

## Rubi 4.16.1.4 Integration Test Results

on the problems in the test-suite directory "3 Logarithms"

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Test results for the 193 problems in "3.1.2 (d x)^m (a+b log(c x^n))^p.m"

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Test results for the 456 problems in "3.1.4 (f x)^m (d+e x^r)^q (a+b log(c x^n))^p.m"

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Test results for the 249 problems in "3.1.5 u (a+b log(c x^n))^p.m"

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Test results for the 314 problems in "3.2.1 (f+g x)^m (A+B log(e ((a+b x) over (c+d x))^n))^p.m"

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Test results for the 263 problems in "3.2.2 (f+g x)^m (h+i x)^q (A+B log(e ((a+b x) over (c+d x))^n))^p.m"

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Test results for the 108 problems in "3.2.3 u log(e (f (a+b x)^p (c+d x)^q)^r)^s.m"

Problem 39: Result valid but suboptimal antiderivative.

$$\int \frac{\text{Log}[e (f (a + b x)^p (c + d x)^q)^r]^2}{g + h x} dx$$

Optimal (type 4, 1471 leaves, ? steps):

$$\begin{aligned}
& \frac{p q r^2 \operatorname{Log}\left[-\frac{bc-ad}{d(a+bx)}\right] \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right]^2}{h} + \frac{p^2 r^2 \operatorname{Log}[a+bx]^2 \operatorname{Log}[g+hx]}{h} + \frac{2 p q r^2 \operatorname{Log}[a+bx] \operatorname{Log}[c+dx] \operatorname{Log}[g+hx]}{h} + \\
& \frac{q^2 r^2 \operatorname{Log}[c+dx]^2 \operatorname{Log}[g+hx]}{h} - \frac{2 p r \operatorname{Log}[a+bx] \operatorname{Log}\left[e\left(f(a+bx)^p (c+dx)^q\right)^r\right] \operatorname{Log}[g+hx]}{h} - \\
& \frac{2 q r \operatorname{Log}[c+dx] \operatorname{Log}\left[e\left(f(a+bx)^p (c+dx)^q\right)^r\right] \operatorname{Log}[g+hx]}{h} + \frac{\operatorname{Log}\left[e\left(f(a+bx)^p (c+dx)^q\right)^r\right]^2 \operatorname{Log}[g+hx]}{h} - \\
& \frac{p^2 r^2 \operatorname{Log}[a+bx]^2 \operatorname{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h} - \frac{2 p q r^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right] \operatorname{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h} + \frac{p q r^2 \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right]^2 \operatorname{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h} - \\
& \frac{2 p q r^2 \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right] \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right] \operatorname{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h} + \frac{p q r^2 \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right]^2 \operatorname{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h} + \\
& \frac{2 p r \operatorname{Log}[a+bx] \operatorname{Log}\left[e\left(f(a+bx)^p (c+dx)^q\right)^r\right] \operatorname{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h} - \frac{2 p q r^2 \operatorname{Log}[a+bx] \operatorname{Log}[c+dx] \operatorname{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h} - \\
& \frac{q^2 r^2 \operatorname{Log}[c+dx]^2 \operatorname{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h} + \frac{2 p q r^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right] \operatorname{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h} - \frac{p q r^2 \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right]^2 \operatorname{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h} + \\
& \frac{2 p q r^2 \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right] \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right] \operatorname{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h} + \frac{2 q r \operatorname{Log}[c+dx] \operatorname{Log}\left[e\left(f(a+bx)^p (c+dx)^q\right)^r\right] \operatorname{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h} - \\
& \frac{p q r^2 \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right]^2 \operatorname{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right]}{h} - \frac{2 p r \left(q r \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right] - \operatorname{Log}\left[e\left(f(a+bx)^p (c+dx)^q\right)^r\right]\right) \operatorname{PolyLog}\left[2, -\frac{h(a+bx)}{bg-ah}\right]}{h} + \\
& \frac{2 q r \left(p r \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right] + \operatorname{Log}\left[e\left(f(a+bx)^p (c+dx)^q\right)^r\right]\right) \operatorname{PolyLog}\left[2, -\frac{h(c+dx)}{dg-ch}\right]}{h} + \frac{2 p q r^2 \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right] \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{d(a+bx)}\right]}{h} - \\
& \frac{2 p q r^2 \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right] \operatorname{PolyLog}\left[2, \frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right]}{h} - \frac{2 p^2 r^2 \operatorname{PolyLog}\left[3, -\frac{h(a+bx)}{bg-ah}\right]}{h} - \frac{2 p q r^2 \operatorname{PolyLog}\left[3, -\frac{h(a+bx)}{bg-ah}\right]}{h} - \\
& \frac{2 p q r^2 \operatorname{PolyLog}\left[3, -\frac{h(c+dx)}{dg-ch}\right]}{h} - \frac{2 q^2 r^2 \operatorname{PolyLog}\left[3, -\frac{h(c+dx)}{dg-ch}\right]}{h} - \frac{2 p q r^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{d(a+bx)}\right]}{h} + \frac{2 p q r^2 \operatorname{PolyLog}\left[3, \frac{(bg-ah)(c+dx)}{(dg-ch)(a+bx)}\right]}{h}
\end{aligned}$$

