right), a x^q\right]}{27 d^2 (3-2q) \sqrt{d x}} + \frac{8 q^2 \text{Log}[1 - a x^q]}{27 d (d x)^{3/2}} - \frac{4 q \text{PolyLog}[2, a x^q]}{9 d (d x)^{3/2}} - \frac{2 \text{PolyLog}[3, a x^q]}{3 d (d x)^{3/2}}$$

Result (type 9, 50 leaves):

$$\frac{x \text{MeijerG}\left[\left\{\left\{1, 1, 1, 1, 1 + \frac{3}{2q}\right\}, \{\}\right\}, \left\{\{1\}, \{0, 0, 0, \frac{3}{2q}\}\right\}, -a x^q\right]}{q (dx)^{5/2}}$$

**Problem 101: Unable to integrate problem.**

$$\int \left( \text{PolyLog}\left[-\frac{3}{2}, a x\right] + \text{PolyLog}\left[-\frac{1}{2}, a x\right] \right) dx$$

Optimal (type 4, 9 leaves, 2 steps):

$$x \text{PolyLog}\left[-\frac{1}{2}, a x\right]$$

Result (type 8, 17 leaves):

$$\int \left( \text{PolyLog}\left[-\frac{3}{2}, a x\right] + \text{PolyLog}\left[-\frac{1}{2}, a x\right] \right) dx$$

**Problem 103: Unable to integrate problem.**

$$\int (dx)^m \text{PolyLog}[3, a x] dx$$

Optimal (type 5, 102 leaves, 4 steps):

$$\frac{a (dx)^{2+m} \text{Hypergeometric2F1}\left[1, 2+m, 3+m, a x\right]}{d^2 (1+m)^3 (2+m)} - \frac{(dx)^{1+m} \text{Log}[1-a x]}{d (1+m)^3} - \frac{(dx)^{1+m} \text{PolyLog}[2, a x]}{d (1+m)^2} + \frac{(dx)^{1+m} \text{PolyLog}[3, a x]}{d (1+m)}$$

Result (type 9, 88 leaves):

$$\frac{-\left(x (dx)^m \text{Gamma}[2+m] \left(a (1+m) x \text{Gamma}[1+m] \text{HypergeometricPFQRegularized}\left[\{1, 2+m\}, \{3+m\}, a x\right] + \text{Log}[1-a x] + (1+m) \text{PolyLog}[2, a x] - \text{PolyLog}[3, a x] - 2 m \text{PolyLog}[3, a x] - m^2 \text{PolyLog}[3, a x]\right)\right)}{\left((1+m)^4 \text{Gamma}[1+m]\right)}$$

**Problem 104: Unable to integrate problem.**

$$\int (dx)^m \text{PolyLog}[4, a x] dx$$

Optimal (type 5, 121 leaves, 5 steps):

$$\frac{a (d x)^{2+m} \text{Hypergeometric2F1}[1, 2+m, 3+m, a x]}{d^2 (1+m)^4 (2+m)} + \frac{(d x)^{1+m} \text{Log}[1-a x]}{d (1+m)^4} +$$

$$\frac{(d x)^{1+m} \text{PolyLog}[2, a x]}{d (1+m)^3} - \frac{(d x)^{1+m} \text{PolyLog}[3, a x]}{d (1+m)^2} + \frac{(d x)^{1+m} \text{PolyLog}[4, a x]}{d (1+m)}$$

Result (type 9, 119 leaves):

$$\frac{1}{(1+m)^5 \text{Gamma}[1+m]} x (d x)^m \text{Gamma}[2+m]$$

$$(a (1+m) x \text{Gamma}[1+m] \text{HypergeometricPFQRegularized}[\{1, 2+m\}, \{3+m\}, a x] + \text{Log}[1-a x] +$$

$$(1+m) \text{PolyLog}[2, a x] - \text{PolyLog}[3, a x] - 2 m \text{PolyLog}[3, a x] - m^2 \text{PolyLog}[3, a x] +$$

$$\text{PolyLog}[4, a x] + 3 m \text{PolyLog}[4, a x] + 3 m^2 \text{PolyLog}[4, a x] + m^3 \text{PolyLog}[4, a x])$$

### Problem 106: Unable to integrate problem.

$$\int (d x)^m \text{PolyLog}[3, a x^2] dx$$

Optimal (type 5, 118 leaves, 5 steps):

$$\frac{8 a (d x)^{3+m} \text{Hypergeometric2F1}\left[1, \frac{3+m}{2}, \frac{5+m}{2}, a x^2\right]}{d^3 (1+m)^3 (3+m)}$$

$$\frac{4 (d x)^{1+m} \text{Log}[1-a x^2]}{d (1+m)^3} - \frac{2 (d x)^{1+m} \text{PolyLog}[2, a x^2]}{d (1+m)^2} + \frac{(d x)^{1+m} \text{PolyLog}[3, a x^2]}{d (1+m)}$$

Result (type 9, 126 leaves):

$$-\left(\left(2 x (d x)^m \text{Gamma}\left[\frac{3+m}{2}\right]\right.\right.$$

$$\left.\left(2 a (1+m) x^2 \text{Gamma}\left[\frac{1+m}{2}\right] \text{HypergeometricPFQRegularized}\left[\left\{1, \frac{3+m}{2}\right\}, \left\{\frac{5+m}{2}\right\}, a x^2\right] +\right.\right.$$

$$\left.4 \text{Log}[1-a x^2] + 2 (1+m) \text{PolyLog}[2, a x^2] - \text{PolyLog}[3, a x^2] -\right.$$

$$\left.2 m \text{PolyLog}[3, a x^2] - m^2 \text{PolyLog}[3, a x^2]\right)\left)/\left((1+m)^4 \text{Gamma}\left[\frac{1+m}{2}\right]\right)\right)$$

### Problem 107: Unable to integrate problem.

$$\int (d x)^m \text{PolyLog}[4, a x^2] dx$$

Optimal (type 5, 142 leaves, 6 steps):

$$\frac{16 a (d x)^{3+m} \text{Hypergeometric2F1}\left[1, \frac{3+m}{2}, \frac{5+m}{2}, a x^2\right]}{d^3 (1+m)^4 (3+m)} + \frac{8 (d x)^{1+m} \text{Log}[1-a x^2]}{d (1+m)^4} +$$

$$\frac{4 (d x)^{1+m} \text{PolyLog}[2, a x^2]}{d (1+m)^3} - \frac{2 (d x)^{1+m} \text{PolyLog}[3, a x^2]}{d (1+m)^2} + \frac{(d x)^{1+m} \text{PolyLog}[4, a x^2]}{d (1+m)}$$

