

## Rubi 4.16.0.4 Integration Test Results

on the problems in the test-suite directory "8 Special functions"

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Test results for the 97 problems in "8.10 Formal derivatives.m"

Problem 24: Result valid but suboptimal antiderivative.

$$\int (g[x] f'[x] + f[x] g'[x]) dx$$

Optimal (type 9, 5 leaves, ? steps):

$$f[x] g[x]$$

Result (type 9, 19 leaves, 1 step):

$$\text{CannotIntegrate}[g[x] f'[x], x] + \text{CannotIntegrate}[f[x] g'[x], x]$$

Problem 43: Result valid but suboptimal antiderivative.

$$\int (\cos[x] g[e^x] f'[\sin[x]] + e^x f[\sin[x]] g'[e^x]) dx$$

Optimal (type 9, 8 leaves, ? steps):

$$f[\sin[x]] g[e^x]$$

Result (type 9, 30 leaves, 1 step):

$$\text{CannotIntegrate}[\cos[x] g[e^x] f'[\sin[x]], x] + \text{CannotIntegrate}[e^x f[\sin[x]] g'[e^x], x]$$

## Test results for the 311 problems in "8.1 Error functions.m"

Problem 40: Result optimal but 2 more steps used.

$$\int x^2 \operatorname{Erf} [d (a + b \operatorname{Log} [c x^n])] dx$$

Optimal (type 4, 102 leaves, 5 steps):

$$\frac{1}{3} x^3 \operatorname{Erf} [d (a + b \operatorname{Log} [c x^n])] - \frac{1}{3} e^{\frac{9-12ab d^2 n}{4b^2 d^2 n^2}} x^3 (c x^n)^{-3/n} \operatorname{Erf} \left[ \frac{2ab d^2 - \frac{3}{n} + 2b^2 d^2 \operatorname{Log} [c x^n]}{2bd} \right]$$

Result (type 4, 102 leaves, 7 steps):

$$\frac{1}{3} x^3 \operatorname{Erf} [d (a + b \operatorname{Log} [c x^n])] - \frac{1}{3} e^{\frac{9-12ab d^2 n}{4b^2 d^2 n^2}} x^3 (c x^n)^{-3/n} \operatorname{Erf} \left[ \frac{2ab d^2 - \frac{3}{n} + 2b^2 d^2 \operatorname{Log} [c x^n]}{2bd} \right]$$

Problem 41: Result optimal but 2 more steps used.

$$\int x \operatorname{Erf} [d (a + b \operatorname{Log} [c x^n])] dx$$

Optimal (type 4, 94 leaves, 5 steps):

$$\frac{1}{2} x^2 \operatorname{Erf} [d (a + b \operatorname{Log} [c x^n])] - \frac{1}{2} e^{\frac{1-2ab d^2 n}{b^2 d^2 n^2}} x^2 (c x^n)^{-2/n} \operatorname{Erf} \left[ \frac{ab d^2 - \frac{1}{n} + b^2 d^2 \operatorname{Log} [c x^n]}{bd} \right]$$

Result (type 4, 94 leaves, 7 steps):

$$\frac{1}{2} x^2 \operatorname{Erf} [d (a + b \operatorname{Log} [c x^n])] - \frac{1}{2} e^{\frac{1-2ab d^2 n}{b^2 d^2 n^2}} x^2 (c x^n)^{-2/n} \operatorname{Erf} \left[ \frac{ab d^2 - \frac{1}{n} + b^2 d^2 \operatorname{Log} [c x^n]}{bd} \right]$$

Problem 42: Result optimal but 2 more steps used.

$$\int \operatorname{Erf} [d (a + b \operatorname{Log} [c x^n])] dx$$

Optimal (type 4, 93 leaves, 5 steps):

$$x \operatorname{Erf} [d (a + b \operatorname{Log} [c x^n])] - e^{\frac{1-4ab d^2 n}{4b^2 d^2 n^2}} x (c x^n)^{-1/n} \operatorname{Erf} \left[ \frac{2ab d^2 - \frac{1}{n} + 2b^2 d^2 \operatorname{Log} [c x^n]}{2bd} \right]$$

Result (type 4, 93 leaves, 7 steps):

$$x \operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] - e^{\frac{1-4 a b d^2 n}{4 b^2 d^2 n^2}} x\left(c x^n\right)^{-1/n} \operatorname{Erf}\left[\frac{2 a b d^2 - \frac{1}{n} + 2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right]$$

**Problem 44: Result optimal but 2 more steps used.**

$$\int \frac{\operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{x^2} dx$$

Optimal (type 4, 92 leaves, 5 steps):

$$-\frac{\operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{x} + \frac{e^{\frac{1}{4 b^2 d^2 n^2} + \frac{a}{b n}}\left(c x^n\right)^{\frac{1}{n}} \operatorname{Erf}\left[\frac{2 a b d^2 + \frac{1}{n} + 2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right]}{x}$$

Result (type 4, 92 leaves, 7 steps):

$$-\frac{\operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{x} + \frac{e^{\frac{1}{4 b^2 d^2 n^2} + \frac{a}{b n}}\left(c x^n\right)^{\frac{1}{n}} \operatorname{Erf}\left[\frac{2 a b d^2 + \frac{1}{n} + 2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right]}{x}$$

**Problem 45: Result optimal but 2 more steps used.**

$$\int \frac{\operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{x^3} dx$$

Optimal (type 4, 95 leaves, 5 steps):

$$-\frac{\operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{2 x^2} + \frac{e^{\frac{1+2 a b d^2 n}{b^2 d^2 n^2}}\left(c x^n\right)^{2/n} \operatorname{Erf}\left[\frac{1+a b d^2 n+b^2 d^2 n \operatorname{Log}\left[c x^n\right]}{b d n}\right]}{2 x^2}$$

Result (type 4, 95 leaves, 7 steps):

$$-\frac{\operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{2 x^2} + \frac{e^{\frac{1+2 a b d^2 n}{b^2 d^2 n^2}}\left(c x^n\right)^{2/n} \operatorname{Erf}\left[\frac{1+a b d^2 n+b^2 d^2 n \operatorname{Log}\left[c x^n\right]}{b d n}\right]}{2 x^2}$$

**Problem 46: Result optimal but 3 more steps used.**

$$\int\left(e x\right)^m \operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 4, 125 leaves, 5 steps):

$$\frac{(e x)^{1+m} \operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{e(1+m)} + \frac{e^{\frac{(1+m)(1+n-4 a b d^2 n)}{4 b^2 d^2 n^2}} x (e x)^m (c x^n)^{-\frac{1+m}{n}} \operatorname{Erf}\left[\frac{1+m-2 a b d^2 n-2 b^2 d^2 n \operatorname{Log}\left[c x^n\right]}{2 b d n}\right]}{1+m}$$

Result (type 4, 125 leaves, 8 steps):

$$\frac{(e x)^{1+m} \operatorname{Erf}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{e(1+m)} + \frac{e^{\frac{(1+m)(1+n-4 a b d^2 n)}{4 b^2 d^2 n^2}} x (e x)^m (c x^n)^{-\frac{1+m}{n}} \operatorname{Erf}\left[\frac{1+m-2 a b d^2 n-2 b^2 d^2 n \operatorname{Log}\left[c x^n\right]}{2 b d n}\right]}{1+m}$$