Result (type 4, 2096 leaves, 29 steps):

$$\begin{aligned}
& \frac{\operatorname{Log}\left[(a+bx)^{pr}\right]^2 \operatorname{Log}[g+hx]}{h} - \frac{2 p q r^2 \operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] \operatorname{Log}[c+dx] \operatorname{Log}[g+hx]}{h} - \frac{2 p q r^2 \operatorname{Log}[a+bx] \operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] \operatorname{Log}[g+hx]}{h} + \\
& \frac{2 q r \left(p r \operatorname{Log}[a+bx] - \operatorname{Log}\left[(a+bx)^{pr}\right]\right) \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right] \operatorname{Log}[g+hx]}{h} + \frac{2 p r \operatorname{Log}\left[-\frac{h(a+bx)}{bg-ah}\right] \left(q r \operatorname{Log}[c+dx] - \operatorname{Log}\left[(c+dx)^{qr}\right]\right) \operatorname{Log}[g+hx]}{h} - \\
& \frac{\operatorname{Log}\left[(c+dx)^{qr}\right]^2 \operatorname{Log}[g+hx]}{h} + \frac{1}{h} 2 p r \operatorname{Log}\left[-\frac{h(a+bx)}{bg-ah}\right] \left(\operatorname{Log}\left[(a+bx)^{pr}\right] + \operatorname{Log}\left[(c+dx)^{qr}\right] - \operatorname{Log}\left[e\left(f(a+bx)^p (c+dx)^q\right)^r\right]\right) \operatorname{Log}[g+hx] +
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{h} 2 q r \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right] \left(\operatorname{Log}\left[(a+bx)^{pr}\right] + \operatorname{Log}\left[(c+dx)^{qr}\right] - \operatorname{Log}\left[e(f(a+bx)^p(c+dx)^q)^r\right]\right) \operatorname{Log}[g+hx] + \\
& \frac{\operatorname{Log}\left[e(f(a+bx)^p(c+dx)^q)^r\right]^2 \operatorname{Log}[g+hx]}{h} + \frac{\operatorname{Log}\left[(a+bx)^{pr}\right]^2 \operatorname{Log}\left[\frac{b(g+hx)}{bg-ah}\right]}{h} + \frac{\operatorname{Log}\left[(c+dx)^{qr}\right]^2 \operatorname{Log}\left[\frac{d(g+hx)}{dg-ch}\right]}{h} - \\
& \frac{pq r^2 \left(\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] + \operatorname{Log}\left[\frac{bg-ah}{b(g+hx)}\right] - \operatorname{Log}\left[\frac{(bg-ah)(c+dx)}{(bc-ad)(g+hx)}\right]\right) \operatorname{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right]^2}{h} + \\
& \frac{pq r^2 \left(\operatorname{Log}\left[\frac{b(c+dx)}{bc-ad}\right] - \operatorname{Log}\left[-\frac{h(c+dx)}{dg-ch}\right]\right) \left(\operatorname{Log}[a+bx] + \operatorname{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right]\right)^2}{h} - \\
& \frac{pq r^2 \left(\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] + \operatorname{Log}\left[\frac{dg-ch}{d(g+hx)}\right] - \operatorname{Log}\left[-\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right]\right) \operatorname{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right]^2}{h} + \\
& \frac{pq r^2 \left(\operatorname{Log}\left[-\frac{d(a+bx)}{bc-ad}\right] - \operatorname{Log}\left[-\frac{h(a+bx)}{bg-ah}\right]\right) \left(\operatorname{Log}[c+dx] + \operatorname{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right]\right)^2}{h} - \frac{2pq r^2 \left(\operatorname{Log}[g+hx] - \operatorname{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right]\right) \operatorname{PolyLog}\left[2, -\frac{d(a+bx)}{bc-ad}\right]}{h} + \\