Result (type 9, 166 leaves):

$$\frac{1}{(1+m)^5 \Gamma\left[\frac{1+m}{2}\right]} 2x (dx)^m \Gamma\left[\frac{3+m}{2}\right] \\ \left( 4a(1+m)x^2 \Gamma\left[\frac{1+m}{2}\right] \text{HypergeometricPFQRegularized}\left[\left\{1, \frac{3+m}{2}\right\}, \left\{\frac{5+m}{2}\right\}, ax^2\right] + \right. \\ \left. 8 \text{Log}[1-ax^2] + 4(1+m) \text{PolyLog}[2, ax^2] - 2 \text{PolyLog}[3, ax^2] - \right. \\ \left. 4m \text{PolyLog}[3, ax^2] - 2m^2 \text{PolyLog}[3, ax^2] + \text{PolyLog}[4, ax^2] + \right. \\ \left. 3m \text{PolyLog}[4, ax^2] + 3m^2 \text{PolyLog}[4, ax^2] + m^3 \text{PolyLog}[4, ax^2] \right)$$

Problem 109: Unable to integrate problem.

$$\int (dx)^m \text{PolyLog}[3, ax^3] dx$$

Optimal (type 5, 118 leaves, 5 steps):

$$\frac{27a(dx)^{4+m} \text{Hypergeometric2F1}\left[1, \frac{4+m}{3}, \frac{7+m}{3}, ax^3\right]}{d^4(1+m)^3(4+m)} - \\ \frac{9(dx)^{1+m} \text{Log}[1-ax^3]}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{PolyLog}[2, ax^3]}{d(1+m)^2} + \frac{(dx)^{1+m} \text{PolyLog}[3, ax^3]}{d(1+m)}$$

Result (type 9, 126 leaves):

$$-\left( \left( 3x(dx)^m \Gamma\left[\frac{4+m}{3}\right] \right. \right. \\ \left. \left( 3a(1+m)x^3 \Gamma\left[\frac{1+m}{3}\right] \text{HypergeometricPFQRegularized}\left[\left\{1, \frac{4+m}{3}\right\}, \left\{\frac{7+m}{3}\right\}, ax^3\right] + \right. \right. \\ \left. \left. 9 \text{Log}[1-ax^3] + 3(1+m) \text{PolyLog}[2, ax^3] - \text{PolyLog}[3, ax^3] - \right. \right. \\ \left. \left. 2m \text{PolyLog}[3, ax^3] - m^2 \text{PolyLog}[3, ax^3] \right) \right) / \left( (1+m)^4 \Gamma\left[\frac{1+m}{3}\right] \right)$$

Problem 110: Unable to integrate problem.

$$\int (dx)^m \text{PolyLog}[4, ax^3] dx$$

Optimal (type 5, 142 leaves, 6 steps):

$$\frac{81a(dx)^{4+m} \text{Hypergeometric2F1}\left[1, \frac{4+m}{3}, \frac{7+m}{3}, ax^3\right]}{d^4(1+m)^4(4+m)} + \frac{27(dx)^{1+m} \text{Log}[1-ax^3]}{d(1+m)^4} + \\ \frac{9(dx)^{1+m} \text{PolyLog}[2, ax^3]}{d(1+m)^3} - \frac{3(dx)^{1+m} \text{PolyLog}[3, ax^3]}{d(1+m)^2} + \frac{(dx)^{1+m} \text{PolyLog}[4, ax^3]}{d(1+m)}$$

Result (type 9, 166 leaves):

$$\frac{1}{(1+m)^5 \Gamma\left[\frac{1+m}{3}\right]} 3 x (d x)^m \Gamma\left[\frac{4+m}{3}\right] \left( 9 a (1+m) x^3 \Gamma\left[\frac{1+m}{3}\right] \text{HypergeometricPFQRegularized}\left[\left\{1, \frac{4+m}{3}\right\}, \left\{\frac{7+m}{3}\right\}, a x^3\right] + 27 \text{Log}\left[1 - a x^3\right] + 9 (1+m) \text{PolyLog}\left[2, a x^3\right] - 3 \text{PolyLog}\left[3, a x^3\right] - 6 m \text{PolyLog}\left[3, a x^3\right] - 3 m^2 \text{PolyLog}\left[3, a x^3\right] + \text{PolyLog}\left[4, a x^3\right] + 3 m \text{PolyLog}\left[4, a x^3\right] + 3 m^2 \text{PolyLog}\left[4, a x^3\right] + m^3 \text{PolyLog}\left[4, a x^3\right] \right)$$

### Problem 112: Unable to integrate problem.

$$\int (d x)^m \text{PolyLog}\left[3, a x^q\right] d x$$

Optimal (type 5, 130 leaves, 5 steps):

$$\frac{a q^3 x^{1+q} (d x)^m \text{Hypergeometric2F1}\left[1, \frac{1+m+q}{q}, \frac{1+m+2q}{q}, a x^q\right]}{(1+m)^3 (1+m+q)} - \frac{q^2 (d x)^{1+m} \text{Log}\left[1 - a x^q\right]}{d (1+m)^3} - \frac{q (d x)^{1+m} \text{PolyLog}\left[2, a x^q\right]}{d (1+m)^2} + \frac{(d x)^{1+m} \text{PolyLog}\left[3, a x^q\right]}{d (1+m)}$$

Result (type 9, 50 leaves):

$$-\frac{1}{q} x (d x)^m \text{MeijerG}\left[\left[\left\{1, 1, 1, 1, 1 - \frac{1+m}{q}\right\}, \{\}\right], \left\{\{1\}, \{0, 0, 0, -\frac{1+m}{q}\}\right\}, -a x^q\right]$$

### Problem 113: Unable to integrate problem.

$$\int (d x)^m \text{PolyLog}\left[4, a x^q\right] d x$$

Optimal (type 5, 154 leaves, 6 steps):

$$\frac{a q^4 x^{1+q} (d x)^m \text{Hypergeometric2F1}\left[1, \frac{1+m+q}{q}, \frac{1+m+2q}{q}, a x^q\right]}{(1+m)^4 (1+m+q)} + \frac{q^3 (d x)^{1+m} \text{Log}\left[1 - a x^q\right]}{d (1+m)^4} + \frac{q^2 (d x)^{1+m} \text{PolyLog}\left[2, a x^q\right]}{d (1+m)^3} - \frac{q (d x)^{1+m} \text{PolyLog}\left[3, a x^q\right]}{d (1+m)^2} + \frac{(d x)^{1+m} \text{PolyLog}\left[4, a x^q\right]}{d (1+m)}$$

Result (type 9, 52 leaves):

$$-\frac{1}{q} x (d x)^m \text{MeijerG}\left[\left[\left\{1, 1, 1, 1, 1, 1 - \frac{1+m}{q}\right\}, \{\}\right], \left\{\{1\}, \{0, 0, 0, 0, -\frac{1+m}{q}\}\right\}, -a x^q\right]$$