**Problem 143: Result optimal but 2 more steps used.**

$$\int x^2 \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 4, 102 leaves, 5 steps):

$$\frac{1}{3} e^{\frac{9-12 a b d^2 n}{4 b^2 d^2 n^2}} x^3 (c x^n)^{-3/n} \operatorname{Erf}\left[\frac{2 a b d^2 - \frac{3}{n} + 2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right] + \frac{1}{3} x^3 \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]$$

Result (type 4, 102 leaves, 7 steps):

$$\frac{1}{3} e^{\frac{9-12 a b d^2 n}{4 b^2 d^2 n^2}} x^3 (c x^n)^{-3/n} \operatorname{Erf}\left[\frac{2 a b d^2 - \frac{3}{n} + 2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right] + \frac{1}{3} x^3 \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]$$

**Problem 144: Result optimal but 2 more steps used.**

$$\int x \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 4, 94 leaves, 5 steps):

$$\frac{1}{2} e^{\frac{1-2 a b d^2 n}{b^2 d^2 n^2}} x^2 (c x^n)^{-2/n} \operatorname{Erf}\left[\frac{a b d^2 - \frac{1}{n} + b^2 d^2 \operatorname{Log}\left[c x^n\right]}{b d}\right] + \frac{1}{2} x^2 \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]$$

Result (type 4, 94 leaves, 7 steps):

$$\frac{1}{2} e^{\frac{1-2 a b d^2 n}{b^2 d^2 n^2}} x^2 (c x^n)^{-2/n} \operatorname{Erf}\left[\frac{a b d^2 - \frac{1}{n} + b^2 d^2 \operatorname{Log}\left[c x^n\right]}{b d}\right] + \frac{1}{2} x^2 \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]$$

**Problem 145: Result optimal but 2 more steps used.**

$$\int \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 4, 92 leaves, 5 steps):

$$e^{\frac{1-4abd^2n}{4b^2d^2n^2}} x (cx^n)^{-1/n} \operatorname{Erf}\left[\frac{2abd^2 - \frac{1}{n} + 2b^2d^2 \operatorname{Log}[cx^n]}{2bd}\right] + x \operatorname{Erfc}\left[d(a + b \operatorname{Log}[cx^n])\right]$$

Result (type 4, 92 leaves, 7 steps):

$$e^{\frac{1-4abd^2n}{4b^2d^2n^2}} x (cx^n)^{-1/n} \operatorname{Erf}\left[\frac{2abd^2 - \frac{1}{n} + 2b^2d^2 \operatorname{Log}[cx^n]}{2bd}\right] + x \operatorname{Erfc}\left[d(a + b \operatorname{Log}[cx^n])\right]$$

**Problem 147: Result optimal but 2 more steps used.**

$$\int \frac{\operatorname{Erfc}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x^2} dx$$

Optimal (type 4, 93 leaves, 5 steps):

$$-\frac{e^{\frac{1}{4b^2d^2n^2} + \frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Erf}\left[\frac{2abd^2 + \frac{1}{n} + 2b^2d^2 \operatorname{Log}[cx^n]}{2bd}\right]}{x} - \frac{\operatorname{Erfc}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x}$$

Result (type 4, 93 leaves, 7 steps):

$$-\frac{e^{\frac{1}{4b^2d^2n^2} + \frac{a}{bn}} (cx^n)^{\frac{1}{n}} \operatorname{Erf}\left[\frac{2abd^2 + \frac{1}{n} + 2b^2d^2 \operatorname{Log}[cx^n]}{2bd}\right]}{x} - \frac{\operatorname{Erfc}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x}$$

**Problem 148: Result optimal but 2 more steps used.**

$$\int \frac{\operatorname{Erfc}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x^3} dx$$

Optimal (type 4, 95 leaves, 5 steps):

$$-\frac{e^{\frac{1+2abd^2n}{b^2d^2n^2}} (cx^n)^{2/n} \operatorname{Erf}\left[\frac{1+abd^2n+b^2d^2n \operatorname{Log}[cx^n]}{bdn}\right]}{2x^2} - \frac{\operatorname{Erfc}\left[d(a + b \operatorname{Log}[cx^n])\right]}{2x^2}$$

Result (type 4, 95 leaves, 7 steps):

$$-\frac{e^{\frac{1+2abd^2n}{b^2d^2n^2}} (cx^n)^{2/n} \operatorname{Erf}\left[\frac{1+abd^2n+b^2d^2n \operatorname{Log}[cx^n]}{bdn}\right]}{2x^2} - \frac{\operatorname{Erfc}\left[d(a + b \operatorname{Log}[cx^n])\right]}{2x^2}$$

### Problem 149: Result optimal but 3 more steps used.

$$\int (e x)^m \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 4, 126 leaves, 5 steps):

$$-\frac{e^{\frac{(1+m)(1+m-4abd^2n)}{4b^2d^2n^2}} x (e x)^m (c x^n)^{-\frac{1+m}{n}} \operatorname{Erfc}\left[\frac{1+m-2abd^2n-2b^2d^2n \operatorname{Log}[c x^n]}{2bdn}\right]}{1+m} + \frac{(e x)^{1+m} \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{e(1+m)}$$

Result (type 4, 126 leaves, 8 steps):

$$-\frac{e^{\frac{(1+m)(1+m-4abd^2n)}{4b^2d^2n^2}} x (e x)^m (c x^n)^{-\frac{1+m}{n}} \operatorname{Erfc}\left[\frac{1+m-2abd^2n-2b^2d^2n \operatorname{Log}[c x^n]}{2bdn}\right]}{1+m} + \frac{(e x)^{1+m} \operatorname{Erfc}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{e(1+m)}$$

### Problem 246: Result optimal but 2 more steps used.

$$\int x^2 \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 4, 102 leaves, 5 steps):

$$\frac{1}{3} x^3 \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] - \frac{1}{3} e^{-\frac{3(3+4abd^2n)}{4b^2d^2n^2}} x^3 (c x^n)^{-3/n} \operatorname{Erfi}\left[\frac{2abd^2 + \frac{3}{n} + 2b^2d^2 \operatorname{Log}[c x^n]}{2bd}\right]$$

Result (type 4, 102 leaves, 7 steps):

$$\frac{1}{3} x^3 \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] - \frac{1}{3} e^{-\frac{3(3+4abd^2n)}{4b^2d^2n^2}} x^3 (c x^n)^{-3/n} \operatorname{Erfi}\left[\frac{2abd^2 + \frac{3}{n} + 2b^2d^2 \operatorname{Log}[c x^n]}{2bd}\right]$$

### Problem 247: Result optimal but 2 more steps used.

$$\int x \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] dx$$

Optimal (type 4, 93 leaves, 5 steps):

$$\frac{1}{2} x^2 \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] - \frac{1}{2} e^{-\frac{1+2abd^2n}{b^2d^2n^2}} x^2 (c x^n)^{-2/n} \operatorname{Erfi}\left[\frac{abd^2 + \frac{1}{n} + b^2d^2 \operatorname{Log}[c x^n]}{bd}\right]$$