& \frac{2pr \operatorname{Log}\left[(a+bx)^{pr}\right] \operatorname{PolyLog}\left[2, -\frac{h(a+bx)}{bg-ah}\right]}{h} - \frac{2pq r^2 \left(\operatorname{Log}[g+hx] - \operatorname{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(c+dx)}{bc-ad}\right]}{h} + \\
& \frac{2qr \operatorname{Log}\left[(c+dx)^{qr}\right] \operatorname{PolyLog}\left[2, -\frac{h(c+dx)}{dg-ch}\right]}{h} + \frac{2pq r^2 \operatorname{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right] \operatorname{PolyLog}\left[2, \frac{h(a+bx)}{b(g+hx)}\right]}{h} - \\
& \frac{2pq r^2 \operatorname{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right] \operatorname{PolyLog}\left[2, -\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right]}{h} + \frac{2pq r^2 \operatorname{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right] \operatorname{PolyLog}\left[2, \frac{h(c+dx)}{d(g+hx)}\right]}{h} - \\
& \frac{2pq r^2 \operatorname{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right] \operatorname{PolyLog}\left[2, \frac{(bg-ah)(c+dx)}{(bc-ad)(g+hx)}\right]}{h} + \frac{2pr \left(qr \operatorname{Log}[c+dx] - \operatorname{Log}\left[(c+dx)^{qr}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right]}{h} + \\
& \frac{2pr \left(\operatorname{Log}\left[(a+bx)^{pr}\right] + \operatorname{Log}\left[(c+dx)^{qr}\right] - \operatorname{Log}\left[e(f(a+bx)^p(c+dx)^q)^r\right]\right) \operatorname{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right]}{h} - \\
& \frac{2pq r^2 \left(\operatorname{Log}[c+dx] + \operatorname{Log}\left[\frac{(bc-ad)(g+hx)}{(bg-ah)(c+dx)}\right]\right) \operatorname{PolyLog}\left[2, \frac{b(g+hx)}{bg-ah}\right]}{h} + \frac{2qr \left(pr \operatorname{Log}[a+bx] - \operatorname{Log}\left[(a+bx)^{pr}\right]\right) \operatorname{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right]}{h} + \\
& \frac{2qr \left(\operatorname{Log}\left[(a+bx)^{pr}\right] + \operatorname{Log}\left[(c+dx)^{qr}\right] - \operatorname{Log}\left[e(f(a+bx)^p(c+dx)^q)^r\right]\right) \operatorname{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right]}{h} - \\
& \frac{2pq r^2 \left(\operatorname{Log}[a+bx] + \operatorname{Log}\left[-\frac{(bc-ad)(g+hx)}{(dg-ch)(a+bx)}\right]\right) \operatorname{PolyLog}\left[2, \frac{d(g+hx)}{dg-ch}\right]}{h} + \frac{2pq r^2 \operatorname{PolyLog}\left[3, -\frac{d(a+bx)}{bc-ad}\right]}{h} - \frac{2p^2 r^2 \operatorname{PolyLog}\left[3, -\frac{h(a+bx)}{bg-ah}\right]}{h} + \\
& \frac{2pq r^2 \operatorname{PolyLog}\left[3, \frac{b(c+dx)}{bc-ad}\right]}{h} - \frac{2q^2 r^2 \operatorname{PolyLog}\left[3, -\frac{h(c+dx)}{dg-ch}\right]}{h} + \frac{2pq r^2 \operatorname{PolyLog}\left[3, \frac{h(a+bx)}{b(g+hx)}\right]}{h} - \frac{2pq r^2 \operatorname{PolyLog}\left[3, -\frac{(dg-ch)(a+bx)}{(bc-ad)(g+hx)}\right]}{h} +
\end{aligned}$$

$$\frac{2 p q r^2 \text{PolyLog}\left[3, \frac{h(c+d x)}{d(g+h x)}\right]}{h} - \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{(b g-a h)(c+d x)}{(b c-a d)(g+h x)}\right]}{h} + \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{b(g+h x)}{b g-a h}\right]}{h} + \frac{2 p q r^2 \text{PolyLog}\left[3, \frac{d(g+h x)}{d g-c h}\right]}{h}$$