### Problem 152: Unable to integrate problem.

$$\int -\frac{\text{Log}\left[1 - e\left(\frac{a+b x}{c+d x}\right)^n\right]}{(a+b x)(c+d x)} d x$$



Optimal (type 4, 33 leaves, 1 step):

$$\frac{\text{PolyLog}\left[2, e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]}{(bc-ad)n}$$

Result (type 8, 40 leaves):

$$-\int \frac{\text{Log}\left[1 - e^{\left(\frac{a+bx}{c+dx}\right)^n}\right]}{(a+bx)(c+dx)} dx$$

**Problem 181: Unable to integrate problem.**

$$\int \frac{(g+h \text{Log}[f(d+ex)^n]) \text{PolyLog}[2, c(a+bx)]}{x^2} dx$$

Optimal (type 4, 2498 leaves, 22 steps):

$$\begin{aligned} & \frac{bg \text{Log}\left[\frac{bcx}{1-ac}\right] \text{Log}[1-ac-bcx]}{a} - \frac{bhn \text{Log}\left[\frac{bcx}{1-ac}\right] \text{Log}[1-ac-bcx] \text{Log}[d+ex]}{a} - \frac{1}{2a} \\ & bhn \left( \text{Log}\left[\frac{bcx}{1-ac}\right] + \text{Log}\left[\frac{bcd+e-ace}{bc(d+ex)}\right] - \text{Log}\left[\frac{(bcd+e-ace)x}{(1-ac)(d+ex)}\right] \right) \text{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right]^2 + \\ & \frac{1}{2a} bhn \left( \text{Log}\left[\frac{bcx}{1-ac}\right] - \text{Log}\left[-\frac{ex}{d}\right] \right) \left( \text{Log}[1-ac-bcx] + \text{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \right)^2 + \\ & \frac{bh \text{Log}\left[\frac{bcx}{1-ac}\right] \text{Log}[1-ac-bcx] (n \text{Log}[d+ex] - \text{Log}[f(d+ex)^n])}{a} + \frac{1}{2a} \\ & bhn \left( \text{Log}[c(a+bx)] + \text{Log}\left[\frac{bcd+e-ace}{bc(d+ex)}\right] - \text{Log}\left[\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right] \right) \\ & \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right]^2 - \frac{1}{2d} \\ & eh n \left( \text{Log}[c(a+bx)] + \text{Log}\left[\frac{bcd+e-ace}{bc(d+ex)}\right] - \text{Log}\left[\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right] \right) \\ & \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right]^2 + \frac{eh n \text{Log}[x] \text{Log}\left[1+\frac{bx}{a}\right] \text{Log}[1-c(a+bx)]}{d} + \\ & \frac{bhn \text{Log}[c(a+bx)] \text{Log}[d+ex] \text{Log}[1-c(a+bx)]}{a} - \\ & \frac{eh n \text{Log}[c(a+bx)] \text{Log}[d+ex] \text{Log}[1-c(a+bx)]}{d} - \\ & \frac{1}{2a} bhn \left( \text{Log}[c(a+bx)] - \text{Log}\left[-\frac{e(a+bx)}{bd-ae}\right] \right) \\ & \left( \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \text{Log}[1-c(a+bx)] \right)^2 + \frac{1}{2d} eh n \end{aligned}$$

$$\begin{aligned}
& \left( \text{Log}[c(a+bx)] - \text{Log}\left[-\frac{e(a+bx)}{bd-ae}\right] \right) \left( \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \text{Log}[1-c(a+bx)] \right)^2 + \\
& \frac{1}{2d} e h n \left( \text{Log}\left[1+\frac{bx}{a}\right] + \text{Log}\left[\frac{1-ac}{1-c(a+bx)}\right] - \text{Log}\left[\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right] \right) \text{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right]^2 + \\
& \frac{e h n \left( \text{Log}[c(a+bx)] - \text{Log}\left[1+\frac{bx}{a}\right] \right) \left( \text{Log}[x] + \text{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \right)^2}{2d} + \\
& \frac{e h n \left( \text{Log}[1-c(a+bx)] - \text{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \right) \text{PolyLog}\left[2, -\frac{bx}{a}\right] - b g \text{PolyLog}\left[2, c(a+bx)\right]}{d} + \\
& \frac{e h n \text{Log}[x] \text{PolyLog}\left[2, c(a+bx)\right]}{d} - \frac{e h n \text{Log}[d+ex] \text{PolyLog}\left[2, c(a+bx)\right]}{d} + \\
& \frac{b h \left( n \text{Log}[d+ex] - \text{Log}[f(d+ex)^n] \right) \text{PolyLog}\left[2, c(a+bx)\right]}{a} - \\
& \frac{(g+h \text{Log}[f(d+ex)^n]) \text{PolyLog}\left[2, c(a+bx)\right] - b g \text{PolyLog}\left[2, 1-\frac{bcx}{1-ac}\right]}{x} - \\
& \frac{b h n \left( \text{Log}[d+ex] - \text{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \right) \text{PolyLog}\left[2, 1-\frac{bcx}{1-ac}\right]}{a} + \\
& \frac{b h \left( n \text{Log}[d+ex] - \text{Log}[f(d+ex)^n] \right) \text{PolyLog}\left[2, 1-\frac{bcx}{1-ac}\right]}{a} - \\
& \frac{b h n \text{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \text{PolyLog}\left[2, \frac{d(1-ac-bcx)}{(1-ac)(d+ex)}\right]}{a} + \\
& \frac{b h n \text{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \text{PolyLog}\left[2, -\frac{e(1-ac-bcx)}{bc(d+ex)}\right]}{a} + \frac{1}{a} \\
& b h n \left( \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \text{Log}[1-c(a+bx)] \right) \text{PolyLog}\left[2, \frac{b(d+ex)}{bd-ae}\right] - \\
& \frac{1}{d} e h n \left( \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \text{Log}[1-c(a+bx)] \right) \text{PolyLog}\left[2, \frac{b(d+ex)}{bd-ae}\right] - \\
& \frac{b h n \left( \text{Log}[1-ac-bcx] + \text{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \right) \text{PolyLog}\left[2, 1+\frac{ex}{d}\right]}{a} + \\
& \frac{e h n \text{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \text{PolyLog}\left[2, -\frac{bx}{a(1-c(a+bx))}\right]}{d} - \\
& \frac{e h n \text{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \text{PolyLog}\left[2, -\frac{bcx}{1-c(a+bx)}\right]}{d} + \frac{1}{a} \\
& b h n \left( \text{Log}[d+ex] - \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \right) \text{PolyLog}\left[2, 1-c(a+bx)\right] - \frac{1}{d} \\
& e h n \left( \text{Log}[d+ex] - \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \right) \text{PolyLog}\left[2, 1-c(a+bx)\right] +
\end{aligned}$$