Result (type 4, 93 leaves, 7 steps):

$$\frac{1}{2} x^2 \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] - \frac{1}{2} e^{-\frac{1+2 a b d^2 n}{b^2 d^2 n^2}} x^2\left(c x^n\right)^{-2 / n} \operatorname{Erfi}\left[\frac{a b d^2+\frac{1}{n}+b^2 d^2 \operatorname{Log}\left[c x^n\right]}{b d}\right]$$

**Problem 248: Result optimal but 2 more steps used.**

$$\int \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] d x$$

Optimal (type 4, 91 leaves, 5 steps):

$$x \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] - e^{-\frac{1+4 a b d^2 n}{4 b^2 d^2 n^2}} x\left(c x^n\right)^{-1 / n} \operatorname{Erfi}\left[\frac{2 a b d^2+\frac{1}{n}+2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right]$$

Result (type 4, 91 leaves, 7 steps):

$$x \operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right] - e^{-\frac{1+4 a b d^2 n}{4 b^2 d^2 n^2}} x\left(c x^n\right)^{-1 / n} \operatorname{Erfi}\left[\frac{2 a b d^2+\frac{1}{n}+2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right]$$

**Problem 250: Result optimal but 2 more steps used.**

$$\int \frac{\operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{x^2} d x$$

Optimal (type 4, 94 leaves, 5 steps):

$$-\frac{\operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{x} + \frac{e^{-\frac{1}{4 b^2 d^2 n^2}+\frac{a}{b n}}\left(c x^n\right)^{\frac{1}{n}} \operatorname{Erfi}\left[\frac{2 a b d^2-\frac{1}{n}+2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right]}{x}$$

Result (type 4, 94 leaves, 7 steps):

$$-\frac{\operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{x} + \frac{e^{-\frac{1}{4 b^2 d^2 n^2}+\frac{a}{b n}}\left(c x^n\right)^{\frac{1}{n}} \operatorname{Erfi}\left[\frac{2 a b d^2-\frac{1}{n}+2 b^2 d^2 \operatorname{Log}\left[c x^n\right]}{2 b d}\right]}{x}$$

**Problem 251: Result optimal but 2 more steps used.**

$$\int \frac{\operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{x^3} d x$$

Optimal (type 4, 95 leaves, 5 steps):

$$-\frac{\operatorname{Erfi}\left[d\left(a+b \operatorname{Log}\left[c x^n\right]\right)\right]}{2 x^2} + \frac{e^{-\frac{1-2 a b d^2 n}{b^2 d^2 n^2}}\left(c x^n\right)^{2 / n} \operatorname{Erfi}\left[\frac{a b d^2-\frac{1}{n}+b^2 d^2 \operatorname{Log}\left[c x^n\right]}{b d}\right]}{2 x^2}$$

Result (type 4, 95 leaves, 7 steps):

$$-\frac{\operatorname{Erfi}\left[d\left(a+b\operatorname{Log}\left[cx^n\right]\right)\right]}{2x^2} + \frac{e^{-\frac{1-2abd^2n}{b^2d^2n^2}}\left(cx^n\right)^{2/n}\operatorname{Erfi}\left[\frac{abd^2-\frac{1}{n}b^2d^2\operatorname{Log}\left[cx^n\right]}{bd}\right]}{2x^2}$$

Problem 252: Result optimal but 3 more steps used.

$$\int (ex)^m \operatorname{Erfi}\left[d\left(a+b\operatorname{Log}\left[cx^n\right]\right)\right] dx$$

Optimal (type 4, 126 leaves, 5 steps):

$$\frac{(ex)^{1+m}\operatorname{Erfi}\left[d\left(a+b\operatorname{Log}\left[cx^n\right]\right)\right]}{e(1+m)} - \frac{e^{-\frac{(1+m)(1+m+4abd^2n)}{4b^2d^2n^2}}x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\operatorname{Erfi}\left[\frac{1+m+2abd^2n+2b^2d^2n\operatorname{Log}\left[cx^n\right]}{2bdn}\right]}{1+m}$$

Result (type 4, 126 leaves, 8 steps):

$$\frac{(ex)^{1+m}\operatorname{Erfi}\left[d\left(a+b\operatorname{Log}\left[cx^n\right]\right)\right]}{e(1+m)} - \frac{e^{-\frac{(1+m)(1+m+4abd^2n)}{4b^2d^2n^2}}x(ex)^m\left(cx^n\right)^{-\frac{1+m}{n}}\operatorname{Erfi}\left[\frac{1+m+2abd^2n+2b^2d^2n\operatorname{Log}\left[cx^n\right]}{2bdn}\right]}{1+m}$$

## Test results for the 218 problems in "8.2 Fresnel integral functions.m"

Problem 54: Result optimal but 4 more steps used.

$$\int x^2 \operatorname{FresnelS}\left[d\left(a+b\operatorname{Log}\left[cx^n\right]\right)\right] dx$$

Optimal (type 4, 231 leaves, 10 steps):

$$\left(\frac{1}{12} - \frac{i}{12}\right) e^{-\frac{3a}{bn} + \frac{9i}{2b^2d^2n^2\pi}} x^3 \left(cx^n\right)^{-3/n} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{3}{n} + iabd^2\pi + ib^2d^2\pi\operatorname{Log}\left[cx^n\right]\right)}{bd\sqrt{\pi}}\right] +$$

$$\left(\frac{1}{12} - \frac{i}{12}\right) e^{\frac{3a}{bn} - \frac{9i}{2b^2d^2n^2\pi}} x^3 \left(cx^n\right)^{-3/n} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\left(\frac{3}{n} - iabd^2\pi - ib^2d^2\pi\operatorname{Log}\left[cx^n\right]\right)}{bd\sqrt{\pi}}\right] + \frac{1}{3} x^3 \operatorname{FresnelS}\left[d\left(a+b\operatorname{Log}\left[cx^n\right]\right)\right]$$

Result (type 4, 231 leaves, 14 steps):



$$\left(\frac{1}{12} - \frac{i}{12}\right) e^{-\frac{3a}{bn} + \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (c x^n)^{-3/n} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} + i a b d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[c x^n]\right)}{b d \sqrt{\pi}}\right] +$$

$$\left(\frac{1}{12} - \frac{i}{12}\right) e^{\frac{3a}{bn} - \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (c x^n)^{-3/n} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{3}{n} - i a b d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[c x^n]\right)}{b d \sqrt{\pi}}\right] + \frac{1}{3} x^3 \operatorname{FresnelS}\left[d (a + b \operatorname{Log}[c x^n])\right]$$

**Problem 55: Result optimal but 4 more steps used.**

$$\int x \operatorname{FresnelS}\left[d (a + b \operatorname{Log}[c x^n])\right] dx$$

Optimal (type 4, 227 leaves, 10 steps):

$$\left(\frac{1}{8} - \frac{i}{8}\right) e^{\frac{2i-2ab d^2 n \pi}{b^2 d^2 n^2 \pi}} x^2 (c x^n)^{-2/n} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[c x^n]\right)}{b d \sqrt{\pi}}\right] +$$

$$\left(\frac{1}{8} - \frac{i}{8}\right) e^{-\frac{2(i+ab d^2 n \pi)}{b^2 d^2 n^2 \pi}} x^2 (c x^n)^{-2/n} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} - i a b d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[c x^n]\right)}{b d \sqrt{\pi}}\right] + \frac{1}{2} x^2 \operatorname{FresnelS}\left[d (a + b \operatorname{Log}[c x^n])\right]$$