**Problem 74: Unable to integrate problem.**

$$\int \left( \frac{1}{(c+d x)(-a+c+(-b+d)x) \text{Log}\left[\frac{a+b x}{c+d x}\right]} + \frac{\text{Log}\left[1-\frac{a+b x}{c+d x}\right]}{(a+b x)(c+d x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2} \right) dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-\frac{\text{Log}\left[1-\frac{a+b x}{c+d x}\right]}{(b c-a d) \text{Log}\left[\frac{a+b x}{c+d x}\right]}$$

Result (type 8, 152 leaves, 3 steps):

$$\frac{b \text{ CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{a+b x}{c+d x}\right]}{(a+b x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2}, x\right]}{b c-a d} - \frac{d \text{ CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{a+b x}{c+d x}\right]}{(c+d x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2}, x\right]}{b c-a d} + \text{Unintegrateable}\left[\frac{1}{(c+d x)(-a+c+(-b+d)x) \text{Log}\left[\frac{a+b x}{c+d x}\right]}, x\right]$$

**Problem 75: Unable to integrate problem.**

$$\int \left( -\frac{1}{(a+b x)(a-c+(b-d)x) \text{Log}\left[\frac{a+b x}{c+d x}\right]} + \frac{\text{Log}\left[1-\frac{c+d x}{a+b x}\right]}{(a+b x)(c+d x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2} \right) dx$$

Optimal (type 3, 45 leaves, ? steps):

$$-\frac{\text{Log}\left[1-\frac{c+d x}{a+b x}\right]}{(b c-a d) \text{Log}\left[\frac{a+b x}{c+d x}\right]}$$

Result (type 8, 154 leaves, 3 steps):

$$\frac{b \text{ CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{c+d x}{a+b x}\right]}{(a+b x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2}, x\right]}{b c-a d} - \frac{d \text{ CannotIntegrate}\left[\frac{\text{Log}\left[1-\frac{c+d x}{a+b x}\right]}{(c+d x) \text{Log}\left[\frac{a+b x}{c+d x}\right]^2}, x\right]}{b c-a d} - \text{Unintegrateable}\left[\frac{1}{(a+b x)(a-c+(b-d)x) \text{Log}\left[\frac{a+b x}{c+d x}\right]}, x\right]$$

## Test results for the 547 problems in "3.3 u (a+b log(c (d+e x)^n))^p.m"

Problem 370: Unable to integrate problem.

$$\int \frac{\text{Log}[f x^m] (a + b \text{Log}[c (d + e x)^n])^2}{x} dx$$

Optimal (type 4, 823 leaves, ? steps):