$$\begin{aligned}
 & \frac{e h n \left( \text{Log}[x] + \text{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \right) \text{PolyLog}\left[2, 1-c(a+bx)\right]}{d} - \\
 & \frac{b h n \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \text{PolyLog}\left[2, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{a} + \\
 & \frac{e h n \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \text{PolyLog}\left[2, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{d} + \\
 & \frac{b h n \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \text{PolyLog}\left[2, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{a} - \\
 & \frac{e h n \text{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \text{PolyLog}\left[2, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{d} - \frac{e h n \text{PolyLog}\left[3, -\frac{bx}{a}\right]}{d} + \\
 & \frac{b h n \text{PolyLog}\left[3, 1-\frac{bcx}{1-ac}\right]}{a} - \frac{b h n \text{PolyLog}\left[3, \frac{d(1-ac-bcx)}{(1-ac)(d+ex)}\right]}{a} + \frac{b h n \text{PolyLog}\left[3, -\frac{e(1-ac-bcx)}{bc(d+ex)}\right]}{a} - \\
 & \frac{b h n \text{PolyLog}\left[3, \frac{b(d+ex)}{bd-ae}\right]}{a} + \frac{e h n \text{PolyLog}\left[3, \frac{b(d+ex)}{bd-ae}\right]}{d} + \frac{b h n \text{PolyLog}\left[3, 1+\frac{ex}{d}\right]}{a} + \\
 & \frac{e h n \text{PolyLog}\left[3, -\frac{bx}{a(1-c(a+bx))}\right]}{d} - \frac{e h n \text{PolyLog}\left[3, -\frac{bcx}{1-c(a+bx)}\right]}{d} - \frac{b h n \text{PolyLog}\left[3, 1-c(a+bx)\right]}{a} - \\
 & \frac{b h n \text{PolyLog}\left[3, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{a} + \frac{e h n \text{PolyLog}\left[3, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{d} + \\
 & \frac{b h n \text{PolyLog}\left[3, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{a} - \frac{e h n \text{PolyLog}\left[3, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{d}
 \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(g + h \text{Log}[f(d+ex)^n]) \text{PolyLog}[2, c(a+bx)]}{x^2} dx$$

**Problem 182: Unable to integrate problem.**

$$\int \frac{(g + h \text{Log}[f(d+ex)^n]) \text{PolyLog}[2, c(a+bx)]}{x^3} dx$$

Optimal (type 4, 3119 leaves, 44 steps):

$$\begin{aligned}
 & \frac{b^2 g \text{Log}\left[\frac{bcx}{1-ac}\right] \text{Log}[1-ac-bcx]}{2a^2} - \frac{b e h n \text{Log}\left[\frac{bcx}{1-ac}\right] \text{Log}[1-ac-bcx]}{ad} + \\
 & \frac{b^2 h n \text{Log}\left[\frac{bcx}{1-ac}\right] \text{Log}[1-ac-bcx] \text{Log}[d+ex]}{2a^2} + \frac{b e h n \text{Log}[1-ac-bcx] \text{Log}\left[\frac{bc(d+ex)}{bcd+e-ace}\right]}{2ad} + \frac{1}{4a^2} \\
 & b^2 h n \left( \text{Log}\left[\frac{bcx}{1-ac}\right] + \text{Log}\left[\frac{bcd+e-ace}{bc(d+ex)}\right] - \text{Log}\left[\frac{(bcd+e-ace)x}{(1-ac)(d+ex)}\right] \right) \text{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right]^2 - \\
 & \frac{1}{4a^2} b^2 h n \left( \text{Log}\left[\frac{bcx}{1-ac}\right] - \text{Log}\left[-\frac{ex}{d}\right] \right) \left( \text{Log}[1-ac-bcx] + \text{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \right)^2 -
 \end{aligned}$$

$$\begin{aligned}
& \frac{b^2 h \operatorname{Log}\left[\frac{bcx}{1-ac}\right] \operatorname{Log}[1-ac-bcx] (n \operatorname{Log}[d+ex] - \operatorname{Log}[f(d+ex)^n])}{2a^2} + \\
& \frac{b^2 c \operatorname{Log}\left[-\frac{ex}{d}\right] (g+h \operatorname{Log}[f(d+ex)^n])}{2a(1-ac)} + \frac{b \operatorname{Log}[1-ac-bcx] (g+h \operatorname{Log}[f(d+ex)^n])}{2ax} - \\
& \frac{b^2 c \operatorname{Log}\left[\frac{e(1-ac-bcx)}{bcd+e-ace}\right] (g+h \operatorname{Log}[f(d+ex)^n])}{2a(1-ac)} - \frac{1}{4a^2} \\
& b^2 h n \left( \operatorname{Log}[c(a+bx)] + \operatorname{Log}\left[\frac{bcd+e-ace}{bc(d+ex)}\right] - \operatorname{Log}\left[\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right] \right) \\
& \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right]^2 + \frac{1}{4d^2} \\
& e^2 h n \left( \operatorname{Log}[c(a+bx)] + \operatorname{Log}\left[\frac{bcd+e-ace}{bc(d+ex)}\right] - \operatorname{Log}\left[\frac{(bcd+e-ace)(a+bx)}{b(d+ex)}\right] \right) \\
& \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right]^2 - \frac{e^2 h n \operatorname{Log}[x] \operatorname{Log}\left[1+\frac{bx}{a}\right] \operatorname{Log}[1-c(a+bx)]}{2d^2} - \\
& \frac{b^2 h n \operatorname{Log}[c(a+bx)] \operatorname{Log}[d+ex] \operatorname{Log}[1-c(a+bx)]}{2a^2} + \\
& \frac{e^2 h n \operatorname{Log}[c(a+bx)] \operatorname{Log}[d+ex] \operatorname{Log}[1-c(a+bx)]}{2d^2} + \frac{1}{4a^2} b^2 h n \\
& \left( \operatorname{Log}[c(a+bx)] - \operatorname{Log}\left[-\frac{e(a+bx)}{bd-ae}\right] \right) \left( \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \operatorname{Log}[1-c(a+bx)] \right)^2 - \\
& \frac{1}{4d^2} e^2 h n \left( \operatorname{Log}[c(a+bx)] - \operatorname{Log}\left[-\frac{e(a+bx)}{bd-ae}\right] \right) \\
& \left( \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \operatorname{Log}[1-c(a+bx)] \right)^2 - \frac{1}{4d^2} \\
& e^2 h n \left( \operatorname{Log}\left[1+\frac{bx}{a}\right] + \operatorname{Log}\left[\frac{1-ac}{1-c(a+bx)}\right] - \operatorname{Log}\left[\frac{(1-ac)(a+bx)}{a(1-c(a+bx))}\right] \right) \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right]^2 - \\
& \frac{1}{4d^2} e^2 h n \left( \operatorname{Log}[c(a+bx)] - \operatorname{Log}\left[1+\frac{bx}{a}\right] \right) \left( \operatorname{Log}[x] + \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \right)^2 - \\
& \frac{e^2 h n \left( \operatorname{Log}[1-c(a+bx)] - \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \right) \operatorname{PolyLog}\left[2, -\frac{bx}{a}\right]}{2d^2} + \\
& \frac{b^2 g \operatorname{PolyLog}\left[2, c(a+bx)\right]}{2a^2} - \frac{b e h n \operatorname{PolyLog}\left[2, c(a+bx)\right]}{2ad} - \frac{e h n \operatorname{PolyLog}\left[2, c(a+bx)\right]}{2dx} - \\
& \frac{e^2 h n \operatorname{Log}[x] \operatorname{PolyLog}\left[2, c(a+bx)\right]}{2d^2} + \frac{e^2 h n \operatorname{Log}[d+ex] \operatorname{PolyLog}\left[2, c(a+bx)\right]}{2d^2} - \\
& \frac{b^2 h (n \operatorname{Log}[d+ex] - \operatorname{Log}[f(d+ex)^n]) \operatorname{PolyLog}\left[2, c(a+bx)\right]}{2a^2} - \\
& \frac{(g+h \operatorname{Log}[f(d+ex)^n]) \operatorname{PolyLog}\left[2, c(a+bx)\right]}{2x^2} + \frac{b e h n \operatorname{PolyLog}\left[2, \frac{e(1-ac-bcx)}{bcd+e-ace}\right]}{2ad} +
\end{aligned}$$