Result (type 4, 227 leaves, 14 steps):

$$\left(\frac{1}{8} - \frac{i}{8}\right) e^{\frac{2i-2ab d^2 n \pi}{b^2 d^2 n^2 \pi}} x^2 (c x^n)^{-2/n} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[c x^n]\right)}{b d \sqrt{\pi}}\right] +$$

$$\left(\frac{1}{8} - \frac{i}{8}\right) e^{-\frac{2(i+ab d^2 n \pi)}{b^2 d^2 n^2 \pi}} x^2 (c x^n)^{-2/n} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} - i a b d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[c x^n]\right)}{b d \sqrt{\pi}}\right] + \frac{1}{2} x^2 \operatorname{FresnelS}\left[d (a + b \operatorname{Log}[c x^n])\right]$$

**Problem 56: Result optimal but 4 more steps used.**

$$\int \operatorname{FresnelS}\left[d (a + b \operatorname{Log}[c x^n])\right] dx$$

Optimal (type 4, 214 leaves, 10 steps):

$$\left(\frac{1}{4} - \frac{i}{4}\right) e^{-\frac{2abn - \frac{1}{d^2 n}}{2b^2 n^2}} x (c x^n)^{-1/n} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + i a b d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[c x^n]\right)}{b d \sqrt{\pi}}\right] +$$

$$\left(\frac{1}{4} - \frac{i}{4}\right) e^{-\frac{2abn + \frac{1}{d^2 n}}{2b^2 n^2}} x (c x^n)^{-1/n} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - i a b d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[c x^n]\right)}{b d \sqrt{\pi}}\right] + x \operatorname{FresnelS}\left[d (a + b \operatorname{Log}[c x^n])\right]$$

Result (type 4, 214 leaves, 14 steps):

$$\left(\frac{1}{4} - \frac{i}{4}\right) e^{-\frac{2abn - \frac{i}{d^2n}}{2b^2n^2}} x (cx^n)^{-1/n} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + i ab d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right] +$$

$$\left(\frac{1}{4} - \frac{i}{4}\right) e^{-\frac{2abn - \frac{i}{d^2n}}{2b^2n^2}} x (cx^n)^{-1/n} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - i ab d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right] + x \operatorname{FresnelS}\left[d(a + b \operatorname{Log}[cx^n])\right]$$

Problem 58: Result optimal but 4 more steps used.

$$\int \frac{\operatorname{FresnelS}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x^2} dx$$

Optimal (type 4, 217 leaves, 10 steps):

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn + \frac{i}{d^2n}}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - i ab d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x} +$$

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn - \frac{i}{d^2n}}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + i ab d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x} - \frac{\operatorname{FresnelS}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x}$$

Result (type 4, 217 leaves, 14 steps):

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn + \frac{i}{d^2n}}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - i ab d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x} +$$

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{2abn - \frac{i}{d^2n}}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + i ab d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x} - \frac{\operatorname{FresnelS}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x}$$

Problem 59: Result optimal but 4 more steps used.

$$\int \frac{\operatorname{FresnelS}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x^3} dx$$

Optimal (type 4, 228 leaves, 10 steps):

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{Erf}\left[\frac{\left(\frac{1+i}{2}\right) \left(\frac{2-i}{n} - iabd^2\pi - i b^2d^2\pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x^2} +$$

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) e^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{Erfi}\left[\frac{\left(\frac{1+i}{2}\right) \left(\frac{2+i}{n} + iabd^2\pi + i b^2d^2\pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x^2} - \frac{\operatorname{FresnelS}\left[d\left(a + b \operatorname{Log}[cx^n]\right)\right]}{2x^2}$$

Result (type 4, 228 leaves, 14 steps):

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{Erf}\left[\frac{\left(\frac{1+i}{2}\right) \left(\frac{2-i}{n} - iabd^2\pi - i b^2d^2\pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x^2} +$$

$$\frac{\left(\frac{1}{8} - \frac{i}{8}\right) e^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{Erfi}\left[\frac{\left(\frac{1+i}{2}\right) \left(\frac{2+i}{n} + iabd^2\pi + i b^2d^2\pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x^2} - \frac{\operatorname{FresnelS}\left[d\left(a + b \operatorname{Log}[cx^n]\right)\right]}{2x^2}$$

Problem 60: Result optimal but 6 more steps used.

$$\int (ex)^m \operatorname{FresnelS}\left[d\left(a + b \operatorname{Log}[cx^n]\right)\right] dx$$

Optimal (type 4, 280 leaves, 10 steps):

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{i(1+m)(1+m+2iab d^2 n \pi)}{2b^2 d^2 n^2 \pi}} x (ex)^m (cx^n)^{-\frac{1+m}{n}} \operatorname{Erf}\left[\frac{\left(\frac{1+i}{2}\right) (1+m+iabd^2n\pi + i b^2d^2n\pi \operatorname{Log}[cx^n])}{bdn\sqrt{\pi}}\right]}{1+m} +$$

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{-\frac{i(1+m)(1+m-2iab d^2 n \pi)}{2b^2 d^2 n^2 \pi}} x (ex)^m (cx^n)^{-\frac{1+m}{n}} \operatorname{Erfi}\left[\frac{\left(\frac{1+i}{2}\right) (1+m-iabd^2n\pi - i b^2d^2n\pi \operatorname{Log}[cx^n])}{bdn\sqrt{\pi}}\right]}{1+m} + \frac{(ex)^{1+m} \operatorname{FresnelS}\left[d\left(a + b \operatorname{Log}[cx^n]\right)\right]}{e(1+m)}$$

Result (type 4, 280 leaves, 16 steps):

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{\frac{i(1+m)(1+m+2iab d^2 n \pi)}{2b^2 d^2 n^2 \pi}} x (ex)^m (cx^n)^{-\frac{1+m}{n}} \operatorname{Erf}\left[\frac{\left(\frac{1+i}{2}\right) (1+m+iabd^2n\pi + i b^2d^2n\pi \operatorname{Log}[cx^n])}{bdn\sqrt{\pi}}\right]}{1+m} +$$

$$\frac{\left(\frac{1}{4} - \frac{i}{4}\right) e^{-\frac{i(1+m)(1+m-2iab d^2 n \pi)}{2b^2 d^2 n^2 \pi}} x (ex)^m (cx^n)^{-\frac{1+m}{n}} \operatorname{Erfi}\left[\frac{\left(\frac{1+i}{2}\right) (1+m-iabd^2n\pi - i b^2d^2n\pi \operatorname{Log}[cx^n])}{bdn\sqrt{\pi}}\right]}{1+m} + \frac{(ex)^{1+m} \operatorname{FresnelS}\left[d\left(a + b \operatorname{Log}[cx^n]\right)\right]}{e(1+m)}$$