$$\begin{aligned} & \frac{1}{2} m \text{Log}[x]^2 (a - b n \text{Log}[d + e x] + b \text{Log}[c (d + e x)^n])^2 + \text{Log}[x] (-m \text{Log}[x] + \text{Log}[f x^m]) (a - b n \text{Log}[d + e x] + b \text{Log}[c (d + e x)^n])^2 + \\ & 2 b n (-m \text{Log}[x] + \text{Log}[f x^m]) (a - b n \text{Log}[d + e x] + b \text{Log}[c (d + e x)^n]) \left( \text{Log}[x] \left( \text{Log}[d + e x] - \text{Log}\left[1 + \frac{e x}{d}\right] \right) - \text{PolyLog}\left[2, -\frac{e x}{d}\right] \right) + \\ & 2 b m n (a - b n \text{Log}[d + e x] + b \text{Log}[c (d + e x)^n]) \left( \frac{1}{2} \text{Log}[x]^2 \left( \text{Log}[d + e x] - \text{Log}\left[1 + \frac{e x}{d}\right] \right) - \text{Log}[x] \text{PolyLog}\left[2, -\frac{e x}{d}\right] + \text{PolyLog}\left[3, -\frac{e x}{d}\right] \right) - \\ & b^2 n^2 (m \text{Log}[x] - \text{Log}[f x^m]) \left( \text{Log}\left[-\frac{e x}{d}\right] \text{Log}[d + e x]^2 + 2 \text{Log}[d + e x] \text{PolyLog}\left[2, 1 + \frac{e x}{d}\right] - 2 \text{PolyLog}\left[3, 1 + \frac{e x}{d}\right] \right) + \\ & \frac{1}{12} b^2 m n^2 \left( \text{Log}\left[-\frac{e x}{d}\right]^4 + 6 \text{Log}\left[-\frac{e x}{d}\right]^2 \text{Log}\left[-\frac{e x}{d + e x}\right]^2 - 4 \left( \text{Log}\left[-\frac{e x}{d}\right] + \text{Log}\left[\frac{d}{d + e x}\right] \right) \text{Log}\left[-\frac{e x}{d + e x}\right]^3 + \right. \\ & \quad \left. \text{Log}\left[-\frac{e x}{d + e x}\right]^4 + 6 \text{Log}[x]^2 \text{Log}[d + e x]^2 + 4 \left( 2 \text{Log}\left[-\frac{e x}{d}\right]^3 - 3 \text{Log}[x]^2 \text{Log}[d + e x] \right) \text{Log}\left[1 + \frac{e x}{d}\right] + \right. \\ & \quad \left. 6 \left( \text{Log}[x] - \text{Log}\left[-\frac{e x}{d}\right] \right) \left( \text{Log}[x] + 3 \text{Log}\left[-\frac{e x}{d}\right] \right) \text{Log}\left[1 + \frac{e x}{d}\right]^2 - 4 \text{Log}\left[-\frac{e x}{d}\right]^2 \text{Log}\left[-\frac{e x}{d + e x}\right] \left( \text{Log}\left[-\frac{e x}{d}\right] + 3 \text{Log}\left[1 + \frac{e x}{d}\right] \right) + \right. \\ & \quad \left. 12 \left( \text{Log}\left[-\frac{e x}{d}\right]^2 - 2 \text{Log}\left[-\frac{e x}{d}\right] \left( \text{Log}\left[-\frac{e x}{d + e x}\right] + \text{Log}\left[1 + \frac{e x}{d}\right] \right) + 2 \text{Log}[x] \left( -\text{Log}[d + e x] + \text{Log}\left[1 + \frac{e x}{d}\right] \right) \right) \text{PolyLog}\left[2, -\frac{e x}{d}\right] - \right. \\ & \quad \left. 12 \text{Log}\left[-\frac{e x}{d + e x}\right]^2 \text{PolyLog}\left[2, \frac{e x}{d + e x}\right] + 12 \left( \text{Log}\left[-\frac{e x}{d}\right] - \text{Log}\left[-\frac{e x}{d + e x}\right] \right)^2 \text{PolyLog}\left[2, 1 + \frac{e x}{d}\right] + 24 \left( \text{Log}[x] - \text{Log}\left[-\frac{e x}{d}\right] \right) \right. \\ & \quad \left. \text{Log}\left[1 + \frac{e x}{d}\right] \text{PolyLog}\left[2, 1 + \frac{e x}{d}\right] + 24 \left( \text{Log}\left[-\frac{e x}{d + e x}\right] + \text{Log}[d + e x] \right) \text{PolyLog}\left[3, -\frac{e x}{d}\right] + 24 \text{Log}\left[-\frac{e x}{d + e x}\right] \text{PolyLog}\left[3, \frac{e x}{d + e x}\right] + \right. \\ & \quad \left. 24 \left( -\text{Log}[x] + \text{Log}\left[-\frac{e x}{d + e x}\right] \right) \text{PolyLog}\left[3, 1 + \frac{e x}{d}\right] - 24 \left( \text{PolyLog}\left[4, -\frac{e x}{d}\right] + \text{PolyLog}\left[4, \frac{e x}{d + e x}\right] - \text{PolyLog}\left[4, 1 + \frac{e x}{d}\right] \right) \right) \end{aligned}$$