$$\begin{aligned}
 & \frac{b^2 g \operatorname{PolyLog}\left[2, 1 - \frac{bcx}{1-ac}\right]}{2a^2} - \frac{b e h n \operatorname{PolyLog}\left[2, 1 - \frac{bcx}{1-ac}\right]}{ad} + \\
 & \frac{b^2 h n \left( \operatorname{Log}[d+ex] - \operatorname{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \right) \operatorname{PolyLog}\left[2, 1 - \frac{bcx}{1-ac}\right]}{2a^2} - \\
 & \frac{b^2 h \left( n \operatorname{Log}[d+ex] - \operatorname{Log}[f(d+ex)^n] \right) \operatorname{PolyLog}\left[2, 1 - \frac{bcx}{1-ac}\right]}{2a^2} + \\
 & \frac{b^2 h n \operatorname{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \operatorname{PolyLog}\left[2, \frac{d(1-ac-bcx)}{(1-ac)(d+ex)}\right]}{2a^2} - \\
 & \frac{b^2 h n \operatorname{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \operatorname{PolyLog}\left[2, -\frac{e(1-ac-bcx)}{bc(d+ex)}\right]}{2a^2} - \frac{1}{2a^2} \\
 & b^2 h n \left( \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \operatorname{Log}[1-c(a+bx)] \right) \operatorname{PolyLog}\left[2, \frac{b(d+ex)}{bd-ae}\right] + \\
 & \frac{1}{2d^2} e^2 h n \left( \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \operatorname{Log}[1-c(a+bx)] \right) \operatorname{PolyLog}\left[2, \frac{b(d+ex)}{bd-ae}\right] - \\
 & \frac{b^2 c h n \operatorname{PolyLog}\left[2, \frac{bc(d+ex)}{bcd+e-ace}\right]}{2a(1-ac)} + \frac{b^2 c h n \operatorname{PolyLog}\left[2, 1 + \frac{ex}{d}\right]}{2a(1-ac)} + \\
 & \frac{b^2 h n \left( \operatorname{Log}[1-ac-bcx] + \operatorname{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \right) \operatorname{PolyLog}\left[2, 1 + \frac{ex}{d}\right]}{2a^2} - \\
 & \frac{e^2 h n \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \operatorname{PolyLog}\left[2, -\frac{bx}{a(1-c(a+bx))}\right]}{2d^2} + \\
 & \frac{e^2 h n \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \operatorname{PolyLog}\left[2, -\frac{bcx}{1-c(a+bx)}\right]}{2d^2} - \frac{1}{2a^2} \\
 & b^2 h n \left( \operatorname{Log}[d+ex] - \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \right) \operatorname{PolyLog}\left[2, 1-c(a+bx)\right] + \\
 & \frac{1}{2d^2} e^2 h n \left( \operatorname{Log}[d+ex] - \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \right) \operatorname{PolyLog}\left[2, 1-c(a+bx)\right] - \\
 & \frac{e^2 h n \left( \operatorname{Log}[x] + \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \right) \operatorname{PolyLog}\left[2, 1-c(a+bx)\right]}{2d^2} + \\
 & \frac{b^2 h n \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \operatorname{PolyLog}\left[2, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{2a^2} - \\
 & \frac{e^2 h n \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \operatorname{PolyLog}\left[2, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{2d^2} - \\
 & \frac{b^2 h n \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \operatorname{PolyLog}\left[2, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{2a^2} + \\
 & \frac{e^2 h n \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \operatorname{PolyLog}\left[2, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{2d^2} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{e^2 h n \operatorname{PolyLog}\left[3, -\frac{b x}{a}\right]}{2 d^2} - \frac{b^2 h n \operatorname{PolyLog}\left[3, 1 - \frac{b c x}{1-a c}\right]}{2 a^2} + \frac{b^2 h n \operatorname{PolyLog}\left[3, \frac{d(1-a c-b c x)}{(1-a c)(d+e x)}\right]}{2 a^2} - \\
 & \frac{b^2 h n \operatorname{PolyLog}\left[3, -\frac{e(1-a c-b c x)}{b c(d+e x)}\right]}{2 a^2} + \frac{b^2 h n \operatorname{PolyLog}\left[3, \frac{b(d+e x)}{b d-a e}\right]}{2 a^2} - \\
 & \frac{e^2 h n \operatorname{PolyLog}\left[3, \frac{b(d+e x)}{b d-a e}\right]}{2 d^2} - \frac{b^2 h n \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{2 a^2} - \frac{e^2 h n \operatorname{PolyLog}\left[3, -\frac{b x}{a(1-c(a+b x))}\right]}{2 d^2} + \\
 & \frac{e^2 h n \operatorname{PolyLog}\left[3, -\frac{b c x}{1-c(a+b x)}\right]}{2 d^2} + \frac{b^2 h n \operatorname{PolyLog}\left[3, 1 - c(a+b x)\right]}{2 a^2} + \\
 & \frac{b^2 h n \operatorname{PolyLog}\left[3, -\frac{e(1-c(a+b x))}{b c(d+e x)}\right]}{2 a^2} - \frac{e^2 h n \operatorname{PolyLog}\left[3, -\frac{e(1-c(a+b x))}{b c(d+e x)}\right]}{2 d^2} - \\
 & \frac{b^2 h n \operatorname{PolyLog}\left[3, \frac{(b d-a e)(1-c(a+b x))}{b(d+e x)}\right]}{2 a^2} + \frac{e^2 h n \operatorname{PolyLog}\left[3, \frac{(b d-a e)(1-c(a+b x))}{b(d+e x)}\right]}{2 d^2}
 \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(g + h \operatorname{Log}[f(d+e x)^n]) \operatorname{PolyLog}[2, c(a+b x)]}{x^3} dx$$