### Problem 163: Result optimal but 4 more steps used.

$$\int x^2 \operatorname{FresnelC}[d (a + b \operatorname{Log}[c x^n])] dx$$

Optimal (type 4, 231 leaves, 10 steps):

$$\left( \frac{1}{12} + \frac{i}{12} \right) e^{-\frac{3a}{bn} + \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (c x^n)^{-3/n} \operatorname{Erf} \left[ \frac{\left( \frac{1}{2} + \frac{i}{2} \right) \left( \frac{3}{n} + i a b d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[c x^n] \right)}{b d \sqrt{\pi}} \right] -$$

$$\left( \frac{1}{12} + \frac{i}{12} \right) e^{-\frac{3a}{bn} - \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (c x^n)^{-3/n} \operatorname{Erfi} \left[ \frac{\left( \frac{1}{2} + \frac{i}{2} \right) \left( \frac{3}{n} - i a b d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[c x^n] \right)}{b d \sqrt{\pi}} \right] + \frac{1}{3} x^3 \operatorname{FresnelC}[d (a + b \operatorname{Log}[c x^n])] ]$$

Result (type 4, 231 leaves, 14 steps):

$$\left( \frac{1}{12} + \frac{i}{12} \right) e^{-\frac{3a}{bn} + \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (c x^n)^{-3/n} \operatorname{Erf} \left[ \frac{\left( \frac{1}{2} + \frac{i}{2} \right) \left( \frac{3}{n} + i a b d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[c x^n] \right)}{b d \sqrt{\pi}} \right] -$$

$$\left( \frac{1}{12} + \frac{i}{12} \right) e^{-\frac{3a}{bn} - \frac{9i}{2b^2 d^2 n^2 \pi}} x^3 (c x^n)^{-3/n} \operatorname{Erfi} \left[ \frac{\left( \frac{1}{2} + \frac{i}{2} \right) \left( \frac{3}{n} - i a b d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[c x^n] \right)}{b d \sqrt{\pi}} \right] + \frac{1}{3} x^3 \operatorname{FresnelC}[d (a + b \operatorname{Log}[c x^n])] ]$$

### Problem 164: Result optimal but 4 more steps used.

$$\int x \operatorname{FresnelC}[d (a + b \operatorname{Log}[c x^n])] dx$$

Optimal (type 4, 227 leaves, 10 steps):

$$\left( \frac{1}{8} + \frac{i}{8} \right) e^{\frac{2i-2ab d^2 n \pi}{b^2 d^2 n^2 \pi}} x^2 (c x^n)^{-2/n} \operatorname{Erf} \left[ \frac{\left( \frac{1}{2} + \frac{i}{2} \right) \left( \frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[c x^n] \right)}{b d \sqrt{\pi}} \right] -$$

$$\left( \frac{1}{8} + \frac{i}{8} \right) e^{-\frac{2(i+ab d^2 n \pi)}{b^2 d^2 n^2 \pi}} x^2 (c x^n)^{-2/n} \operatorname{Erfi} \left[ \frac{\left( \frac{1}{2} + \frac{i}{2} \right) \left( \frac{2}{n} - i a b d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[c x^n] \right)}{b d \sqrt{\pi}} \right] + \frac{1}{2} x^2 \operatorname{FresnelC}[d (a + b \operatorname{Log}[c x^n])] ]$$

Result (type 4, 227 leaves, 14 steps):

$$\left( \frac{1}{8} + \frac{i}{8} \right) e^{\frac{2i-2ab d^2 n \pi}{b^2 d^2 n^2 \pi}} x^2 (c x^n)^{-2/n} \operatorname{Erf} \left[ \frac{\left( \frac{1}{2} + \frac{i}{2} \right) \left( \frac{2}{n} + i a b d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[c x^n] \right)}{b d \sqrt{\pi}} \right] -$$

$$\left( \frac{1}{8} + \frac{i}{8} \right) e^{-\frac{2(i+ab d^2 n \pi)}{b^2 d^2 n^2 \pi}} x^2 (c x^n)^{-2/n} \operatorname{Erfi} \left[ \frac{\left( \frac{1}{2} + \frac{i}{2} \right) \left( \frac{2}{n} - i a b d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[c x^n] \right)}{b d \sqrt{\pi}} \right] + \frac{1}{2} x^2 \operatorname{FresnelC}[d (a + b \operatorname{Log}[c x^n])] ]$$

### Problem 165: Result optimal but 4 more steps used.

$$\int \text{FresnelC}[d (a + b \text{Log}[c x^n])] dx$$

Optimal (type 4, 214 leaves, 10 steps):

$$\left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{2abn - \frac{1}{d^2n}}{2b^2n^2} x} (cx^n)^{-1/n} \text{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + i ab d^2 \pi + i b^2 d^2 \pi \text{Log}[c x^n]\right)}{bd \sqrt{\pi}}\right] -$$

$$\left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{2abn + \frac{1}{d^2n}}{2b^2n^2} x} (cx^n)^{-1/n} \text{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - i ab d^2 \pi - i b^2 d^2 \pi \text{Log}[c x^n]\right)}{bd \sqrt{\pi}}\right] + x \text{FresnelC}[d (a + b \text{Log}[c x^n])] ]$$

Result (type 4, 214 leaves, 14 steps):

$$\left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{2abn - \frac{1}{d^2n}}{2b^2n^2} x} (cx^n)^{-1/n} \text{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + i ab d^2 \pi + i b^2 d^2 \pi \text{Log}[c x^n]\right)}{bd \sqrt{\pi}}\right] -$$

$$\left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{2abn + \frac{1}{d^2n}}{2b^2n^2} x} (cx^n)^{-1/n} \text{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - i ab d^2 \pi - i b^2 d^2 \pi \text{Log}[c x^n]\right)}{bd \sqrt{\pi}}\right] + x \text{FresnelC}[d (a + b \text{Log}[c x^n])] ]$$

### Problem 167: Result optimal but 4 more steps used.

$$\int \frac{\text{FresnelC}[d (a + b \text{Log}[c x^n])]}{x^2} dx$$

Optimal (type 4, 217 leaves, 10 steps):

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{2abn + \frac{1}{d^2n}}{2b^2n^2} x} (cx^n)^{\frac{1}{n}} \text{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - i ab d^2 \pi - i b^2 d^2 \pi \text{Log}[c x^n]\right)}{bd \sqrt{\pi}}\right]}{x} -$$

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{2abn - \frac{1}{d^2n}}{2b^2n^2} x} (cx^n)^{\frac{1}{n}} \text{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + i ab d^2 \pi + i b^2 d^2 \pi \text{Log}[c x^n]\right)}{bd \sqrt{\pi}}\right]}{x} - \frac{\text{FresnelC}[d (a + b \text{Log}[c x^n])]}{x}$$

Result (type 4, 217 leaves, 14 steps):

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{2abn + \frac{i}{d^2\pi}}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} - i ab d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x} - \frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{2abn - \frac{i}{d^2\pi}}{2b^2n^2}} (cx^n)^{\frac{1}{n}} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{1}{n} + i ab d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x} - \frac{\operatorname{FresnelC}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x}$$

Problem 168: Result optimal but 4 more steps used.

$$\int \frac{\operatorname{FresnelC}\left[d(a + b \operatorname{Log}[cx^n])\right]}{x^3} dx$$

Optimal (type 4, 228 leaves, 10 steps):

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} - i ab d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x^2} - \frac{\left(\frac{1}{8} + \frac{i}{8}\right) e^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} + i ab d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x^2} - \frac{\operatorname{FresnelC}\left[d(a + b \operatorname{Log}[cx^n])\right]}{2x^2}$$