Result (type 8, 72 leaves, 1 step):

$$\frac{\text{Log}[f x^m]^2 (a + b \text{Log}[c (d + e x)^n])^2}{2 m} - \frac{b e n \text{Unintegrable}\left[\frac{\text{Log}[f x^m]^2 (a + b \text{Log}[c (d + e x)^n])}{d + e x}, x\right]}{m}$$

Problem 371: Unable to integrate problem.

$$\int \frac{\text{Log}[f x^m] (a + b \text{Log}[c (d + e x)^n])^2}{x^2} dx$$

Optimal (type 4, 607 leaves, ? steps):

$$\begin{aligned}
& - \frac{b^2 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[d + e x]}{d} + \frac{2 b^2 e m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]}{d} + \frac{2 b^2 e n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{d} - \frac{b^2 e m n^2 \operatorname{Log}[d + e x]^2}{d} \\
& - \frac{b^2 m n^2 \operatorname{Log}[d + e x]^2}{x} + \frac{b^2 e m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]^2}{d} - \frac{b^2 e n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{d} - \frac{b^2 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{x} \\
& - \frac{1}{d x} 2 b n (m \operatorname{Log}[x] - \operatorname{Log}[f x^m]) \left( e x \operatorname{Log}\left[-\frac{e x}{d}\right] - (d + e x) \operatorname{Log}[d + e x] \right) (a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}[c (d + e x)^n]) - \\
& \frac{m \operatorname{Log}[x] (a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}[c (d + e x)^n])^2}{x} - \frac{(m - m \operatorname{Log}[x] + \operatorname{Log}[f x^m]) (a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}[c (d + e x)^n])^2}{x} + \\
& \frac{b^2 e m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d} - \frac{2 b^2 e n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d} - \\
& \frac{2 b^2 e n^2 \operatorname{Log}[f x^m] \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d} + \frac{1}{d x} b m n (a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}[c (d + e x)^n]) \\
& \left( 2 e x \operatorname{Log}\left[-\frac{e x}{d}\right] - 2 (d + e x) \operatorname{Log}[d + e x] - 2 d \operatorname{Log}[x] \operatorname{Log}[d + e x] + e x \left( \operatorname{Log}[x]^2 - 2 \left( \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right) \right) \right) + \\
& \frac{2 b^2 e m n^2 (1 + \operatorname{Log}[d + e x]) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{d} + \frac{2 b^2 e m n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d} - \frac{2 b^2 e m n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{d}
\end{aligned}$$

Result (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])^2}{x^2}, x\right]$$

**Problem 372: Unable to integrate problem.**

$$\int \frac{\operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])^2}{x^3} dx$$

Optimal (type 4, 939 leaves, ? steps):