### Problem 183: Unable to integrate problem.

$$\int \frac{(g + h \operatorname{Log}[f(d+e x)^n]) \operatorname{PolyLog}[2, c(a+b x)]}{x^4} dx$$

Optimal (type 4, 3733 leaves, 78 steps):

$$\begin{aligned}
 & \frac{b^2 c e h n \operatorname{Log}[x]}{2 a(1-a c) d} - \frac{b^2 c e h n \operatorname{Log}[1-a c-b c x]}{3 a(1-a c) d} + \\
 & \frac{b e h n \operatorname{Log}[1-a c-b c x]}{3 a d x} - \frac{b^3 g \operatorname{Log}\left[\frac{b c x}{1-a c}\right] \operatorname{Log}[1-a c-b c x]}{3 a^3} + \\
 & \frac{b^2 e h n \operatorname{Log}\left[\frac{b c x}{1-a c}\right] \operatorname{Log}[1-a c-b c x]}{2 a^2 d} + \frac{b e^2 h n \operatorname{Log}\left[\frac{b c x}{1-a c}\right] \operatorname{Log}[1-a c-b c x]}{2 a d^2} - \\
 & \frac{b^2 c e h n \operatorname{Log}[d+e x]}{6 a(1-a c) d} - \frac{b^3 h n \operatorname{Log}\left[\frac{b c x}{1-a c}\right] \operatorname{Log}[1-a c-b c x] \operatorname{Log}[d+e x]}{3 a^3} - \\
 & \frac{b^2 e h n \operatorname{Log}[1-a c-b c x] \operatorname{Log}\left[\frac{b c(d+e x)}{b c d+e-a c e}\right]}{3 a^2 d} - \frac{b e^2 h n \operatorname{Log}[1-a c-b c x] \operatorname{Log}\left[\frac{b c(d+e x)}{b c d+e-a c e}\right]}{6 a d^2} - \frac{1}{6 a^3} \\
 & b^3 h n \left( \operatorname{Log}\left[\frac{b c x}{1-a c}\right] + \operatorname{Log}\left[\frac{b c d+e-a c e}{b c(d+e x)}\right] - \operatorname{Log}\left[\frac{(b c d+e-a c e) x}{(1-a c)(d+e x)}\right] \right) \operatorname{Log}\left[\frac{(1-a c)(d+e x)}{d(1-a c-b c x)}\right]^2 + \\
 & \frac{1}{6 a^3} b^3 h n \left( \operatorname{Log}\left[\frac{b c x}{1-a c}\right] - \operatorname{Log}\left[-\frac{e x}{d}\right] \right) \left( \operatorname{Log}[1-a c-b c x] + \operatorname{Log}\left[\frac{(1-a c)(d+e x)}{d(1-a c-b c x)}\right] \right)^2 + \\
 & \frac{b^3 h \operatorname{Log}\left[\frac{b c x}{1-a c}\right] \operatorname{Log}[1-a c-b c x] (n \operatorname{Log}[d+e x] - \operatorname{Log}[f(d+e x)^n])}{3 a^3} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{b^2 c (g + h \operatorname{Log}[f (d + e x)^n])}{6 a (1 - a c) x} + \frac{b^3 c^2 \operatorname{Log}\left[-\frac{e x}{d}\right] (g + h \operatorname{Log}[f (d + e x)^n])}{6 a (1 - a c)^2} - \\
 & \frac{b^3 c \operatorname{Log}\left[-\frac{e x}{d}\right] (g + h \operatorname{Log}[f (d + e x)^n])}{3 a^2 (1 - a c)} + \frac{b \operatorname{Log}[1 - a c - b c x] (g + h \operatorname{Log}[f (d + e x)^n])}{6 a x^2} - \\
 & \frac{b^2 \operatorname{Log}[1 - a c - b c x] (g + h \operatorname{Log}[f (d + e x)^n])}{3 a^2 x} - \frac{b^3 c^2 \operatorname{Log}\left[\frac{e(1-a c-b c x)}{b c d+e-a c e}\right] (g + h \operatorname{Log}[f (d + e x)^n])}{6 a (1 - a c)^2} + \\
 & \frac{b^3 c \operatorname{Log}\left[\frac{e(1-a c-b c x)}{b c d+e-a c e}\right] (g + h \operatorname{Log}[f (d + e x)^n])}{3 a^2 (1 - a c)} + \frac{1}{6 a^3} \\
 & b^3 h n \left( \operatorname{Log}[c (a + b x)] + \operatorname{Log}\left[\frac{b c d + e - a c e}{b c (d + e x)}\right] - \operatorname{Log}\left[\frac{(b c d + e - a c e) (a + b x)}{b (d + e x)}\right] \right) \\
 & \operatorname{Log}\left[\frac{b (d + e x)}{(b d - a e) (1 - c (a + b x))}\right]^2 - \frac{1}{6 d^3} \\
 & e^3 h n \left( \operatorname{Log}[c (a + b x)] + \operatorname{Log}\left[\frac{b c d + e - a c e}{b c (d + e x)}\right] - \operatorname{Log}\left[\frac{(b c d + e - a c e) (a + b x)}{b (d + e x)}\right] \right) \\
 & \operatorname{Log}\left[\frac{b (d + e x)}{(b d - a e) (1 - c (a + b x))}\right]^2 + \frac{e^3 h n \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{b x}{a}\right] \operatorname{Log}[1 - c (a + b x)]}{3 d^3} + \\
 & \frac{b^3 h n \operatorname{Log}[c (a + b x)] \operatorname{Log}[d + e x] \operatorname{Log}[1 - c (a + b x)]}{3 a^3} - \\
 & \frac{e^3 h n \operatorname{Log}[c (a + b x)] \operatorname{Log}[d + e x] \operatorname{Log}[1 - c (a + b x)]}{3 d^3} - \frac{1}{6 a^3} b^3 h n \\
 & \left( \operatorname{Log}[c (a + b x)] - \operatorname{Log}\left[-\frac{e (a + b x)}{b d - a e}\right] \right) \left( \operatorname{Log}\left[\frac{b (d + e x)}{(b d - a e) (1 - c (a + b x))}\right] + \operatorname{Log}[1 - c (a + b x)] \right)^2 + \\
 & \frac{1}{6 d^3} e^3 h n \left( \operatorname{Log}[c (a + b x)] - \operatorname{Log}\left[-\frac{e (a + b x)}{b d - a e}\right] \right) \\
 & \left( \operatorname{Log}\left[\frac{b (d + e x)}{(b d - a e) (1 - c (a + b x))}\right] + \operatorname{Log}[1 - c (a + b x)] \right)^2 + \frac{1}{6 d^3} \\
 & e^3 h n \left( \operatorname{Log}\left[1 + \frac{b x}{a}\right] + \operatorname{Log}\left[\frac{1 - a c}{1 - c (a + b x)}\right] - \operatorname{Log}\left[\frac{(1 - a c) (a + b x)}{a (1 - c (a + b x))}\right] \right) \operatorname{Log}\left[-\frac{a (1 - c (a + b x))}{b x}\right]^2 + \\
 & \frac{1}{6 d^3} e^3 h n \left( \operatorname{Log}[c (a + b x)] - \operatorname{Log}\left[1 + \frac{b x}{a}\right] \right) \left( \operatorname{Log}[x] + \operatorname{Log}\left[-\frac{a (1 - c (a + b x))}{b x}\right] \right)^2 + \\
 & \frac{e^3 h n \left( \operatorname{Log}[1 - c (a + b x)] - \operatorname{Log}\left[-\frac{a (1 - c (a + b x))}{b x}\right] \right) \operatorname{PolyLog}\left[2, -\frac{b x}{a}\right]}{3 d^3} - \\
 & \frac{b^3 g \operatorname{PolyLog}\left[2, c (a + b x)\right]}{3 a^3} + \frac{b^2 e h n \operatorname{PolyLog}\left[2, c (a + b x)\right]}{6 a^2 d} + \\
 & \frac{b e^2 h n \operatorname{PolyLog}\left[2, c (a + b x)\right]}{3 a d^2} - \frac{e h n \operatorname{PolyLog}\left[2, c (a + b x)\right]}{6 d x^2} + \frac{e^2 h n \operatorname{PolyLog}\left[2, c (a + b x)\right]}{3 d^2 x} +
 \end{aligned}$$