Result (type 4, 228 leaves, 14 steps):

$$\frac{\left(\frac{1}{8} + \frac{i}{8}\right) e^{\frac{2i+2abd^2n\pi}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} - i ab d^2 \pi - i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x^2} - \frac{\left(\frac{1}{8} + \frac{i}{8}\right) e^{-\frac{2(i-abd^2n\pi)}{b^2d^2n^2\pi}} (cx^n)^{2/n} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) \left(\frac{2}{n} + i ab d^2 \pi + i b^2 d^2 \pi \operatorname{Log}[cx^n]\right)}{bd\sqrt{\pi}}\right]}{x^2} - \frac{\operatorname{FresnelC}\left[d(a + b \operatorname{Log}[cx^n])\right]}{2x^2}$$

Problem 169: Result optimal but 6 more steps used.

$$\int (ex)^m \operatorname{FresnelC}\left[d(a + b \operatorname{Log}[cx^n])\right] dx$$

Optimal (type 4, 280 leaves, 10 steps):

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{i(1+m)(1+m-2iab d^2 n \pi)}{2b^2 d^2 n^2 \pi}} x (e x)^m (c x^n)^{-\frac{1+m}{n}} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (1+m+iab d^2 n \pi + i b^2 d^2 n \pi \operatorname{Log}[c x^n])}{b d n \sqrt{\pi}}\right]}{1+m} -$$

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{i(1+m)(1+m-2iab d^2 n \pi)}{2b^2 d^2 n^2 \pi}} x (e x)^m (c x^n)^{-\frac{1+m}{n}} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (1+m-iab d^2 n \pi - i b^2 d^2 n \pi \operatorname{Log}[c x^n])}{b d n \sqrt{\pi}}\right]}{1+m} + \frac{(e x)^{1+m} \operatorname{FresnelC}\left[d (a + b \operatorname{Log}[c x^n])\right]}{e (1+m)}$$

Result (type 4, 280 leaves, 16 steps):

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{\frac{i(1+m)(1+m-2iab d^2 n \pi)}{2b^2 d^2 n^2 \pi}} x (e x)^m (c x^n)^{-\frac{1+m}{n}} \operatorname{Erf}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (1+m+iab d^2 n \pi + i b^2 d^2 n \pi \operatorname{Log}[c x^n])}{b d n \sqrt{\pi}}\right]}{1+m} -$$

$$\frac{\left(\frac{1}{4} + \frac{i}{4}\right) e^{-\frac{i(1+m)(1+m-2iab d^2 n \pi)}{2b^2 d^2 n^2 \pi}} x (e x)^m (c x^n)^{-\frac{1+m}{n}} \operatorname{Erfi}\left[\frac{\left(\frac{1}{2} + \frac{i}{2}\right) (1+m-iab d^2 n \pi - i b^2 d^2 n \pi \operatorname{Log}[c x^n])}{b d n \sqrt{\pi}}\right]}{1+m} + \frac{(e x)^{1+m} \operatorname{FresnelC}\left[d (a + b \operatorname{Log}[c x^n])\right]}{e (1+m)}$$

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Test results for the 208 problems in "8.3 Exponential integral functions.m"

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Test results for the 136 problems in "8.4 Trig integral functions.m"

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Test results for the 136 problems in "8.5 Hyperbolic integral functions.m"

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Test results for the 233 problems in "8.6 Gamma functions.m"

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Test results for the 14 problems in "8.7 Zeta function.m"

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Test results for the 198 problems in "8.8 Polylogarithm function.m"

Problem 170: Result valid but suboptimal antiderivative.

$$\int x^2 (g + h \operatorname{Log}[1 - c x]) \operatorname{PolyLog}[2, c x] dx$$

Optimal (type 4, 423 leaves, 25 steps):

$$\begin{aligned}
& \frac{121 h x}{108 c^2} + \frac{13 h x^2}{216 c} + \frac{h x^3}{81} + \frac{h (1-c x)^2}{6 c^3} - \frac{2 h (1-c x)^3}{81 c^3} + \frac{13 h \operatorname{Log}[1-c x]}{108 c^3} - \frac{h x^2 \operatorname{Log}[1-c x]}{12 c} - \frac{1}{27} h x^3 \operatorname{Log}[1-c x] + \frac{h (1-c x) \operatorname{Log}[1-c x]}{3 c^3} + \\
& \frac{h \operatorname{Log}[1-c x]^2}{9 c^3} - \frac{h \operatorname{Log}[c x] \operatorname{Log}[1-c x]^2}{3 c^3} + \frac{1}{9} x^3 \operatorname{Log}[1-c x] (g + h \operatorname{Log}[1-c x]) + \frac{(1-c x) (g + 2 h \operatorname{Log}[1-c x])}{3 c^3} - \frac{(1-c x)^2 (g + 2 h \operatorname{Log}[1-c x])}{6 c^3} + \\
& \frac{(1-c x)^3 (g + 2 h \operatorname{Log}[1-c x])}{27 c^3} - \frac{\operatorname{Log}[1-c x] (g + 2 h \operatorname{Log}[1-c x])}{9 c^3} - \frac{h x \operatorname{PolyLog}[2, c x]}{3 c^2} - \frac{h x^2 \operatorname{PolyLog}[2, c x]}{6 c} - \frac{1}{9} h x^3 \operatorname{PolyLog}[2, c x] - \\
& \frac{h \operatorname{Log}[1-c x] \operatorname{PolyLog}[2, c x]}{3 c^3} + \frac{1}{3} x^3 (g + h \operatorname{Log}[1-c x]) \operatorname{PolyLog}[2, c x] - \frac{2 h \operatorname{Log}[1-c x] \operatorname{PolyLog}[2, 1-c x]}{3 c^3} + \frac{2 h \operatorname{PolyLog}[3, 1-c x]}{3 c^3}
\end{aligned}$$

Result (type 4, 366 leaves, 37 steps):

$$\begin{aligned}
& \frac{107 h x}{108 c^2} + \frac{23 h x^2}{216 c} + \frac{2 h x^3}{81} + \frac{h (1-c x)^2}{12 c^3} - \frac{h (1-c x)^3}{81 c^3} + \frac{23 h \operatorname{Log}[1-c x]}{108 c^3} - \frac{5 h x^2 \operatorname{Log}[1-c x]}{36 c} - \frac{2}{27} h x^3 \operatorname{Log}[1-c x] + \frac{4 h (1-c x) \operatorname{Log}[1-c x]}{9 c^3} - \\
& \frac{h \operatorname{Log}[c x] \operatorname{Log}[1-c x]^2}{3 c^3} + \frac{1}{9} x^3 \operatorname{Log}[1-c x] (g + h \operatorname{Log}[1-c x]) + \frac{1}{54} \left( \frac{18 (1-c x)}{c^3} - \frac{9 (1-c x)^2}{c^3} + \frac{2 (1-c x)^3}{c^3} - \frac{6 \operatorname{Log}[1-c x]}{c^3} \right) (g + h \operatorname{Log}[1-c x]) - \\
& \frac{h x \operatorname{PolyLog}[2, c x]}{3 c^2} - \frac{h x^2 \operatorname{PolyLog}[2, c x]}{6 c} - \frac{1}{9} h x^3 \operatorname{PolyLog}[2, c x] - \frac{h \operatorname{Log}[1-c x] \operatorname{PolyLog}[2, c x]}{3 c^3} + \\
& \frac{1}{3} x^3 (g + h \operatorname{Log}[1-c x]) \operatorname{PolyLog}[2, c x] - \frac{2 h \operatorname{Log}[1-c x] \operatorname{PolyLog}[2, 1-c x]}{3 c^3} + \frac{2 h \operatorname{PolyLog}[3, 1-c x]}{3 c^3}
\end{aligned}$$