$$\begin{aligned}
& \frac{b^2 e^2 m n^2 \operatorname{Log}[x]}{d^2} - \frac{b^2 e^2 m n^2 \operatorname{Log}[x]^2}{2 d^2} + \frac{b^2 e^2 m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right]}{2 d^2} + \frac{b^2 e^2 n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m]}{d^2} - \frac{3 b^2 e^2 m n^2 \operatorname{Log}[d + e x]}{2 d^2} - \\
& \frac{3 b^2 e m n^2 \operatorname{Log}[d + e x]}{2 d x} + \frac{b^2 e^2 m n^2 \operatorname{Log}[x] \operatorname{Log}[d + e x]}{d^2} + \frac{b^2 e^2 m n^2 \operatorname{Log}[x]^2 \operatorname{Log}[d + e x]}{2 d^2} - \frac{b^2 e^2 m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]}{2 d^2} - \\
& \frac{b^2 e^2 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{d^2} - \frac{b^2 e n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{d x} - \frac{b^2 e^2 n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]}{d^2} + \frac{b^2 e^2 m n^2 \operatorname{Log}[d + e x]^2}{4 d^2} - \\
& \frac{b^2 m n^2 \operatorname{Log}[d + e x]^2}{4 x^2} - \frac{b^2 e^2 m n^2 \operatorname{Log}\left[-\frac{e x}{d}\right] \operatorname{Log}[d + e x]^2}{2 d^2} + \frac{b^2 e^2 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{2 d^2} - \frac{b^2 n^2 \operatorname{Log}[f x^m] \operatorname{Log}[d + e x]^2}{2 x^2} + \frac{1}{d^2 x^2} \\
& b n (m \operatorname{Log}[x] - \operatorname{Log}[f x^m]) \left( e^2 x^2 \operatorname{Log}\left[-\frac{e x}{d}\right] + (d + e x) (e x + (d - e x) \operatorname{Log}[d + e x]) \right) (a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}[c (d + e x)^n]) - \\
& \frac{m \operatorname{Log}[x] (a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}[c (d + e x)^n])^2}{2 x^2} - \frac{(m - 2 m \operatorname{Log}[x] + 2 \operatorname{Log}[f x^m]) (a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}[c (d + e x)^n])^2}{4 x^2} - \\
& \frac{b^2 e^2 m n^2 \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} - \frac{b^2 e^2 m n^2 \operatorname{Log}[x]^2 \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{2 d^2} + \frac{b^2 e^2 n^2 \operatorname{Log}[x] \operatorname{Log}[f x^m] \operatorname{Log}\left[1 + \frac{e x}{d}\right]}{d^2} - \frac{b^2 e^2 n^2 (m - \operatorname{Log}[f x^m]) \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right]}{d^2} - \\
& \frac{1}{2 d^2 x^2} b m n (a - b n \operatorname{Log}[d + e x] + b \operatorname{Log}[c (d + e x)^n]) \left( e x (d + e x) + e^2 x^2 \operatorname{Log}\left[-\frac{e x}{d}\right] + (d^2 - e^2 x^2) \operatorname{Log}[d + e x] + \right. \\
& \left. 2 d^2 \operatorname{Log}[x] \operatorname{Log}[d + e x] + e x \left( e x \operatorname{Log}[x]^2 + 2 d (1 + \operatorname{Log}[x]) - 2 e x \left( \operatorname{Log}[x] \operatorname{Log}\left[1 + \frac{e x}{d}\right] + \operatorname{PolyLog}\left[2, -\frac{e x}{d}\right] \right) \right) \right) - \\
& \frac{b^2 e^2 m n^2 (1 + 2 \operatorname{Log}[d + e x]) \operatorname{PolyLog}\left[2, 1 + \frac{e x}{d}\right]}{2 d^2} - \frac{b^2 e^2 m n^2 \operatorname{PolyLog}\left[3, -\frac{e x}{d}\right]}{d^2} + \frac{b^2 e^2 m n^2 \operatorname{PolyLog}\left[3, 1 + \frac{e x}{d}\right]}{d^2}
\end{aligned}$$

Result (type 8, 28 leaves, 0 steps):

$$\text{Unintegrable}\left[\frac{\operatorname{Log}[f x^m] (a + b \operatorname{Log}[c (d + e x)^n])^2}{x^3}, x\right]$$

**Problem 374: Unable to integrate problem.**

$$\int \frac{\operatorname{Log}[x] \operatorname{Log}[a + b x]^2}{x} dx$$

Optimal (type 4, 519 leaves, ? steps):