$$\begin{aligned}
 & \frac{e^3 h n \operatorname{Log}[x] \operatorname{PolyLog}\left[2, c(a+bx)\right]}{3 d^3} - \frac{e^3 h n \operatorname{Log}[d+ex] \operatorname{PolyLog}\left[2, c(a+bx)\right]}{3 d^3} + \\
 & \frac{b^3 h \left(n \operatorname{Log}[d+ex] - \operatorname{Log}\left[f(d+ex)^n\right]\right) \operatorname{PolyLog}\left[2, c(a+bx)\right]}{3 a^3} - \\
 & \frac{\left(g+h \operatorname{Log}\left[f(d+ex)^n\right]\right) \operatorname{PolyLog}\left[2, c(a+bx)\right]}{3 x^3} - \frac{b^2 e h n \operatorname{PolyLog}\left[2, \frac{e(1-ac-bcx)}{bcd+e-ace}\right]}{3 a^2 d} - \\
 & \frac{b e^2 h n \operatorname{PolyLog}\left[2, \frac{e(1-ac-bcx)}{bcd+e-ace}\right]}{6 a d^2} - \frac{b^3 g \operatorname{PolyLog}\left[2, 1-\frac{bcx}{1-ac}\right]}{3 a^3} + \frac{b^2 e h n \operatorname{PolyLog}\left[2, 1-\frac{bcx}{1-ac}\right]}{2 a^2 d} + \\
 & \frac{b e^2 h n \operatorname{PolyLog}\left[2, 1-\frac{bcx}{1-ac}\right]}{2 a d^2} - \frac{b^3 h n \left(\operatorname{Log}[d+ex] - \operatorname{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right]\right) \operatorname{PolyLog}\left[2, 1-\frac{bcx}{1-ac}\right]}{3 a^3} + \\
 & \frac{b^3 h \left(n \operatorname{Log}[d+ex] - \operatorname{Log}\left[f(d+ex)^n\right]\right) \operatorname{PolyLog}\left[2, 1-\frac{bcx}{1-ac}\right]}{3 a^3} - \\
 & \frac{b^3 h n \operatorname{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \operatorname{PolyLog}\left[2, \frac{d(1-ac-bcx)}{(1-ac)(d+ex)}\right]}{3 a^3} + \\
 & \frac{b^3 h n \operatorname{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right] \operatorname{PolyLog}\left[2, -\frac{e(1-ac-bcx)}{bc(d+ex)}\right]}{3 a^3} + \frac{1}{3 a^3} \\
 & b^3 h n \left(\operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \operatorname{Log}[1-c(a+bx)]\right) \operatorname{PolyLog}\left[2, \frac{b(d+ex)}{bd-ae}\right] - \\
 & \frac{1}{3 d^3} e^3 h n \left(\operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] + \operatorname{Log}[1-c(a+bx)]\right) \operatorname{PolyLog}\left[2, \frac{b(d+ex)}{bd-ae}\right] - \\
 & \frac{b^3 c^2 h n \operatorname{PolyLog}\left[2, \frac{bc(d+ex)}{bcd+e-ace}\right]}{6 a(1-ac)^2} + \frac{b^3 c h n \operatorname{PolyLog}\left[2, \frac{bc(d+ex)}{bcd+e-ace}\right]}{3 a^2(1-ac)} + \frac{b^3 c^2 h n \operatorname{PolyLog}\left[2, 1+\frac{ex}{d}\right]}{6 a(1-ac)^2} - \\
 & \frac{b^3 c h n \operatorname{PolyLog}\left[2, 1+\frac{ex}{d}\right]}{3 a^2(1-ac)} - \frac{b^3 h n \left(\operatorname{Log}[1-ac-bcx] + \operatorname{Log}\left[\frac{(1-ac)(d+ex)}{d(1-ac-bcx)}\right]\right) \operatorname{PolyLog}\left[2, 1+\frac{ex}{d}\right]}{3 a^3} + \\
 & \frac{e^3 h n \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \operatorname{PolyLog}\left[2, -\frac{bx}{a(1-c(a+bx))}\right]}{3 d^3} - \\
 & \frac{e^3 h n \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right] \operatorname{PolyLog}\left[2, -\frac{bcx}{1-c(a+bx)}\right]}{3 d^3} + \frac{1}{3 a^3} \\
 & b^3 h n \left(\operatorname{Log}[d+ex] - \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right]\right) \operatorname{PolyLog}\left[2, 1-c(a+bx)\right] - \\
 & \frac{1}{3 d^3} e^3 h n \left(\operatorname{Log}[d+ex] - \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right]\right) \operatorname{PolyLog}\left[2, 1-c(a+bx)\right] + \\
 & \frac{e^3 h n \left(\operatorname{Log}[x] + \operatorname{Log}\left[-\frac{a(1-c(a+bx))}{bx}\right]\right) \operatorname{PolyLog}\left[2, 1-c(a+bx)\right]}{3 d^3} - \\
 & \frac{b^3 h n \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \operatorname{PolyLog}\left[2, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{3 a^3} +
 \end{aligned}$$