**Problem 171: Result valid but suboptimal antiderivative.**

$$\int x (g + h \operatorname{Log}[1-c x]) \operatorname{PolyLog}[2, c x] dx$$

Optimal (type 4, 330 leaves, 21 steps):

$$\begin{aligned}
& \frac{13 h x}{8 c} + \frac{h x^2}{16} + \frac{h (1-c x)^2}{8 c^2} + \frac{h \operatorname{Log}[1-c x]}{8 c^2} - \frac{1}{8} h x^2 \operatorname{Log}[1-c x] + \frac{h (1-c x) \operatorname{Log}[1-c x]}{2 c^2} + \frac{h \operatorname{Log}[1-c x]^2}{4 c^2} - \\
& \frac{h \operatorname{Log}[c x] \operatorname{Log}[1-c x]^2}{2 c^2} + \frac{1}{4} x^2 \operatorname{Log}[1-c x] (g + h \operatorname{Log}[1-c x]) + \frac{(1-c x) (g + 2 h \operatorname{Log}[1-c x])}{2 c^2} - \frac{(1-c x)^2 (g + 2 h \operatorname{Log}[1-c x])}{8 c^2} - \\
& \frac{\operatorname{Log}[1-c x] (g + 2 h \operatorname{Log}[1-c x])}{4 c^2} - \frac{h x \operatorname{PolyLog}[2, c x]}{2 c} - \frac{1}{4} h x^2 \operatorname{PolyLog}[2, c x] - \frac{h \operatorname{Log}[1-c x] \operatorname{PolyLog}[2, c x]}{2 c^2} + \\
& \frac{1}{2} x^2 (g + h \operatorname{Log}[1-c x]) \operatorname{PolyLog}[2, c x] - \frac{h \operatorname{Log}[1-c x] \operatorname{PolyLog}[2, 1-c x]}{c^2} + \frac{h \operatorname{PolyLog}[3, 1-c x]}{c^2}
\end{aligned}$$

Result (type 4, 287 leaves, 30 steps):



$$\begin{aligned} & \frac{3 h x}{2 c} + \frac{h x^2}{8} + \frac{h (1 - c x)^2}{16 c^2} + \frac{h \operatorname{Log}[1 - c x]}{4 c^2} - \frac{1}{4} h x^2 \operatorname{Log}[1 - c x] + \frac{3 h (1 - c x) \operatorname{Log}[1 - c x]}{4 c^2} - \frac{h \operatorname{Log}[c x] \operatorname{Log}[1 - c x]^2}{2 c^2} + \\ & \frac{1}{4} x^2 \operatorname{Log}[1 - c x] (g + h \operatorname{Log}[1 - c x]) + \frac{1}{8} \left( \frac{4 (1 - c x)}{c^2} - \frac{(1 - c x)^2}{c^2} - \frac{2 \operatorname{Log}[1 - c x]}{c^2} \right) (g + h \operatorname{Log}[1 - c x]) - \\ & \frac{h x \operatorname{PolyLog}[2, c x]}{2 c} - \frac{1}{4} h x^2 \operatorname{PolyLog}[2, c x] - \frac{h \operatorname{Log}[1 - c x] \operatorname{PolyLog}[2, c x]}{2 c^2} + \\ & \frac{1}{2} x^2 (g + h \operatorname{Log}[1 - c x]) \operatorname{PolyLog}[2, c x] - \frac{h \operatorname{Log}[1 - c x] \operatorname{PolyLog}[2, 1 - c x]}{c^2} + \frac{h \operatorname{PolyLog}[3, 1 - c x]}{c^2} \end{aligned}$$

**Problem 174: Result valid but suboptimal antiderivative.**

$$\int \frac{(g + h \operatorname{Log}[1 - c x]) \operatorname{PolyLog}[2, c x]}{x^2} dx$$

Optimal (type 4, 156 leaves, 12 steps):

$$\begin{aligned} & c h \operatorname{Log}[c x] \operatorname{Log}[1 - c x]^2 + \frac{\operatorname{Log}[1 - c x] (g + h \operatorname{Log}[1 - c x])}{x} + c (g + 2 h \operatorname{Log}[1 - c x]) \operatorname{Log}\left[1 - \frac{1}{1 - c x}\right] + \\ & c h \operatorname{Log}[1 - c x] \operatorname{PolyLog}[2, c x] - \frac{(g + h \operatorname{Log}[1 - c x]) \operatorname{PolyLog}[2, c x]}{x} - 2 c h \operatorname{PolyLog}\left[2, \frac{1}{1 - c x}\right] + \\ & 2 c h \operatorname{Log}[1 - c x] \operatorname{PolyLog}[2, 1 - c x] - c h \operatorname{PolyLog}[3, c x] - 2 c h \operatorname{PolyLog}[3, 1 - c x] \end{aligned}$$

Result (type 4, 165 leaves, 19 steps):

$$\begin{aligned} & c g \operatorname{Log}[x] - \frac{1}{2} c h \operatorname{Log}[1 - c x]^2 + c h \operatorname{Log}[c x] \operatorname{Log}[1 - c x]^2 + \frac{\operatorname{Log}[1 - c x] (g + h \operatorname{Log}[1 - c x])}{x} - \\ & \frac{c (g + h \operatorname{Log}[1 - c x])^2}{2 h} - 2 c h \operatorname{PolyLog}[2, c x] + c h \operatorname{Log}[1 - c x] \operatorname{PolyLog}[2, c x] - \frac{(g + h \operatorname{Log}[1 - c x]) \operatorname{PolyLog}[2, c x]}{x} + \\ & 2 c h \operatorname{Log}[1 - c x] \operatorname{PolyLog}[2, 1 - c x] - c h \operatorname{PolyLog}[3, c x] - 2 c h \operatorname{PolyLog}[3, 1 - c x] \end{aligned}$$

**Problem 175: Result valid but suboptimal antiderivative.**

$$\int \frac{(g + h \operatorname{Log}[1 - c x]) \operatorname{PolyLog}[2, c x]}{x^3} dx$$

Optimal (type 4, 266 leaves, 20 steps):

$$\begin{aligned}
& -c^2 h \operatorname{Log}[x] + \frac{1}{2} c^2 h \operatorname{Log}[1-cx] - \frac{c h \operatorname{Log}[1-cx]}{2x} + \frac{1}{2} c^2 h \operatorname{Log}[cx] \operatorname{Log}[1-cx]^2 + \\
& \frac{\operatorname{Log}[1-cx] (g+h \operatorname{Log}[1-cx])}{4x^2} - \frac{c(1-cx)(g+2h \operatorname{Log}[1-cx])}{4x} + \frac{1}{4} c^2 (g+2h \operatorname{Log}[1-cx]) \operatorname{Log}\left[1-\frac{1}{1-cx}\right] + \\
& \frac{c h \operatorname{PolyLog}[2, cx]}{2x} + \frac{1}{2} c^2 h \operatorname{Log}[1-cx] \operatorname{PolyLog}[2, cx] - \frac{(g+h \operatorname{Log}[1-cx]) \operatorname{PolyLog}[2, cx]}{2x^2} - \\
& \frac{1}{2} c^2 h \operatorname{PolyLog}\left[2, \frac{1}{1-cx}\right] + c^2 h \operatorname{Log}[1-cx] \operatorname{PolyLog}[2, 1-cx] - \frac{1}{2} c^2 h \operatorname{PolyLog}[3, cx] - c^2 h \operatorname{PolyLog}[3, 1-cx]
\end{aligned}$$