$$\begin{aligned}
& \frac{1}{12} \left( \text{Log}\left[-\frac{bx}{a}\right]^4 + 6 \text{Log}\left[-\frac{bx}{a}\right]^2 \text{Log}\left[-\frac{bx}{a+bx}\right]^2 - 4 \left( \text{Log}\left[-\frac{bx}{a}\right] + \text{Log}\left[\frac{a}{a+bx}\right] \right) \text{Log}\left[-\frac{bx}{a+bx}\right]^3 + \right. \\
& \left. \text{Log}\left[-\frac{bx}{a+bx}\right]^4 + 6 \text{Log}[x]^2 \text{Log}[a+bx]^2 + 4 \left( 2 \text{Log}\left[-\frac{bx}{a}\right]^3 - 3 \text{Log}[x]^2 \text{Log}[a+bx] \right) \text{Log}\left[1 + \frac{bx}{a}\right] + \right. \\
& \left. 6 \left( \text{Log}[x] - \text{Log}\left[-\frac{bx}{a}\right] \right) \left( \text{Log}[x] + 3 \text{Log}\left[-\frac{bx}{a}\right] \right) \text{Log}\left[1 + \frac{bx}{a}\right]^2 - 4 \text{Log}\left[-\frac{bx}{a}\right]^2 \text{Log}\left[-\frac{bx}{a+bx}\right] \left( \text{Log}\left[-\frac{bx}{a}\right] + 3 \text{Log}\left[1 + \frac{bx}{a}\right] \right) + \right. \\
& \left. 12 \left( \text{Log}\left[-\frac{bx}{a}\right]^2 - 2 \text{Log}\left[-\frac{bx}{a}\right] \left( \text{Log}\left[-\frac{bx}{a+bx}\right] + \text{Log}\left[1 + \frac{bx}{a}\right] \right) + 2 \text{Log}[x] \left( -\text{Log}[a+bx] + \text{Log}\left[1 + \frac{bx}{a}\right] \right) \right) \text{PolyLog}\left[2, -\frac{bx}{a}\right] - \right. \\
& \left. 12 \text{Log}\left[-\frac{bx}{a+bx}\right]^2 \text{PolyLog}\left[2, \frac{bx}{a+bx}\right] + 12 \left( \text{Log}\left[-\frac{bx}{a}\right] - \text{Log}\left[-\frac{bx}{a+bx}\right] \right)^2 \text{PolyLog}\left[2, 1 + \frac{bx}{a}\right] + \right. \\
& \left. 24 \left( \text{Log}[x] - \text{Log}\left[-\frac{bx}{a}\right] \right) \text{Log}\left[1 + \frac{bx}{a}\right] \text{PolyLog}\left[2, 1 + \frac{bx}{a}\right] + 24 \left( \text{Log}\left[-\frac{bx}{a+bx}\right] + \text{Log}[a+bx] \right) \text{PolyLog}\left[3, -\frac{bx}{a}\right] + \right. \\
& \left. 24 \text{Log}\left[-\frac{bx}{a+bx}\right] \text{PolyLog}\left[3, \frac{bx}{a+bx}\right] + 24 \left( -\text{Log}[x] + \text{Log}\left[-\frac{bx}{a+bx}\right] \right) \text{PolyLog}\left[3, 1 + \frac{bx}{a}\right] - \right. \\
& \left. 24 \left( \text{PolyLog}\left[4, -\frac{bx}{a}\right] + \text{PolyLog}\left[4, \frac{bx}{a+bx}\right] - \text{PolyLog}\left[4, 1 + \frac{bx}{a}\right] \right) \right)
\end{aligned}$$

Result (type 8, 40 leaves, 1 step):

$$\frac{1}{2} \text{Log}[x]^2 \text{Log}[a+bx]^2 - b \text{Unintegrable}\left[\frac{\text{Log}[x]^2 \text{Log}[a+bx]}{a+bx}, x\right]$$

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Test results for the 641 problems in "3.4 u (a+b log(c (d+e x^m)^n))^p.m"

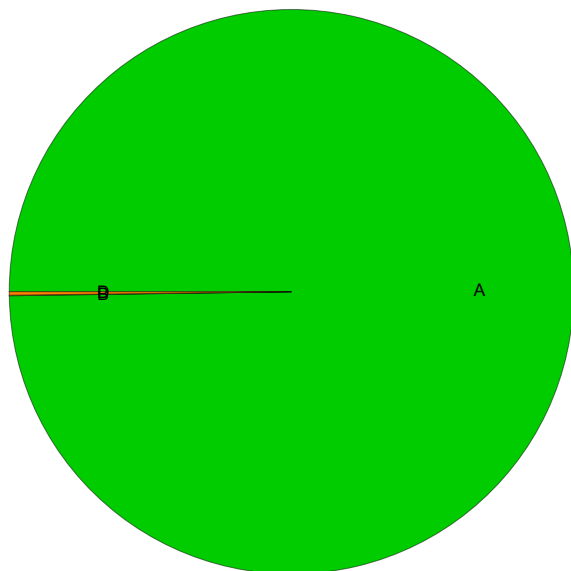
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Test results for the 314 problems in "3.5 Logarithm functions.m"



## Summary of Integration Test Results

3085 integration problems



A - 3078 optimal antiderivatives

B - 1 valid but suboptimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 6 unable to integrate problems

E - 0 integration timeouts

F - 0 invalid antiderivatives