$$\begin{aligned}
 & \frac{e^3 h n \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \operatorname{PolyLog}\left[2, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{3d^3} + \\
 & \frac{b^3 h n \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \operatorname{PolyLog}\left[2, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{3a^3} - \\
 & \frac{e^3 h n \operatorname{Log}\left[\frac{b(d+ex)}{(bd-ae)(1-c(a+bx))}\right] \operatorname{PolyLog}\left[2, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{3d^3} - \\
 & \frac{e^3 h n \operatorname{PolyLog}\left[3, -\frac{bx}{a}\right]}{3d^3} + \frac{b^3 h n \operatorname{PolyLog}\left[3, 1 - \frac{bcx}{1-ac}\right]}{3a^3} - \frac{b^3 h n \operatorname{PolyLog}\left[3, \frac{d(1-ac-bcx)}{(1-ac)(d+ex)}\right]}{3a^3} + \\
 & \frac{b^3 h n \operatorname{PolyLog}\left[3, -\frac{e(1-ac-bcx)}{bc(d+ex)}\right]}{3a^3} - \frac{b^3 h n \operatorname{PolyLog}\left[3, \frac{b(d+ex)}{bd-ae}\right]}{3a^3} + \\
 & \frac{e^3 h n \operatorname{PolyLog}\left[3, \frac{b(d+ex)}{bd-ae}\right]}{3d^3} + \frac{b^3 h n \operatorname{PolyLog}\left[3, 1 + \frac{ex}{d}\right]}{3a^3} + \frac{e^3 h n \operatorname{PolyLog}\left[3, -\frac{bx}{a(1-c(a+bx))}\right]}{3d^3} - \\
 & \frac{e^3 h n \operatorname{PolyLog}\left[3, -\frac{bcx}{1-c(a+bx)}\right]}{3d^3} - \frac{b^3 h n \operatorname{PolyLog}\left[3, 1 - c(a+bx)\right]}{3a^3} - \\
 & \frac{b^3 h n \operatorname{PolyLog}\left[3, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{3a^3} + \frac{e^3 h n \operatorname{PolyLog}\left[3, -\frac{e(1-c(a+bx))}{bc(d+ex)}\right]}{3d^3} + \\
 & \frac{b^3 h n \operatorname{PolyLog}\left[3, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{3a^3} - \frac{e^3 h n \operatorname{PolyLog}\left[3, \frac{(bd-ae)(1-c(a+bx))}{b(d+ex)}\right]}{3d^3}
 \end{aligned}$$

Result (type 8, 29 leaves):

$$\int \frac{(g + h \operatorname{Log}[f(d+ex)^n]) \operatorname{PolyLog}[2, c(a+bx)]}{x^4} dx$$

Problem 196: Unable to integrate problem.

$$\int \frac{(a + bx + cx^2) \operatorname{Log}[1-dx] \operatorname{PolyLog}[2, dx]}{x^3} dx$$

Optimal (type 4, 343 leaves, 32 steps):

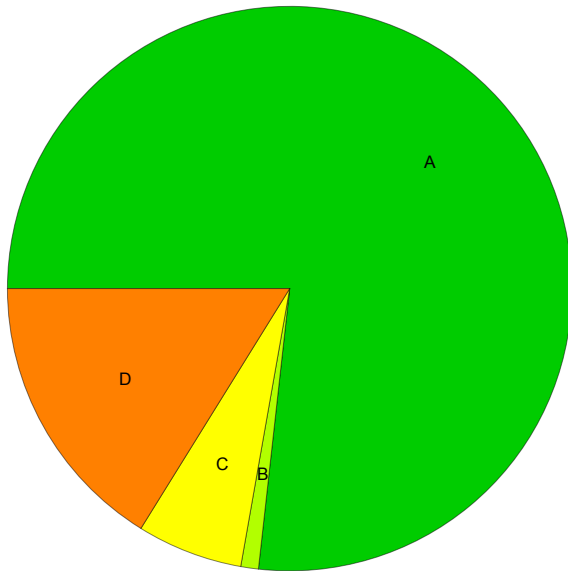
$$\begin{aligned}
 & -a d^2 \operatorname{Log}[x] + a d^2 \operatorname{Log}[1-dx] - \frac{a d \operatorname{Log}[1-dx]}{x} - \frac{1}{4} a d^2 \operatorname{Log}[1-dx]^2 + \\
 & \frac{a \operatorname{Log}[1-dx]^2}{4 x^2} + \frac{b(1-dx) \operatorname{Log}[1-dx]^2}{x} - \frac{b^2 \operatorname{Log}[dx] \operatorname{Log}[1-dx]^2}{2 a} + \\
 & \frac{(b+a d)^2 \operatorname{Log}[dx] \operatorname{Log}[1-dx]^2}{2 a} - 2 b d \operatorname{PolyLog}[2, dx] - \frac{1}{2} a d^2 \operatorname{PolyLog}[2, dx] + \\
 & \frac{a d \operatorname{PolyLog}[2, dx]}{2 x} + \frac{(b+a d)^2 \operatorname{Log}[1-dx] \operatorname{PolyLog}[2, dx]}{2 a} - \\
 & \frac{(a+b x)^2 \operatorname{Log}[1-dx] \operatorname{PolyLog}[2, dx]}{2 a x^2} - \frac{1}{2} c \operatorname{PolyLog}[2, dx]^2 - \\
 & \frac{b^2 \operatorname{Log}[1-dx] \operatorname{PolyLog}[2, 1-dx]}{a} + \frac{(b+a d)^2 \operatorname{Log}[1-dx] \operatorname{PolyLog}[2, 1-dx]}{a} - \\
 & \frac{1}{2} d(2 b+a d) \operatorname{PolyLog}[3, dx] + \frac{b^2 \operatorname{PolyLog}[3, 1-dx]}{a} - \frac{(b+a d)^2 \operatorname{PolyLog}[3, 1-dx]}{a}
 \end{aligned}$$

Result (type 8, 28 leaves):

$$\int \frac{(a+b x+c x^2) \operatorname{Log}[1-dx] \operatorname{PolyLog}[2, dx]}{x^3} dx$$

## Summary of Integration Test Results

198 integration problems



- A - 152 optimal antiderivatives
- B - 2 more than twice size of optimal antiderivatives
- C - 12 unnecessarily complex antiderivatives
- D - 32 unable to integrate problems
- E - 0 integration timeouts