Result (type 4, 278 leaves, 31 steps):

$$\begin{aligned}
& \frac{1}{4} c^2 g \operatorname{Log}[x] - c^2 h \operatorname{Log}[x] + \frac{3}{4} c^2 h \operatorname{Log}[1-cx] - \frac{3 c h \operatorname{Log}[1-cx]}{4x} - \frac{1}{8} c^2 h \operatorname{Log}[1-cx]^2 + \\
& \frac{1}{2} c^2 h \operatorname{Log}[cx] \operatorname{Log}[1-cx]^2 - \frac{c(1-cx)(g+h \operatorname{Log}[1-cx])}{4x} + \frac{\operatorname{Log}[1-cx] (g+h \operatorname{Log}[1-cx])}{4x^2} - \\
& \frac{c^2 (g+h \operatorname{Log}[1-cx])^2}{8h} - \frac{1}{2} c^2 h \operatorname{PolyLog}[2, cx] + \frac{c h \operatorname{PolyLog}[2, cx]}{2x} + \frac{1}{2} c^2 h \operatorname{Log}[1-cx] \operatorname{PolyLog}[2, cx] - \\
& \frac{(g+h \operatorname{Log}[1-cx]) \operatorname{PolyLog}[2, cx]}{2x^2} + c^2 h \operatorname{Log}[1-cx] \operatorname{PolyLog}[2, 1-cx] - \frac{1}{2} c^2 h \operatorname{PolyLog}[3, cx] - c^2 h \operatorname{PolyLog}[3, 1-cx]
\end{aligned}$$

**Problem 176: Result valid but suboptimal antiderivative.**

$$\int \frac{(g+h \operatorname{Log}[1-cx]) \operatorname{PolyLog}[2, cx]}{x^4} dx$$

Optimal (type 4, 340 leaves, 28 steps):

$$\begin{aligned}
& \frac{7 c^2 h}{36x} - \frac{3}{4} c^3 h \operatorname{Log}[x] + \frac{19}{36} c^3 h \operatorname{Log}[1-cx] - \frac{c h \operatorname{Log}[1-cx]}{12x^2} - \frac{c^2 h \operatorname{Log}[1-cx]}{3x} + \frac{1}{3} c^3 h \operatorname{Log}[cx] \operatorname{Log}[1-cx]^2 + \\
& \frac{\operatorname{Log}[1-cx] (g+h \operatorname{Log}[1-cx])}{9x^3} - \frac{c(g+2h \operatorname{Log}[1-cx])}{18x^2} - \frac{c^2(1-cx)(g+2h \operatorname{Log}[1-cx])}{9x} + \frac{1}{9} c^3 (g+2h \operatorname{Log}[1-cx]) \operatorname{Log}\left[1-\frac{1}{1-cx}\right] + \\
& \frac{c h \operatorname{PolyLog}[2, cx]}{6x^2} + \frac{c^2 h \operatorname{PolyLog}[2, cx]}{3x} + \frac{1}{3} c^3 h \operatorname{Log}[1-cx] \operatorname{PolyLog}[2, cx] - \frac{(g+h \operatorname{Log}[1-cx]) \operatorname{PolyLog}[2, cx]}{3x^3} - \\
& \frac{2}{9} c^3 h \operatorname{PolyLog}\left[2, \frac{1}{1-cx}\right] + \frac{2}{3} c^3 h \operatorname{Log}[1-cx] \operatorname{PolyLog}[2, 1-cx] - \frac{1}{3} c^3 h \operatorname{PolyLog}[3, cx] - \frac{2}{3} c^3 h \operatorname{PolyLog}[3, 1-cx]
\end{aligned}$$

Result (type 4, 351 leaves, 42 steps):

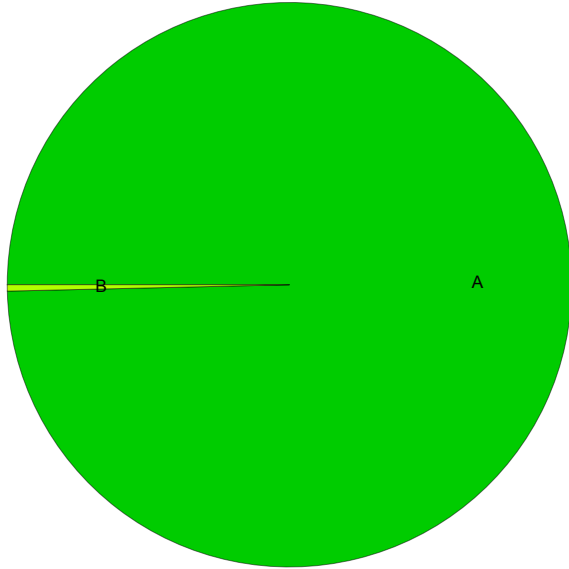
$$\begin{aligned}
& \frac{7c^2h}{36x} + \frac{1}{9}c^3g \operatorname{Log}[x] - \frac{3}{4}c^3h \operatorname{Log}[x] + \frac{23}{36}c^3h \operatorname{Log}[1-cx] - \frac{5ch \operatorname{Log}[1-cx]}{36x^2} - \frac{4c^2h \operatorname{Log}[1-cx]}{9x} - \frac{1}{18}c^3h \operatorname{Log}[1-cx]^2 + \\
& \frac{1}{3}c^3h \operatorname{Log}[cx] \operatorname{Log}[1-cx]^2 - \frac{c(g+h \operatorname{Log}[1-cx])}{18x^2} - \frac{c^2(1-cx)(g+h \operatorname{Log}[1-cx])}{9x} + \frac{\operatorname{Log}[1-cx](g+h \operatorname{Log}[1-cx])}{9x^3} - \\
& \frac{c^3(g+h \operatorname{Log}[1-cx])^2}{18h} - \frac{2}{9}c^3h \operatorname{PolyLog}[2, cx] + \frac{ch \operatorname{PolyLog}[2, cx]}{6x^2} + \frac{c^2h \operatorname{PolyLog}[2, cx]}{3x} + \frac{1}{3}c^3h \operatorname{Log}[1-cx] \operatorname{PolyLog}[2, cx] - \\
& \frac{(g+h \operatorname{Log}[1-cx]) \operatorname{PolyLog}[2, cx]}{3x^3} + \frac{2}{3}c^3h \operatorname{Log}[1-cx] \operatorname{PolyLog}[2, 1-cx] - \frac{1}{3}c^3h \operatorname{PolyLog}[3, cx] - \frac{2}{3}c^3h \operatorname{PolyLog}[3, 1-cx]
\end{aligned}$$

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Test results for the 398 problems in "8.9 Product logarithm function.m"

## Summary of Integration Test Results

1949 integration problems



A - 1942 optimal antiderivatives

B - 7 valid but suboptimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives